ANALYSIS OF THE INSTANTANEOUS FRICTION COEFFICIENT OF THE TROCHOIDAL GEAR PAIR

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ABSTRACT

Wear of contact surfaces is one of the most common causes of gear failure. In addition to physical and chemical processes that occur in the contact zone, kinematic meshing parameters have a great effect on the wear process. This paper analyses the influence of geometric and kinematic parameters of the gear profile on the instantaneous friction coefficient of the trochoidal gearing in the gerotor pump. The friction coefficient is not a constant value and it varies during the teeth meshing as a result of a constant change in the lubrication regime. There are many empirical expressions for calculation of the instantaneous friction coefficient, but due to numerous limitations, many of these expressions can be applied only to certain operating conditions and loads. The analysis of the friction coefficient was performed through two contact periods defined by the change in direction of the rolling velocity of the meshing profiles contact point. The obtained results enable identification of dominant factors that influence the instantaneous friction coefficient and show possibilities for further development of analytical models of mechanical losses in the gerotor pump.

Keywords: gerotor, trochoid, sliding velocity, contact stress, friction coefficient, wear.

AIMS AND BACKGROUND

There is a wide range of pumps used in modern industrial practice. One of the simplest is a gerotor pump. Gerotor (trochoidal) pumps are rotary pumps with in-

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ternal gearing characterised by compactness, simple construction and possibility of wide application in various areas. The gerotor pump consists of an internal and an external gear. The external gear has one more tooth than the internal one. Both these gears rotate in the same direction with all the teeth involved in meshing at the same time, which is a significant advantage because it decreases the contact load. During the gear rotation, the suction zone is in the area where the teeth leave the intertooth space, and the discharge zone is in the area where they enter the intertooth space forcing the fluid through the discharge.

Gear pair parameters have a significant influence on the functional characteristics of the pump. Resistance to wear has an important role in the performance and working life of the pump. Hydrostatic gerotor pumps offer many advantages, but fatigue damage has been noticed on the surface of the teeth in the hydrostatic gerotor pumps used in aeroplanes[1]. Therefore, scientific investigations are becoming more and more focused on friction and wear problems, as well as on destruction of the working surfaces that result in a great number of gear failures.

There has been a significant advance in analysis of friction and wear in evolvent gears. In that sense, development of elastohydrodynamic (EHD) lubrication theory is of special importance because it studies the minimum oil layer thickness between the meshing surfaces in order to define the right lubrication regime. Based on this theory, numerous empirical formulas for calculation of the instantaneous friction coefficient of the contact surfaces have been developed[2–8]. However, there are no data in the available literature on such investigations being conducted for trochoidal gearing, hence this paper makes an original contribution to this scientific field of study.

The aim of this paper is to determine the character of the change of the instantaneous friction coefficient in the trochoidal pump using available formulas.

GEAR GEOMETRY DEFINING

The geometric and kinematic gearing model used in this paper is based on the investigation results given in the references[9–13].

The internal gear profile of the studied gerotor pump is an equidistant epitrochoid, while the external gear profile is a combination of circular arches. The basic geometric relations between the meshing gear profiles in different meshing phases are given in Fig. 1.

The coordinate contact points in the coordinate system of the epitrochoid $O_t x_t y_t$, are:

$$x_t = e\left[\left(\cos z\varphi + \lambda z \cos \varphi\right) - c \cos\left(\varphi + \delta\right)\right],$$
$$y_t = e\left[\left(\sin z\varphi + \lambda z \sin \varphi\right) - c \sin\left(\varphi + \delta\right)\right],$$
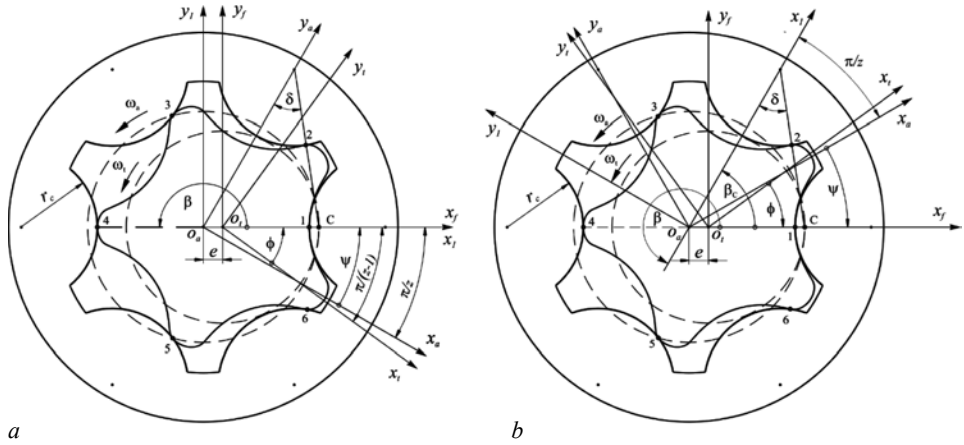
(1)

Fig. 1. A geometric model of the trochoidal gear pair with fixed axes: *a* – initial meshing moment, *b* – the moment when the next tooth enters the meshing

where z is the number of external gear teeth, c – the equidistant radius coefficient:

$$c = \frac{r_c}{e} \tag{2}$$

and δ is the leaning angle:

$$\delta = \arctan\frac{\sin\beta_c}{\lambda - \cos\beta_c}. \tag{3}$$

The rest of the parameters used in equation (1) are explained in details in the references[14,15].

Since the internal gear profile has the form of an equidistant epitrochoid, the expression for determination of the curvature radius ρ_c can be written in the form[14]:

$$\rho_c = \frac{ez\left[1 + \lambda^2 - 2\lambda\cos\beta_c\right]^{3/2}}{z + \lambda^2 - \lambda(z+1)\cos\beta_c} - ec. \tag{4}$$

The change of the curvature radius sign occurs in the inflection point, where the curvature changes from convex into concave, and vice versa. Therefore, it is easy to calculate the equivalent radius of the meshing profiles based on the following expression:

$$\rho_e = \frac{\rho_c r_c}{\rho_c \pm r_c}, \tag{5}$$

where the sign (+) is used for meshing of two convex surfaces, while the sign (−) is used for the meshing of a concave and a convex surface.

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KINEMATIC ANALYSIS OF GEAR MESHING

This section analyses the movement of the meshing profiles contact point. The internal gear rotates with an angular velocity ω_t, and the external gear rotates with an angular velocity ω_a. The transmission ratio is given with the expression:

$$u_{at} = \frac{\omega_a}{\omega_t} = \frac{z-1}{z}.\tag{6}$$

The trochoidal gear tooth starts meshing at the point 1 (Fig. 1*a*) when the rotation angle of the generating coordinate system is $\beta_c=0$. The meshing continues in the direction of rotation and when the meshing point comes to the position 2 (Fig. 1*b*), the following tooth enters the meshing. At that moment the value of the angle β_c is $2\pi/z$, hence, based on the angular displacement, we can calculate the value of the angle β_c for which the contact at characteristic contact points $i = 1, 2, 3, ...$ is realised (Fig. 1*a*):

$$\beta_{ci} = \frac{2\pi}{z}(i-1).\tag{7}$$

If we consider the movement of a tooth meshing point, we can write its coordinates in the fixed coordinate system in the form of[14]:

$$\begin{aligned}x_P &= e\{\lambda z \cos\beta_c - 1 - c\cos(\beta_c + \delta)\}\\y_P &= e\{\lambda z \sin\beta_c - c\sin(\beta_c + \delta)\}\end{aligned}.\tag{8}$$

The equations (8) describe the contact line of the meshing profiles. When gears mesh, the profiles roll and slide against each other. The rolling speed equals the relative velocity of the contact point $\vec{v}_{rt}$, i.e. $\vec{v}_{ra}$. The sliding of the profile at the contact point P is a result of the difference in relative velocities of the points at the internal, i.e. external gear profiles. Based on this and the geometric relations given in Fig. 2, the expression for determination of the sliding velocity of the meshing profiles can be written as:

$$v_s = e\omega_t\left\{z\left(1+\lambda^2 - 2\lambda\cos\beta_c\right)^{\frac{1}{2}} - \frac{c}{z}\right\}.\tag{9}$$

In addition to the sliding velocity, a sum of rolling velocities is also important for wear analysis. The sum of rolling velocities is a vector sum of relative velocities $\vec{v}_{rt}$ and $\vec{v}_{ra}$ at the meshing profiles contact point, and it can be expressed as:

$$v_r = e\omega_t\left\{\left(1+\lambda^2 - 2\lambda\cos\beta_c\right)^{\frac{1}{2}} - \frac{c}{z}\left(1+2\delta'\right)\right\},\tag{10}$$

Fig. 2. Kinematic parameters of the trochoidal gear pair

where

$$\delta' = \frac{(z-1)\left[1-\lambda\cos\beta_c\right]}{1+\lambda^2 - 2\lambda\cos\beta_c}.$$ (11)

The sliding velocity equals zero only when the profiles are in contact at the pitch point, where it changes its direction. With the trochoidal gearing, the contact line does not always go through the pitch point, which depends on the relation of the geometrical parameters of the gear profile[16].

ANALYSIS OF CONTACT FORCES AND STRESSES
WITH THE TROCHOIDAL PUMP

While the gears are in motion, the total normal load is transferred through the simultaneous meshing of the gear pair. However, the load is not equally shared among the teeth pairs because they have different stiffness, size, shape and position of the tooth profile contact point. Under the effect of forces, elastic teeth deformations occur at the instantaneous contact points and the size of teeth changes during the meshing. If we assume that all the profiles have regular geometric forms and that the equation of the mesh stiffness is constant, the further analysis will be based on the fact that the normal force is proportional to deformation of the gear pair in contact. Based on this and the calculations of the torque in Refs 14, 15, the expression that defines the contact force at the instantaneous con-

tact point can be written as:

$$F_{ni} = \frac{M_{F_n(O_t)} \sin \alpha_{ni}}{e(z-1) \sum_{j=p}^{q} \sin^2 \alpha_{nj}}, \tag{12}$$

where p and q are the ordinal numbers of the first and the last tooth of the external gear that transfer the load, $M_{F_n(O_t)}$ is the total moment of normal forces around the centre of the gear[15] that is equal to the sum of moments of normal forces for individual teeth pairs.

In order to define the necessary geometric values, it is necessary to write the expressions for the projections of the line segment $\overline{CP}_i$ (Ref. 14):

$$x_{CPi} = e\{\lambda z \cos\beta_{ci} - z - c\cos(\beta_{ci} + \delta_i)\}, \tag{13}$$
$$y_{CPi} = e\{\lambda z \sin\beta_{ci} - c\sin(\beta_{ci} + \delta_i)\}$$

where

$$\beta_{ci} = \frac{\pi(2i-1)}{z} - \psi, \tag{14}$$

and where i is the ordinal number of the analysed gear pair, and y is reference angle.

Now the size of the angle α_{ni} (Fig. 2) needed for calculation of the contact force can be determined:

$$\alpha_{ni} = \arctan \frac{y_{CPi}}{x_{CPi}}. \tag{15}$$

Finally, when the contact force is known, the maximum Hertzian contact pressure $P_{\max}$ can be calculated according to the following formula:

$$P_{\max} = \sqrt{\frac{WE_q}{2\pi \rho_e}}, \tag{16}$$

where W is the specific load, and E_q – the equivalent elastic modulus.

EMPIRICAL EXPRESSION FOR CALCULATION OF THE FRICTION COEFFICIENT

The friction coefficient is not a constant value and it varies during teeth meshing due to constant changes in the contact surface lubrication regime. EHD lubrication theory that has been developed in the last 50 years has enabled significant advances in understanding of the processes that occur between the contact surfaces. However, determination of the instantaneous friction coefficient value is still a considerate problem. There are many empirical expressions for calculation

of the instantaneous friction coefficient, but due to numerous limitations, many of these expressions can be applied only to certain operating conditions and loads. Most of these empirical expressions are for the boundary or mixed (partial) EHD lubrication regime. Since the trochoidal pump operates at rather low values of Hertz pressure and low circumferential velocities, some of the experimental expressions[2,3] are not considered in this paper.

The boundary and partial EHD lubrication regimes are most commonly used in gear meshing[17]. Shipheng and Cheng[18,19] have given a great contribution to development of the theory of partial EHD lubrication. They suggested that the partial EHD lubrication regime should be used for gear meshing because the thickness of the elastohydrodynamic layer between the contact surfaces had the same value as roughness parameter. The paper by Xu[2] is especially important because it studies the friction coefficient in the EHD lubrication regime, even though his approach has some disadvantages and needs to be further developed. The first disadvantage is that this model requires a lot of calculation, i.e. processing time. He modelled a cylindrical gear with spur teeth and an ideal load distribution, and considered the teeth stiff bodies. Xu thought that empirical formulae could not be used to determine the friction coefficient at gear meshing, but he oversaw the fact that all these expressions were defined for the boundary and partial EHD lubrication regime. Most of the expressions given in the paper[2] cannot be applied to the partial EHD regime, and particularly not to the boundary EHD lubrication regime. An additional disadvantage of the Xu expression is that he worked with the surface roughness of $R_a = 0.07$ μm which is difficult to accomplish in a gear.

Misharin[3] proposed an expression applicable to larger loads, where there are some limitations concerning the lowest value of the Hertzian pressure ($P_{max} \geq$ 245 MPa) and some limitations concerning the ratio of the sliding velocity to the summary rolling velocity v_s/v_r from 0.4 to 1.3. These values are much higher than the ones in the trochoidal pump, thus the Misharin expression cannot be applied. In conditions of larger loads, like with the trochoidal pump, it is possible to apply the Drozdov expression[4] even though there are some limitations concerning the maximum Hertzian pressure. This expression is used for determination of the instantaneous coefficient for the meshing surfaces that roll and slide against each other and for the boundary lubrication regime.

O'Donoghue and Cameron[5] studied friction during the rolling and sliding contacts on the Amsler machine[20]. Benedict and Kelley[6] proposed an empirical formula for determination of the friction coefficient under the mixed lubrication regime based on the experimental tests performed on a tribological machine with discs. Höhn et al.[7] calculated the average friction coefficient as a function of the load, surface roughness, viscosity, velocity and geometric properties. A similar expression is given in the ISO TC60 Standard[8], where the sliding velocity has not been taken into account.

Marjanović[21] used pin on disc tests for tribological investigations. During the testing, variable velocities and normal forces with different pins were used in order to investigate the influence of the equivalent radius, sliding velocity and contact pressure. Lubrication was performed using the standard gear lubrication oil (GALAX HIPOL B, SAE 90). The following expression was obtained:

$$\mu = 0.00234 \frac{P_{max}^{0.584}}{v_s^{0.231}}. \tag{17}$$

ANALYSIS OF THE INSTANTANEOUS FRICTION COEFFICIENT

The analysis was performed using Matlab® and the program procedure that enabled monitoring of the changes of the studied values. In order to perform a better analysis of the changes of the instantaneous friction coefficient and the effect of different parameters on the lubrication regime, four models of the trochoidal pump with different basic parameters were used[14]:

Model A: $z = 6$, $\lambda = 1.375$, $r_c = 9.75$ mm, $c = 2.75$;
Model B: $z = 6$, $\lambda = 1.575$, $r_c = 14.06$ mm, $c = 3.95$;
Model C: $z = 6$, $\lambda = 1.675$, $r_c = 16.20$ mm, $c = 4.55$;
Model D: $z = 5$, $\lambda = 1.85$, $r_c = 13.35$ mm, $c = 3.75$.

Other initial data are given in Table 1.

Table 1. Initial data

Parameter	Sign	Value
Volumetric capacity (m³/rev)	q	14×10^{-6}
Working pressure (bar)	p	6
Number of revolutions of the pump shaft (rpm)	n_t	1500
Angular velocity of the pump shaft (s⁻¹)	ω_t	50π
Dynamic viscosity of hydraulic fluid (kg/ms)	η	0.02
Density of hydraulic fluid (kg/m³)	ρ_f	900
Gears width (mm)	b	16.46
Eccentricity of pump (mm)	e	3.56
Young modulus of gear material (N/m²)	E	2×10^{11}
Poisson coefficient of gear material (–)	ν	0.3
Arithmetic average of the roughness profile (mm)	R_a	0.8

The analysis of the friction coefficient was performed through two contact periods or two contact phases. Figure 3 shows the distribution of velocities and the leaning angle for the model B. The first phase refers to the meshing from the beginning to the moment when the rolling velocity of the external gear changes its direction and when the maximum leaning angle δ_{max} is reached.
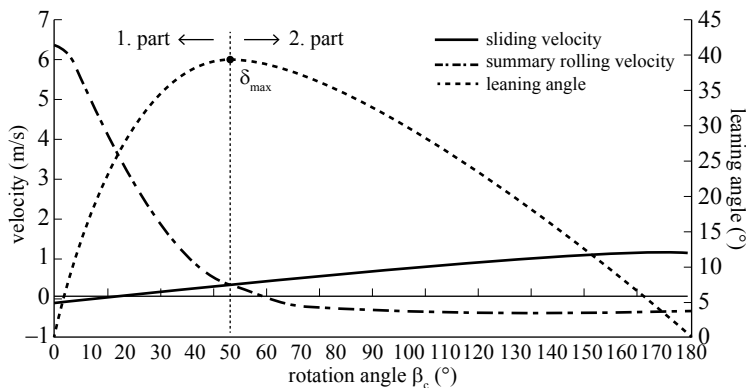
Fig. 3. Distribution of velocities and the leaning angle for the model *B*

The second phase starts at the moment when the maximum leaning angle is achieved and lasts till the end of meshing, when the angle is δ=0. The curves of the sliding velocity and the sum of rolling velocities intersect at the point when the angle reaches the maximum value.

For analysis of the instantenous friction coefficient for the second contact period, the formula (17) was used since it is meant for the half-dry and dry friction area. This formula does not take into account the rolling velocity, and it has no influence on the value of the instantenous friction coefficient. The experiments were performed on a pin/disk system[21], where the disc performed rotary movement, so the meshing was similar to the one in the second contact period.

The rest of the formulae were used for the analysis of the instantaneous friction coefficient for the first contact period, because they are meant for the area
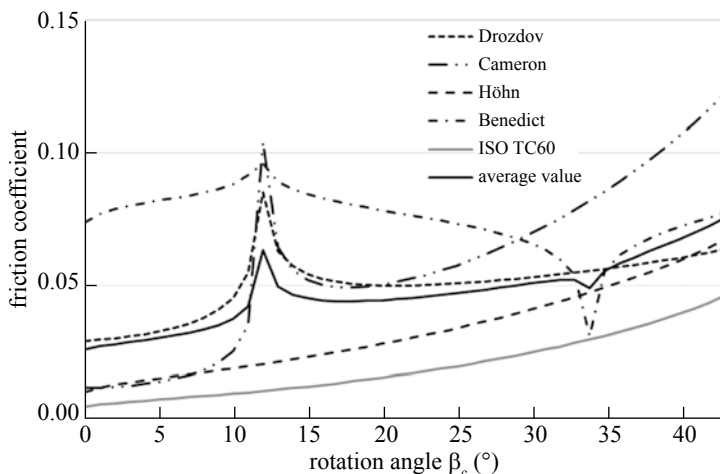
Fig. 4. Distribution of the instantaneous friction coefficient for the model *A*

with a mixed or full EHD lubrication regime. These formulae were obtained based on the experiments conducted on the disc/disc or ball/disc system, where both parts roll into the same direction during the meshing, i.e. where their circumferential velocities had the same direction at the contact point. This kind of meshing corresponds to the first contact period.

Figures 4, 5 and 6 show distributions of the friction coefficient and the average arithmetic value of the coefficient for the models *A, B* and *C,* respectively. There are two characteristic positions where the curves have peaks. The first one is when the contact point passes through the pitch point, where the friction coefficient abruptly increases until it reaches its static value. The second characteristic
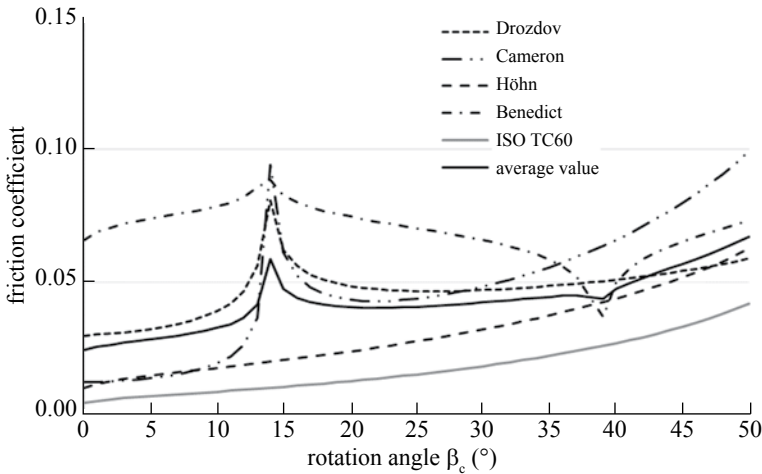
Fig. 5. Distribution of the instantaneous friction coefficient for the model *B*
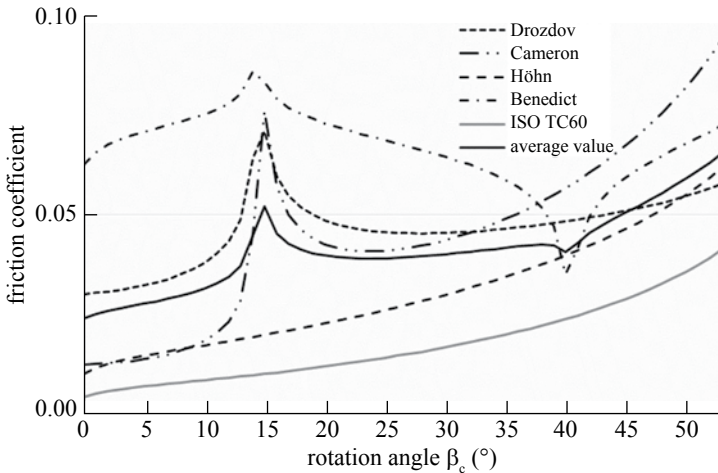

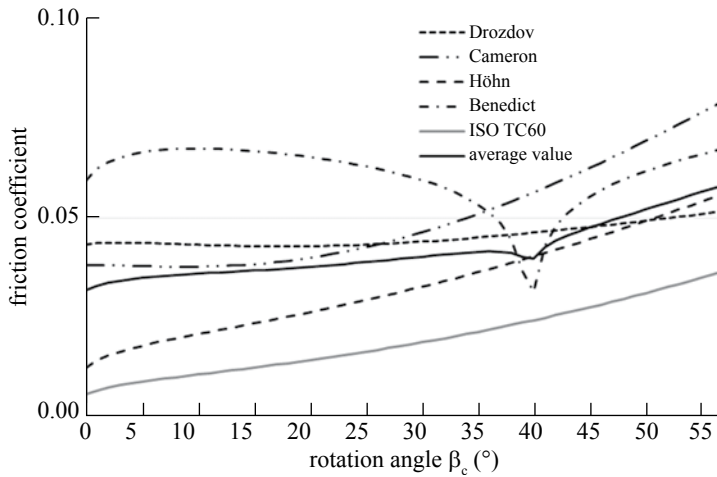**Fig. 6**. Distribution of the instantaneous friction coefficient for the model *C*

Fig. 7. Distribution of the instantaneous friction coefficient for the model *D*
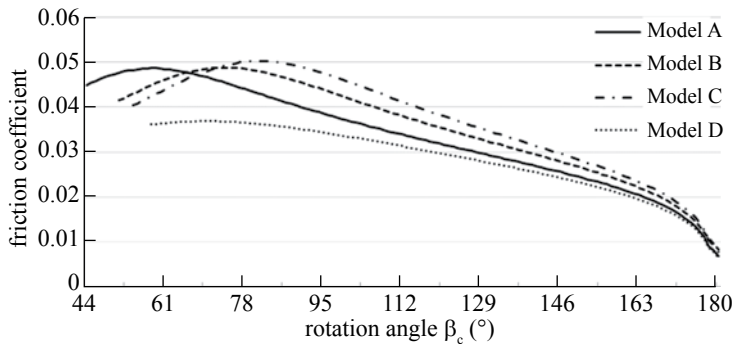

Fig. 8. Distribution of the friction coefficient for the second contact period

position is when the contact point overlaps with the inflection point of the tro-choidal gear curve, when the curves reach their local minimum.

The curve for the average arithmetic value of the friction coefficient is very similar to the curves obtained using the Drozdov and Cameron expressions (for models *A, B* and *C*).

With the model *D* (Fig. 7), the teeth start meshing at the pitch point, so there is no peak that appears with the other models. Maximum values of the friction coefficient are lower with this model, which leads to a conclusion that the friction coefficient decreases with the decrease in the number of teeth. With models that have the external gear with 6 teeth, the friction coefficient decreases with the increase in the value of λ.

In the second meshing period (Fig. 8), the friction coefficient increases up to its maximum value and then it mainly decreases linearly. In this phase, the model *D* exhibits the lowest friction coefficient.

CONCLUSIONS

It has been determined that the friction coefficient value varies in the range from 0.02 to 0.12 for the studied trochoidal pump models. The maximum value (with the models *A*, *B* and *C*) occurs when the contact point goes through the pitch point. The next characteristic point of an abrupt change in the value of the friction coefficient is the trochoidal gear inflection point. The lowest value of the friction coefficient during the distribution is noticed in the model *D*, which shows that the decrease in the number of teeth has a favourable effect on the decrease of friction between the contact surfaces.

If we consider the decrease in the friction coefficient, the model *D* has given the best results for the first contact period while the model *A* has the lowest friction coefficient in the second period.

The empirical expressions given by Drozdov and Cameron have shown the least deviations from the average friction coefficient. It is necessary to study some newer investigations in the field of EHD lubrication, so that a more accurate analysis and a more precise lubrication regime for the second contact period can be defined. A special attention should also be paid to pitting in the upper part of the teeth since in this meshing period the gear rolling velocities, i.e. their relative velocities have opposite directions.

ACKNOWLEDGEMENTS

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