THERMOMECHANICS OF SOFT INELASTICS BODIES WITH APPLICATION TO ASPHALT BEHAVIOR

by

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Thermomechanical behavior of hot mix asphalt (HMA) is considered. Its highly irregular microstructure is covered by the hierarchical approach. A brief survey of endochronic thermodynamics precedes constitutive consideration. Two constitutive models are discussed: classical Perzyna's approach and tensor representation based approach. The second is superior due to its possibility to cover properly diverse multiaxial non-proportional stress-strain histories. However, due to availability of experimental data the first model is applied to rutting problem through Abaqus FEM code with material user subroutine developed by the authors. Vakulenko's thermodynamic time appropriate for aging is incorporated. Hyper elastic-viscoplastic behavior is considered and some preliminary results are presented.

Key words: *endochronic memory, nonlinear hyper elasto-plasticity, thermo-dynamic restrictions, asphalt rutting*

Introduction

The aim of this paper is twofold. First, we will consider in some detail an asphalt pavement - its microstructure and mechanical behavior are described in [2, 3]. Unlike concrete being a rigid pavement, asphalt belongs to flexible pavements. A characteristic permanent strain with spatial distribution along the road is called rutting. While the initial rut is caused by densification of the pavement under the path of the wheel, the subsequent history of rut is a result of shear flow of the *hot mix asphalt* (HMA). Thus, a proper constitutive model for asphalt is needed - able to cover not only tension but shear as well.

Concerning the microstructure of asphalt we see that HMA contains a very high volume fraction of irregularly shaped particulate inclusions [4]. Typically, HMA consists of 93-97% of gravel and sand (usually called *aggregates*) bonded by bitumen. It has been shown by Lakes et al. [5] that for some larger volume fraction of irregular particulates, Eshelbian approach based on ellipsoidal inclusions does not give acceptable results - the predictions of effective properties underestimate the actual properties significantly. This fact pointed out by Lakes et al. [5] is logical due to large discrepancy from ellipsoidal form. As well known and discovered by Eshelby (cf. [6]) within range of isotropic elasticity for ellipsoidal inclusions a

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homogeneous external field exerted outside the inclusion produces a corresponding homogeneous strain field in the inclusion. Such a remarkable feature has made possible a development of large number of papers dealing with so called self consistent field theories where inclusions have form close to ellipsoids. Such an approach (cf. also [7]) here, to our regret, is not promising. Instead, the so called *hierarchical approach* is proposed. By this approach, HMA is modeled as a two phase composite, the first phase having aggregates with principal diameter larger than 0*:*3*mm* while the second phase was taken to be a homogenized mixture of smaller aggregates and binder. The enclosed fig. 1 (a) taken from [4] is convenient to illustrate the essence of this method. It should be noted that for such an approach a proper choice of the size of the Representative Volume Element (RVE) is very important and an image processing technique is needed.

For a body with manifold of immersed ellipsoidal inclusions this issue has been discussed in some detail in [8]. Calculations needed involve solving large number of the coupled integral equations. These calculations would become particularly cumbersome if for calculation of Eshelby tensor instead of average stiffness we insert the effective stiffness tensor (cf. [9]). Due to this complexity if we want explicit information about RVE size then the approach must be oversimplified. For instance in the paper [10] the authors obtained the most valuable information about RVE size. However, due to all the difficulties mentioned above for obtaining explicit solutions they assumed that inclusions are non-overlapping spheres and that the medium is isotropic. The finite element analysis applied in [11] is even simpler neglecting eigen-strains escorting implantation of grains into the polycrystalline RVE. It should be noted here that asphalt is a tar like substance obtained during fractional destillation of crude natural oil. At high temperature it behaves like a viscous fluid. For this reason a proper thermodynamic analysis of the deformation process is indispensable. The above discussed microstructural analysis dealing with windowing of RVE is not even touched here due to high irregularity of large often overlapping grains having irregular shapes. Due to these reasons, although unwillingly we will apply here a phenomenological approach smoothing inclusions and matrix into a homogeneous representative substance. On the other hand, the thermodynamical approach applied in the sequel is adequate and without simplifications.

Endochronic thermodynamics

The main idea in the so-called *endochronic thermodynamics* (cf. [14, 15]) is to replace actual time by means of some non-decreasing scalar function of inelastic strain history responsible for aging whose ultimate value leads finally to rupture of the body. Vakulenko called this function - *thermodynamic time* ([14]). In such a concept purely elastic strain does not contribute to the thermodynamic time. Such a time was introduced in [14] by means of the inelastic entropy source accumulation as follows. This source may be obtained by making use of

$$\psi = (\rho / T)\mathrm{d}s / \mathrm{d}t + div(\mathbf{q}/T), \tag{1}$$

where ρ*;* s*;* $\mathbf{q}$ and T are mass density, specific entropy, specific heat flux vector and absolute temperature. In the range of thermoelasticity ψ vanishes in a homogeneous temperature field.

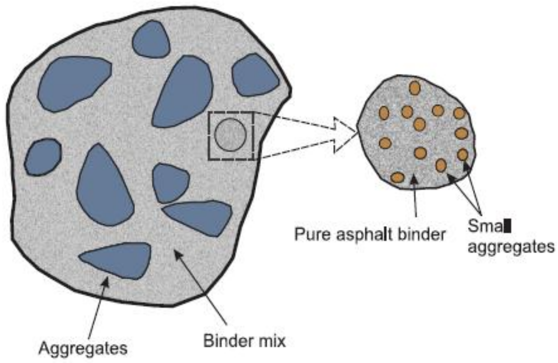

Figure 1. Microstructure of asphalt.

Thus, the inelastic part of the source (1) i.e. $\psi^P \equiv \psi + \mathbf{q} \cdot gradT / T^2$ reads ($\mathrm{d}/\mathrm{d}t$ is used here to denote material time derivative):

$$T\psi^P = \boldsymbol{\sigma} : \mathrm{d}\boldsymbol{\varepsilon} / \mathrm{d}t + \rho \mathrm{d}s / \mathrm{d}t - \rho \mathrm{d}u / \mathrm{d}t, \tag{2}$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, and u stand for Cauchy stress tensor, total strain tensor and internal energy density. As a partial result the reduced dissipation inequality $\mathrm{q} \cdot \mathrm{grad}(1/\mathrm{T}) \geq 0$ shows that heat "flows" to the particle with lower temperature. Now, passing to the point, Vakulenko assumed that $\rho\psi_s^P \geq 0$ and defined the thermodynamic time by means of (cf. also [13])

$$\zeta(\mathrm{t}) = \int_0^{\mathrm{t}} \rho\psi_s^P(t')\mathrm{d}t'. \tag{3}$$

The function $\zeta(t)$ is piecewise continuous and non-decreasing in that $D_t\zeta(\mathrm{t}) \equiv \mathrm{d}\zeta(\mathrm{t})/\mathrm{d}t = 0$ within elastic ranges and $D_t\zeta(\mathrm{t}) > 0$ when plastic deformation takes place.

Assuming that at each time instant the considered state can be obtained either by *instantaneous loading or by unloading* and splitting the whole time history into a sequence of infinitesimal segments Vakulenko claimed that a superposition and causality exists – extending in such a way Boltzmann's and Volterra's superposition to nonlinear inelastic phenomena. Proceeding with accumulation of infinitesimal memory he obtained an integral relationship between Eulerian plastic strain deviator and stress deviator history as follows:

$$\boldsymbol{\varepsilon}_P(\zeta) = \int_0^{\zeta} \boldsymbol{\Phi}\left(\zeta - \xi, \boldsymbol{\sigma}(\xi), \frac{\mathrm{d}}{\mathrm{d}\xi}\boldsymbol{\sigma}(\xi)\right)\mathrm{d}\xi. \tag{4}$$

Therefore, in this setting the plastic strain tensor is obtained as a functional of stress and stress rate history.

In the books [1, 13] the authors applied Vakulenko's approach to diverse media splitting the internal energy into a part dependent on temperature and elastic strain and a part which contains as arguments inelastic internal parameters and temperature. For our purpose in this paper we will consider a possibility to replace actual time with such a thermodynamic time. If we compare the hereinabove function of thermodynamic time to accumulated plastic strain, then we see that ζ is able to account for nonlinear evolution equations in a simple way.

MAM model with tensor representation

According to ([12]) the increment of plastic strain tensor is perpendicular to a loading surface Ω = const where Ω depends on stress, temperature and *Pattern of Internal Rearrangement* (PIR). Translating this statement into the language of the previous section an evolution equation for plastic stretching should hold in the following form ([1]):

$$D_t\boldsymbol{\varepsilon}_P = \partial_{\boldsymbol{\sigma}}\Omega(\boldsymbol{\sigma}, T, PIR). \tag{5}$$

Here PIR is described by a holonomic internal variables representing crystal slips over active slip systems.

The plastic distortion tensor is incompatible (cf. [1]), represents slips and may reflect transformation of a holonomic coordinates. Thus, taking into account that plastic rotation tensor may be either fixed or taken to be unity, it was assumed in [16] that in the above equation PIR may be represented by the plastic strain tensor. Moreover, we extend the above evolution equation inserting in it a scalar function Λ, which must account for the linear connection between rates of Mises equivalent stress and equivalent plastic strain rate. The structure of Λ may be related to Ziegler's principle of least irreversible force [1]. Therefore

$$D_t\boldsymbol{\varepsilon}_P\left(D_t\boldsymbol{\sigma}, \boldsymbol{\sigma}, \boldsymbol{\varepsilon}_P, T\right) = \Lambda\partial_{\boldsymbol{\sigma}}\Omega(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_P, T), \tag{6}$$

where Rice's loading function $\Omega = \Omega(\gamma, T)$ depends on temperature and following invariants:

$$\gamma \equiv \{s_1, s_2, s_3, \pi_2, \pi_3, \mu_1, \mu_2, \mu_3, \mu_4\},$$

with

$$s_1 = \mathrm{tr}\boldsymbol{\sigma}, \ s_2 = \mathrm{tr}\boldsymbol{\sigma}_d^2, \ s_3 = \mathrm{tr}\boldsymbol{\sigma}_d^3, \ \pi_2 = \mathrm{tr}\boldsymbol{\varepsilon}_P^3, \ \pi_3 = \mathrm{tr}\boldsymbol{\varepsilon}_P^3,$$

$$\mu_1 = \mathrm{tr}\{\boldsymbol{\sigma}_d\boldsymbol{\varepsilon}_P\}, \ \mu_2 = \mathrm{tr}\{\boldsymbol{\sigma}_d\boldsymbol{\varepsilon}_P^2\}, \ \mu_3 = \mathrm{tr}\{\boldsymbol{\sigma}_d^2\boldsymbol{\varepsilon}_P\}, \ \mu_4 = \mathrm{tr}\{\boldsymbol{\sigma}_d^2\boldsymbol{\varepsilon}_P^2\}.$$

If Ω is approximated by a fourth order polynomial with respect to σ and first order in $\boldsymbol{\varepsilon}_P$, then a very simple polynomial is obtained as follows (details are given in [1]).

$$2\Omega = a_1 s_2 + (a_2 + a_4\mu_1)(s_1 s_2 - s_3) + \frac{1}{2}a_3 s_2^2 + \frac{1}{3}a_5(3\mu_3 s_2 - 2\mu_1 s_3). \tag{7}$$

Here only five independent constants are needed. Thus we arrive at the next equation:

$$D_t\boldsymbol{\varepsilon}_P = \Lambda\sum_{\alpha=1}^{4}\Gamma_\alpha(\gamma)\mathbf{H}_\alpha \tag{8}$$

with tensor generators

$$Y_0\mathbf{H}_1 = \boldsymbol{\sigma} - \mathbf{1}\mathrm{tr}\boldsymbol{\sigma}/\mathbf{3} \equiv \mathrm{dev}\boldsymbol{\sigma} \equiv \boldsymbol{\sigma}_d, \ \mathbf{H}_3 = \boldsymbol{\varepsilon}_P, \ Y_0^2\mathbf{H}_2 = \mathrm{dev}(\boldsymbol{\sigma}_d^2), \ Y_0\mathbf{H}_4 = \mathrm{dev}(\boldsymbol{\sigma}_d\boldsymbol{\varepsilon}_P + \boldsymbol{\varepsilon}_P\boldsymbol{\sigma}_d)$$

and corresponding scalar coefficients depending on the listed invariants are:

$$\Gamma_1 = a_1 + a_2 s_1 + a_3 s_2 + a_4\mu_1 + a_5\mu_3, \Gamma_2 = -3(a_2 + a_4\mu_1)/2 - 2a_5\mu_1,$$

$$\Gamma_3 = a_4(s_1 s_2 - s_3)/2 - 2a_5 s_3/3, \ \Gamma_4 = a_5 s_2.$$

At this point let us turn our attention to the scalar coefficient Λ appearing in (6). If we want to cover the effect of Rabotnov's yielding delay, the best way is to propose this scalar in the form:

$$\Lambda = \eta\left(\sigma_{eq} - Y\right)\left(\frac{\sigma_{eq}}{Y_0} - 1\right)^{\lambda} D_t\sigma_{eq}\exp\left(-M\right). \tag{9}$$

Here Y is the dynamic initial equivalent yield stress, Y_0 is its static counterpart, $\eta(x)$ is Heaviside's function, λ is a material constant and M is a material constant which covers multiaxial stress-strain histories with broad range of strain rates – from static to impact rates. It is worth noting that inserting of (9) into (6) leads to an evolution equation of incremental form seemingly characteristic for rate–independent materials. At first sight the evolution equation for plastic stretching looks rate–independent since it can be transformed into an incremental equation if it is multiplied by an infinitesimal time increment. However, the rate dependence appears in stress–rate dependent value of the initial yield stress Y which has a triggering role for inelasticity onset. The model could be termed *quasi rate independent*.

Here, a special case of the loading function leading to reduced forms of the evolution equation (8) has remarkable simplicity. If a_4 =0 and a_5 =0, then the plastic stretching is of third–order power of stress. The loading function becomes (this may be called MAM reduced model) $2\Omega = a_1 s_2 + a_2\left(s_1 s_2 - s_3\right) + a_3 s_2^2/2$.

However, although this approach has proved its superiority above all other considered in [1] we believe that the function Λ could be successfully applied in its simpler forms to other constitutive models by the plastic working i.e. $\hat{d}\xi \equiv \boldsymbol{\sigma} : d\boldsymbol{\varepsilon}_p$. Here the symbol $\hat{d}$ shows that the infinitesimal increment of plastic work is not a total differential. This will be applied to the next constitutive model.

Perzyna's model for asphalt

The most frequently used model of viscoplastic mechanical behavior has been proposed by Perzyna. The corresponding set of field equations as listed in [3] is encompassed by plastic strain time rate $D_t\boldsymbol{\varepsilon}_P$ and scalar hardening variable time rate $D_t\kappa$

$$D_t\boldsymbol{\varepsilon}_P = \left(\frac{\langle f\rangle}{\eta}\right)^m \frac{1}{x+\kappa^L}\mathbf{v}, \tag{10}$$

$$D_t\kappa = \left(\frac{\langle f\rangle}{\eta}\right)^m \frac{1}{x+\kappa^l}, \tag{11}$$

where normal $\mathbf{v}$ to loading surface ,$\bar{\Omega}$,

$$\bar{\Omega} = \sqrt{J_2} + \gamma I_1, \tag{12}$$

reads:

$$\mathbf{v} = \frac{\partial\bar{\Omega}}{\partial\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}_d}{2\sqrt{J_2}} + \gamma\mathbf{1}. \tag{13}$$

The elastic range is interior of a closed yield surface in the stress space. It is limited by the sign of yield function

$$f = \bar{s}_1\bar{s}_2 + \alpha\bar{s}_3 - H\kappa. \tag{14}$$

Here $f < 0$ means elastic behavior inside the elastic range, $f = 0$ stands for the yield surface and $f > 0$ is characteristic for viscoplastic range. The hyperelastic nonlinear constitutive equation connects stress and elastic strain as follows:

$$\sigma = \mathsf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_P) \equiv \mathsf{D}(\boldsymbol{\varepsilon}_E) = \left(2b_2 I_1 + 3b_3 I_1^2 + b_4 I_2\right)1 + \left(b_5 I_1 + b_4 I_1\right)\boldsymbol{\varepsilon}_E + b_6 \boldsymbol{\varepsilon}_E^2 , \tag{15}$$

whereas the above used stress and elastic strain invariants are collected into the set:

$$\begin{aligned}\bar{\gamma} &\equiv \{I_1, I_2, I_3, \bar{s}_1, \bar{s}_2, s_3\} \\ &= \left\{tr\boldsymbol{\varepsilon}_E, tr\boldsymbol{\varepsilon}_E^2, tr\boldsymbol{\varepsilon}_E^3, tr\boldsymbol{\sigma}, \left(tr\boldsymbol{\sigma}^2 - s_1^2\right)/2, \det\boldsymbol{\sigma}\right\},\end{aligned} \tag{16}$$

being slightly different from the set used in the previous model. Here it is of utmost importance to underline that the material model constants, $\mathsf{A} = \{b_2, b_3, b_4, b_5, b_6, H, \alpha, \eta, m, l\}$, when calibrated, are completely different for shear and tension experiments (cf. [3]). This is not a shortcoming of this thesis but rather enlightens the fact that all constitutive models which are not based on tensor generators are misleading.

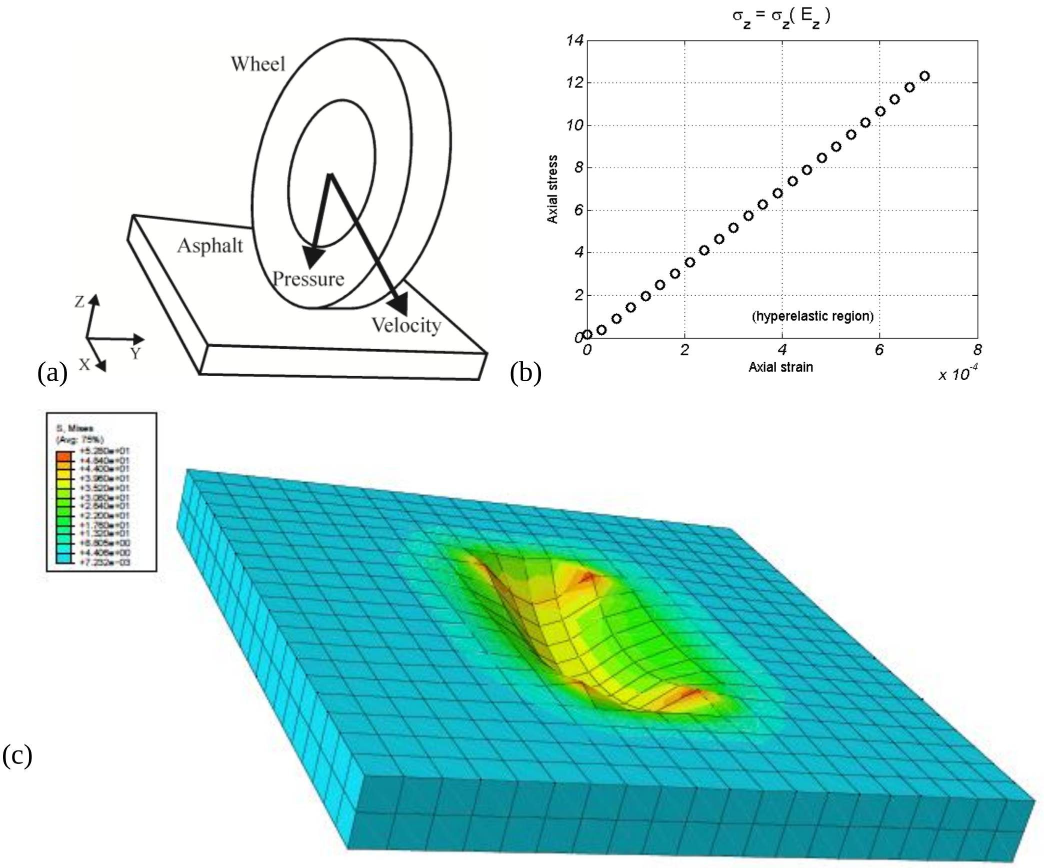

Figure 2. FEM results for pavement rutting (a) A moving wheel on a rutting asphalt surface (b) Axial stress vs axial strain for 1 element asphalt box in hyperelastic range according to material constants in [3] (c) FEM result for deformed pavement upper surface.

At present, we have made an analysis by the FEM for a parallelepiped under compression using the Perzyna's model. However, an important modification is made: instead of time rates $D_t\boldsymbol{\varepsilon}_P, D_t\kappa,$ corresponding thermodynamic time rates $D_\xi\boldsymbol{\varepsilon}_P, D_\xi\kappa$ are used. Work towards full application of the MAM model is in progress. Partial results are depicted in the Figs. 2- 3.

Analysis and some conclusions

Let us note that the model of hyper elasto-viscoplasticity considered here is suitable for numerical applications. Due to this, and following the procedure often applied in papers dealing with FEM analysis, first we have analyzed a single element with homogeneous stress and strain histories. Due to geometry of loading of the considered wheel we analyze here a strain controlled history when only z-component of total strain grows. Results are shown in Figs. 2 and 3. One should notice that contrary to the algorithm applied in [3], here thermodynamic time is used instead of the commonly observed and measured time. As mentioned above, this time progresses only when inelastic strain is changed. Thus, in the hyper elastic range the usual time must be used. Stress-strain curve for direction normal to the pavement surface in its undeformed state is given in the Fig. 2b. When strains grow the inelastic beahviour appears. Curves for the calculated yield function *f* as well hardening function κ (cf. (11)) are further given in the Figures 2a and 2b. Initial zero values of the yield function encompass the hyper elastic range where the hardening function has constant value. A smooth behavior of the curves of the axial stress versus total axial strain is rather close to experimental data reported in [3]. Anyway, a further comparison with the MAM model must be done for more complex multiaxial and non-proportional stress and strain histories. Deformed upper surface of the pavement surface is shown in the Fig. 2c.

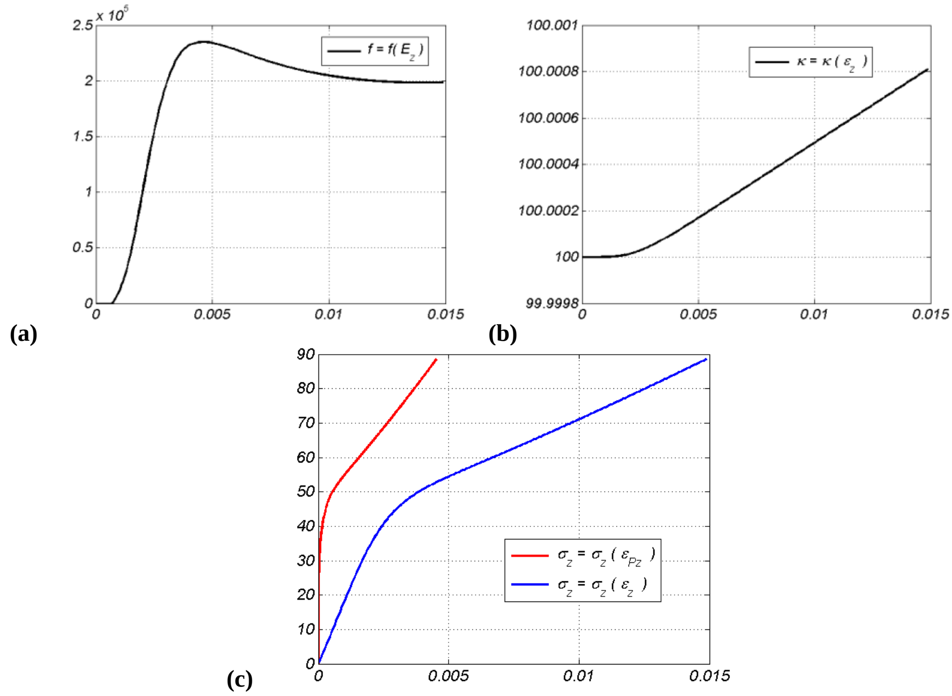

Figure 3. Numerically found histories for 1 element asphalt box (a) Yield function (b) Hardening function (c) Results in hyperelastic-viscoplastic region

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References

[1] Micunovic M., Thermomechanics of Viscoplasticity - Fundamentals and Applications, Advances in Mechanics and Mathematics AMMA, Vol. 20, 2009, eds. R. W. Ogden and D. Gao, Springer, New York, USA

[2] Bahuguna S., Panoskaltsis V. P., Papoulia K. D. Identification and Modeling of Permanent Deformations of Asphalt Concrete., J. Engng. Mech., Vol. 132, 2006, Issue 3, pp. 231-239.

[3] Panneerselvam D., Mechanics of Asphalt Concrete: Analytical and Computational Studies, PhD thesis, Case Western Reserve Univ., 2005, Cleveland, O., USA

[4] Kahan Mitra, Animesh Das, Sumit Basu, Mechanical behavior of asphalt mix: An experimental and numerical study, Construction and Building Materials, Vol. 27, 2012, 1, pp. 545-552.

[5] Lakes RS, Kose S, Bahia H.,Analysis of high volume fraction irregular particulate damping composites, J Eng Mater Technol, Vol.124, 2002, 2, p.174-178.

[6] Eshelby, J. D., The determination of the elastic field of an ellipsoidal inclusion, and related problems, Proc. Roy. Soc., 241, 1957, pp. 376–396.

[7] Micunovic M., Kudrjavceva L, Šumarac, Thermoinealsticity of Damaged Elastomers by Self-consistent Method, Int. J. Damage Mech.,17/6, 2008, pp. 539-565.

[8] Micunovic, M., Self-consistent method applied to quasi-rate independent polycrystals, Phil.Magazine, 85, 2005, 33-35, pp. 4031–4054.

[9] Yaguchi, M., Busso, E. P., A Self-Consistent Constitutive Formulation for a Directionally Solifified Polycristal, Int. J. Solids Structures, 42, 2005, 3-4, pp. 1073-1089

[10] Drugan, W. J. and Willis, J. R, A Micromechanics-Based Nonlocal Constitutive Equation and Estimates of Representative Volume Element Size for Elastic Composites, J. Mech. Phys.Solids, 44, 1996, 497-524.

[11] Gusev, A. A., Representative volume element size for elastic composites: a numerical study, J. Mech. Phys.Solids, 45, 1997, 9, pp. 1449 - 1459.

[12] Rice, J. R., Inelastic constitutive relations for solids: an internal variable theory and its application to metal plasticity, J. Mech. Phys. Solids, 19, 1971, 433–455

[13] Fomin, V. L.: Continuum Mechanics for Engineers (in Russian), St. Peterburg University Publ., St. Peterburg (1975)

[14] Vakulenko, A. A., Superposition in continuum rheology (in Russian), Izv. AN SSSR Mekhanika Tverdogo Tela, 1, 1970, pp. 69–74

[15] Valanis, K. C., A theory of viscoplasticity without a yield surface, Arch. Mech., 23, 1971, pp. 517–533

[16] Micunovic , M., Albertini, C., Montagnani, M., High strain rate viscoplasticity of AISI 316 stainless steel from tension and shear experiments, In: Miljani´c, P. (ed) Solid Mechanics, Serbian Acad. Sci. Meetings - LXXXVII, Dept. Techn. Sci., 3, 1997, 97–106.
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