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Robust fault detection filter design for a class of discrete-time conic-type non-linear Markov jump systems with jump fault signals

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Xuefei Dong[1], *Shuping He*[1] ✉, *Vladimir Stojanovic*[2]

[1]*Key Laboratory of Intelligent Computing and Signal Processing (Ministry of Education), School of Electrical Engineering and Automation, Anhui University, Hefei 230601, People's Republic of China*

[2]*Department of Automatic Control, Robotics and Fluid Technique, Faculty of Mechanical and Civil Engineering, University of Kragujevac, Kraljevo 36000, Serbia*

✉ *E-mail: shuping.he@ahu.edu.cn*

Abstract: This study investigates the robust fault detection filter design problem for a class of discrete-time conic-type non-linear Markov jump systems with jump fault signals. The conic-type non-linearities satisfy a restrictive condition that lies in an *n*-dimensional hyper-sphere with an uncertain centre. A crucial idea is to formulate the robust fault detection filter design problem of non-linear Markov jump systems as H_∞ filtering problem. The authors aim to design a fault detection filter such that the augmented Markov jump systems with conic-type non-linearities are stochastically stable and satisfy the given H_∞ performance against the external disturbances. By means of the appropriate mode-dependent Lyapunov functional method, sufficient conditions for the existence of the designed fault detection filter are presented in terms of linear matrix inequalities. Finally, a practical circuit model example is employed to demonstrate the availability of the main results.

Nomenclature

$(Y)*$	$Y + Y^{\mathrm{T}}$
I	unit matrix
0	zero matrix
A^{-1}	matrix inverse
A^{T}	matrix transpose
$*$	symmetric matrix
$\mathscr{E}\{A\}$	expectation of A
$\mathfrak{R}^{n\times p}$	$n \times p$ real matrix
$\Vert \star \Vert$	Euclidean vector norm
$\mathfrak{R}^{n}$	n-dimensional Euclidean space
diag$\{A\ B\}$	block-diagonal matrix of A and B
$P > (<, \geq, \leq)0$	positive (negative, semi-positive, semi-negative) -definite matrix

1 Introduction

Markov jump systems (MJSs) were firstly proposed by Krasovskii and Lidskii [1] in 1960s. As a special kind of hybrid systems, MJSs consist of a finite number of subsystems converted by a Markov chain. In fact, MJSs can be used to describe some dynamic systems in which the structures are subjected to randomly sudden variables due to abrupt external disturbance, shifts of the action spots of non-linear systems and repairs of components. During the past several decades, MJSs have received considerable attention and many results are available such as stability analysis [2, 3], sliding mode control [4], robust filtering [5] etc.

In fact, some environmental factors (such as modelling errors, external disturbances and uncertainties) usually bring many problems to the dynamic system (poor behaviour and unstable performance) in controlling. Therefore, the non-linearities always should be considered when we model an actual engineering system. As a kind of complex non-linearities, conic-type non-linear dynamic has attracted increasing attention in theoretical analysis and practical application. Aiming at a class of time-delay conic non-linear systems, Song and co-authors [6] presented the finite-time bounded controller design by using the sliding mode control strategy. Moreover, the conic-type non-linear MJSs have become a hot spot recently. In [7], Cheng and He investigated the observer-based finite-time asynchronous control problem for a class of hidden MJSs with conic-type non-linearities. The challenge is how to deal with the conic-type non-linearities, which prompts us to study this topic. For more details about this topic, please see [8–10] and the references therein.

On another research forefront, the research on fault detection (FD) for dynamic systems has attracted increasing attention during the past three decades owing to the rising demand of product quality [11], effectiveness [12] and safety [13] in modern industries. Many results related to FD have been shown in the literature, see for example [14–19] and the references therein. Among these methods, the model-based method is the most common scheme, that is the state observers or filters are usually constructed as a residual generator. By comparing the value of residual evaluation function with the prescribed threshold, one could make a judgment whether a fault occurs or not. With the development of these theories and techniques for various FD system designs, one of the commonly adopted ways is to introduce an H_∞ performance index and to formulate an H_∞ filtering [20] to solve the issue of robustness.

In the past few decades, there exist extensive researches on FD related to MJSs and their applications in a wide range of industrial processes [21–26]. For a class of non-linear MJSs, Dong *et al*. [22] solved the problem of dissipativity-based asynchronous FD for Takagi–Sugeno (T–S) fuzzy MJSs with network data dropouts. In [24], the finite-frequency FD filter (FDF) design problem based on derandomisation for MJSs is proposed. In [26], the research was concerned with the problem of partially mode-dependent $l_2 - l_\infty$ filtering for discrete-time non-homogeneous MJSs with repeated scalar non-linearities. However, to the best of our knowledge, the problem of H_∞ descriptor FDF design for conic-type non-linear MJSs with jump fault signals has not yet been fully investigated in the open literature and the research remains significant and challenging, which further motivates the present work.

Inspired by above research motivations, this paper focuses on the FDF design problem for a class of discrete-time conic-type non-linear MJSs with jump fault signals. Firstly, to linearise the non-linear model. In [27], a T–S fuzzy model is adopted to reconstruct the non-linear systems. Different from the general non-linearities, the conic-type non-linearities satisfy a restrictive condition that lies in an n-dimensional hyper-sphere with an uncertain centre. In order to deal with the predefined conic-type non-linear term, we introduce additional inequality. Based on the relevant matrix inequality transformations, the conic-type non-linear term can be expressed as a linear representation. Secondly, FDF working as the residual generator will be proposed, and further, its design will be cast into a stochastic H_∞ filtering problem. Fourthly, the proposed FDF gains are obtained by solving a set of linear matrix inequalities (LMIs). Finally, to demonstrate the feasibility and effectiveness of the proposed method, a simulation example is included.

Throughout this paper, all matrices are assumed to have compatible dimensions and all notations are quite standard. The implication of the symbols are given in the Nomenclature section.

2 System description and preliminaries

Consider the following discrete-time conic-type non-linear MJSs with jump fault signals:

$$\begin{cases} x_{k+1} = f(x_k, d_k, \bar{f}_k) + B(\theta_k)u_k \\ y_k = C(\theta_k)x_k + D(\theta_k)d_k + H(\theta_k)\bar{f}_k \\ x_k = x_0, \theta_k = \theta_0, k = 0, \end{cases} \tag{1}$$

where $x_k \in \mathfrak{R}^n$ is the state, $d_k \in \mathfrak{R}^r$ is the external disturbance, $u_k \in \mathfrak{R}^p$ is the controlled input and $y_k \in \mathfrak{R}^q$ is the measurement output. u_k and d_k belong to $L_2[0, \infty)$. x_0 and θ_0 are, respectively, the initial state and mode. $B(\theta_k)$, $C(\theta_k)$, $D(\theta_k)$ and $H(\theta_k)$ are the known real matrices with proper dimensions. The stochastic variable θ_k stands for a discrete Markov chain which takes value in the finite set $\mathbf{U} = \{1, 2, 3, \ldots, U\}$ with transition probability matrix $\Pi = \{\pi_{ij}\} \in \mathfrak{R}^{U \times U}$ given by

$$\Pr\{\theta_{k+1} = j | \theta_k = i\} = \pi_{ij} \tag{2}$$

where $0 \le \pi_{ij} \le 1, \forall i, j \in \mathbf{U}$ and $\sum_{j=1}^{U} \pi ij = 1, \forall i \in \mathbf{U}$.

$\bar{f}_k$ is a random jump fault signal described by

$$\bar{f}_k = \alpha_k f_k \tag{3}$$

where α_k is a random jump signal which composes of two values, i.e. 0 and 1, and f_k is a deterministic fault signal.

Remark 1: By investigating the existing references about FD as many as possible, it is found that the fault signal f_k is deterministic. In fact, its fault is always added in a system. When the underlying system is not deterministic but with jump processes, the problem of FD [28] for the jump fault signal $\bar{f}_k$ was considered.

Remark 2: f_k is a fixed fault signal in a specific interval. Combining with the deterministic fault signal f_k and the jump value α_k, a jump fault signal is designed in (3). Particularly, $\bar{f}_k$ exists in system (1) when $\alpha_k = 1$. On the contrary, there is no fault in system (1) with $\alpha_k = 0$. Equation (3) represents that $\bar{f}_k$ is a jump fault signal in a specific interval. In order to describe such random jump fault signal, a discrete Markov chain with two modes is considered in this paper. Then, the transition probability matrix is assumed to be

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} 1 - \mathscr{X} & \mathscr{X} \\ \mathscr{Y} & 1 - \mathscr{Y} \end{bmatrix} \tag{4}$$

where $\mathscr{X}$ and $\mathscr{Y}$ are independent conditional probabilities which satisfy $\mathscr{X} = \Pr\{\theta_{k+1} = 2 | \theta_k = 1\}$ and $\mathscr{Y} = \Pr\{\theta_{k+1} = 1 | \theta_k = 2\}$. The allowed range for these two parameters are [0, 1].

$f(x_k, d_k, \bar{f}_k)$ is an unknown non-linear function with the following conic-type sector description:

$$\begin{aligned} &\| f(x_k, d_k, \bar{f}_k) - \big(A(\theta_k)x_k + E(\theta_k)d_k + F(\theta_k)\bar{f}_k\big) \| \\ &\le \| Gx_k + G_d d_k + L\bar{f}_k \| \end{aligned} \tag{5}$$

where $A(\theta_k)$, $E(\theta_k)$, $F(\theta_k)$, G, G_d and L are the known matrices with appropriate dimensions.

Letting $\xi_k = \| f(x_k, d_k, \bar{f}_k) - (A(\theta_k)x_k + E(\theta_k)d_k + F(\theta_k)\bar{f}_k) \|$, we have

$$\xi_k^{\mathrm{T}} \xi_k \le (Gx_k + G_d d_k + L\bar{f}_k)^{\mathrm{T}}(Gx_k + G_d d_k + L\bar{f}_k) \tag{6}$$

Remark 3: In general, many non-linearities fall within the scope of the conic-type function $f(x_k, d_k, \bar{f}_k)$ in (1). Without d_k, $\bar{f}_k$, the centre of hyper-sphere is set as the origin of n-dimensional space. Then the inequality in (5) will reduce to $\| f(x_k) \| \le \| G(\theta_k)x_k \|$. In this case, the non-linear function $f(x_k)$ satisfies the global Lipschitz conditions.

Then, the discrete-time conic-type non-linear MJSs (1) can be rewritten as:

$$\begin{cases} x_{k+1} = A(\theta_k)x_k + B(\theta_k)u_k + E(\theta_k)d_k + F(\theta_k)\bar{f}_k + \xi_k \\ y_k = C(\theta_k)x_k + D(\theta_k)d_k + H(\theta_k)\bar{f}_k \\ x_k = x_0, \theta_k = \theta_0, k = 0. \end{cases} \tag{7}$$

In order to detect the fault for system (1), we propose the following FDF:

$$\begin{cases} x_f(k+1) = A_f(\theta_k)x_{fk} + B_f(\theta_k)y_k \\ r_{fk} = C_f(\theta_k)x_{fk} \end{cases} \tag{8}$$

where $x_{fk} \in \mathfrak{R}^{\mathbf{n}}$ is the filter state and $r_{fk} \in \mathfrak{R}^{\mathbf{f}}$ is the filter output. $A_f(\theta_k)$, $B_f(\theta_k)$ and $C_f(\theta_k)$ are the filter parameters to be determined. According to the references in [29, 30], we introduce the residual error $r_k = y_k - r_{fk}$ to enhance the sensitivity of faults. Therefore, we can get the following augmented system:

$$\begin{cases} \tilde{x}_{k+1} = \tilde{A}(\theta_k)\tilde{x}_k + \tilde{B}(\theta_k)\omega_k + \bar{\xi}_k \\ r_k = \tilde{C}(\theta_k)\tilde{x}_k + \tilde{D}(\theta_k)\omega_k \end{cases} \tag{9}$$

where $\tilde{x}_k = \mathrm{col}\big[x_k \quad x_{fk}\big]$, $\omega_k = \mathrm{col}\big[u_k \quad d_k \quad \bar{f}_k\big]$

$$\tilde{A}(\theta_k) = \begin{bmatrix} A(\theta_k) & 0 \\ B_f(\theta_k)C(\theta_k) & A_f(\theta_k) \end{bmatrix},$$

$$\tilde{B}(\theta_k) = \begin{bmatrix} B(\theta_k) & E(\theta_k) & F(\theta_k) \\ 0 & B_f(\theta_k)D(\theta_k) & B_f(\theta_k)H(\theta_k) \end{bmatrix},$$

$$\tilde{C}(\theta_k) = \big[C(\theta_k) \quad -C_f(\theta_k)\big],$$

$$\tilde{D}(\theta_k) = \big[0 \quad D(\theta_k) \quad H(\theta_k)\big],$$

$$\bar{\xi}_k = \begin{bmatrix} \xi_k \\ 0 \end{bmatrix}.$$

Considering inequality (6), we have

$$\bar{\xi}_k^{\mathrm{T}} \bar{\xi}_k = \xi_k^{\mathrm{T}} \xi_k \le (\bar{G}\tilde{x}_k + \bar{L}\omega_k)^{\mathrm{T}}(\bar{G}\tilde{x}_k + \bar{L}\omega_k) \tag{10}$$

where $\bar{G} = [G \quad 0]$, $\bar{L} = [0 \quad G_d \quad L]$.

For convenience, we denote the matrices associated with $\theta_k = i \in \mathbf{U}$ as:

$A(\theta_k) = A_i$, $B(\theta_k) = B_i$, $C(\theta_k) = C_i$, $D(\theta_k) = D_i$, $E(\theta_k) = E_i$, $F(\theta_k) = F_i$, $H(\theta_k) = H_i$, $A_f(\theta_k) = A_{fi}$, $B_f(\theta_k) = B_{fi}$, $C_f(\theta_k) = C_{fi}$, $\tilde{A}(\theta_k) = \tilde{A}_i$, $\tilde{B}(\theta_k) = \tilde{B}_i$, $\tilde{C}(\theta_k) = \tilde{C}_i$, $\tilde{D}(\theta_k) = \tilde{D}_i$, $\mathscr{P}(\theta_k) = \mathscr{P}_i$, $\mathscr{P}(\theta_{k+1}) = \bar{\mathscr{P}}_i$.

The next step for FD is to determine the residual evaluation function and the detection threshold. In this paper, we select the residual evaluation function $J_L(r)$ and the threshold J_{th} as:

$$J_L(r) = \sqrt{\sum_{k=k_0}^{k=k_0+L} r_k^{\mathrm{T}} r_k} \tag{11}$$

$$J_{\text{th}} = \sup_{d_k \in L_2, u_k \in L_{2, f_k = 0}} \mathscr{E}\{J_L(r)\} \tag{12}$$

where k_0 is the initial evaluation time instant. L denotes the evaluation time step. Then, the random fault $\bar{f}_k$ can be detected by comparing $J_L(r)$ and J_{th} according to the following test:

$$J_L(r) > J_{\text{th}} \rightarrow \text{alarm of fault} \tag{13}$$

$$J_L(r) \le J_{\text{th}} \rightarrow \text{no fault} \tag{14}$$

To proceed with the study, we give the following lemmas and definitions which can be seen in [31, 32, 33].

Lemma 1: For given appropriate dimension matrices A and B, there exist a positive-definite matrix $\mathscr{P}$ such that the following matrix inequality is satisfied:

$$A^{\mathrm{T}}B + B^{\mathrm{T}}A \le A^{\mathrm{T}}\mathscr{P}A + B^{\mathrm{T}}\mathscr{P}^{-1}B \tag{15}$$

Definition 1: The augmented system (9) is stochastically stable if for $\omega_k = 0, k > 0$, we have

$$\mathscr{E}\left\{\sum_{k=0}^{\infty} \| \tilde{x}_k \|^2 | \tilde{x}_0, \theta_0\right\} < \infty \tag{16}$$

where $\tilde{x}_0 \in \mathfrak{R}^{\mathbf{n}}$, $\theta_0 \in S$.

Definition 2: Given a positive scalar γ, the augmented system (9) is stochastically stable and satisfies the given H_∞ performance index if it is stochastically stable and the following condition under zero initial condition holds for all non-zero $\omega_k \in L_2[0, \infty)$:

$$\mathscr{E}\left\{\sum_{k=0}^{\infty} \| r_k \|^2 | \tilde{x}_0, \theta_0\right\} - \gamma^2 \mathscr{E}\left\{\sum_{k=0}^{\infty} \| \omega_k \|^2\right\} < 0 \tag{17}$$

3 Main results

In this section, the stability analysis and the H_∞ performance index against external disturbance are given.

Theorem 1: Given positive scalars γ, a and b with $a - b < 0$, the augmented system (9) is stochastically stable and satisfies the given H_∞ performance index, if there exist a positive-definite matrix $\mathscr{P}_i$ such that

$$\begin{bmatrix} -\mathscr{P}_i & 0 & 3\tilde{A}_i^{\mathrm{T}} & 0 & \tilde{C}_i^{\mathrm{T}} & b\bar{G}^{\mathrm{T}} \\ * & -\gamma^2 I & 0 & 3\tilde{B}_i^{\mathrm{T}} & \tilde{D}_i^{\mathrm{T}} & b\bar{L}^{\mathrm{T}} \\ * & * & -3\bar{\mathscr{P}}_i^{-1} & 0 & 0 & 0 \\ * & * & * & -3\bar{\mathscr{P}}_i^{-1} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\frac{b}{2}I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} -\mathscr{P}_i & 2\tilde{A}_i^{\mathrm{T}} & 2\bar{G}^{\mathrm{T}} \\ * & -2\bar{\mathscr{P}}_i^{-1} & 0 \\ * & * & -2\bar{\mathscr{P}}_i^{-1} \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} -\bar{\mathscr{P}}_i & \bar{\mathscr{P}}_i \\ * & -\frac{a}{3}I \end{bmatrix} < 0 \tag{20}$$

where $\bar{\mathscr{P}}_i = \sum_{j \in \mathbf{U}} \pi_{ij} \mathscr{P}_j$.

Proof: First, we prove the stochastic stability of the augmented system (9) with $\omega_k = 0$. Selecting a stochastic Lyapunov functional as

$$V(\tilde{x}_k, \theta_k) = \tilde{x}_k^{\mathrm{T}} \mathscr{P}(\theta_k) \tilde{x}_k \tag{21}$$

For every $\theta_k = i, i \in S$, we obtain the difference of (21) as

$$\begin{aligned} \Delta V(\tilde{x}_k, \theta_k) &= \mathscr{E}\{V(\tilde{x}_{k+1}, \theta_{k+1})\} - V(\tilde{x}_k, \theta_k) \\ &= \tilde{x}_{k+1}^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{x}_{k+1} - \tilde{x}_k^{\mathrm{T}} \mathscr{P}_i \tilde{x}_k \\ &= \left(\tilde{A}_i \tilde{x}_k + \bar{\xi}_k\right)^{\mathrm{T}} \bar{\mathscr{P}}_i \left(\tilde{A}_i \tilde{x}_k + \bar{\xi}_k\right) \\ &\quad - \tilde{x}_k^{\mathrm{T}} \mathscr{P}_i \tilde{x}_k \\ &= \tilde{x}_k^{\mathrm{T}} \tilde{A}^{\mathrm{T}}{}_i \bar{\mathscr{P}}_i \tilde{A}_i \tilde{x}_k + \tilde{x}_k^{\mathrm{T}} \tilde{A}^{\mathrm{T}}{}_i \bar{\mathscr{P}}_i \bar{\xi}_k \\ &\quad + \bar{\xi}_k^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{A}_i \tilde{x}_k + \bar{\xi}_k^{\mathrm{T}} \bar{\mathscr{P}}_i \bar{\xi}_k \\ &\quad - \tilde{x}_k^{\mathrm{T}} \mathscr{P}_i \tilde{x}_k < 0 \end{aligned} \tag{22}$$

Considering Lemma 1, we have

$$\tilde{x}_k^{\mathrm{T}} \tilde{A}_i^{\mathrm{T}} \bar{\mathscr{P}}_i \bar{\xi}_k + \bar{\xi}_k^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{A}_i \tilde{x}_k \le \tilde{x}_k^{\mathrm{T}} \tilde{A}_i^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{A}_i \tilde{x}_k + \bar{\xi}_k^{\mathrm{T}} \bar{\mathscr{P}}_i \bar{\xi}_k \tag{23}$$

Then, we can get

$$\begin{aligned} \Delta V(\tilde{x}_k, \theta_k) &\le 2\tilde{x}_k^{\mathrm{T}} \tilde{A}_i^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{A}_i \tilde{x}_k + 2\bar{\xi}_k^{\mathrm{T}} \bar{\mathscr{P}}_i \bar{\xi}_k - \tilde{x}_k^{\mathrm{T}} \mathscr{P}_i \tilde{x}_k \\ &= \tilde{x}_k^{\mathrm{T}} \left(2\tilde{A}_i^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{A}_i + 2\bar{G}^{\mathrm{T}} \bar{\mathscr{P}}_i \bar{G} - \mathscr{P}_i\right) \tilde{x}_k \\ &< 0 \end{aligned} \tag{24}$$

That is

$$2\tilde{A}_i^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{A}_i + 2\bar{G}^{\mathrm{T}} \bar{\mathscr{P}}_i \bar{G} - \mathscr{P}_i < 0 \tag{25}$$

which is implied by condition (19). Based on condition (22), we have

$$\mathscr{E}\{V(\tilde{x}_{k+1}, \theta_{k+1}) | x_k, \theta_k\} \le V(\tilde{x}_k, \theta_k) - \vartheta \| \tilde{x}_k \|^2 \tag{26}$$

where $\vartheta = \min_{i \in S} \{\lambda_{\min}(-\wp_i)\}$ and $\wp_i = 2\tilde{A}_i^{\mathrm{T}} \bar{\mathscr{P}}_i \tilde{A}_i + 2\bar{G}^{\mathrm{T}} \bar{\mathscr{P}}_i \bar{G} - \mathscr{P}_i$. Taking the expectation on both sides of (26) and continuing the iterative procedure as $k \rightarrow \infty$, we can get

$$\begin{aligned} &\mathscr{E}\{V(\tilde{x}_\infty, \theta_\infty) | x_0, \theta_0\} \\ &\le V(\tilde{x}_0, \theta_0) - \vartheta \sum_{k=0}^{\infty} \mathscr{E}\{\| \tilde{x}_k \|^2 | x_0, \theta_0\} \end{aligned} \tag{27}$$

which implies

$$\begin{aligned} \mathscr{E}\left\{\sum_{k=0}^{\infty} \| \tilde{x}_k \|^2 | x_0, \theta_0\right\} &\le \sum_{k=0}^{\infty} \mathscr{E}\{\| \tilde{x}_k \|^2 | x_0, \theta_0\} \\ &\le \frac{1}{\vartheta} V(x_0, \theta_0) < \infty \end{aligned} \tag{28}$$

Thus, the augmented system (9) is stochastically stable.

Then, we introduce the following auxiliary function to prove the augment system (9) with an H_∞ performance index:

$$J = \mathscr{E}\sum_{k=0}^{T-1}\{r_k^{\mathrm{T}} r_k - \gamma^2 \omega_k^{\mathrm{T}}\omega_k\} < 0 \tag{29}$$

where T is an arbitrary positive integer. For any non-zero $\omega_k \in L_2[0, \infty)$ and under zero initial condition $\tilde{x}_0 = 0$, we have

$$\begin{aligned} J &= \mathscr{E}\sum_{k=0}^{T-1}\{r_k^{\mathrm{T}} r_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k + \Delta V(\tilde{x}_k)\} - V(T) \\ &\le \sum_{k=0}^{T-1}\mathscr{E}\{r_k^{\mathrm{T}} r_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k + \Delta V(\tilde{x}_k)\} \end{aligned} \tag{30}$$

Then, we obtain

$$\begin{aligned} &\Delta V(\tilde{x}_k, \theta_k) \\ &= \mathscr{E}\{V(\tilde{x}_{k+1}, \theta_{k+1})\} - V(\tilde{x}_k, \theta_k) \\ &= \tilde{x}_{k+1}^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{x}_{k+1} - \tilde{x}_k^{\mathrm{T}}\mathscr{P}_i\tilde{x}_k \\ &= \left(\tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i + \omega_k^{\mathrm{T}}\tilde{B}^{\mathrm{T}}{}_i + \bar{\xi}_k{}^{\mathrm{T}}\right)\bar{\mathscr{P}}_i\left(\tilde{A}_i\tilde{x}_k + \tilde{B}_i\omega_k + \bar{\xi}_k\right) \\ &\quad -\tilde{x}_k^{\mathrm{T}}\mathscr{P}_i\tilde{x}_k \\ &= \tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k + \tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i\bar{\mathscr{P}}_i\tilde{B}_i\omega_k + \tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i\bar{\mathscr{P}}_i\bar{\xi}_k \\ &\quad +\omega_k^{\mathrm{T}}\tilde{B}^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k + \omega_k^{\mathrm{T}}\tilde{B}^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i\omega_k + \omega_k^{\mathrm{T}}\tilde{B}_i{}^{\mathrm{T}}\bar{\mathscr{P}}_i\bar{\xi}_k \\ &\quad +\bar{\xi}_k{}^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k + \bar{\xi}_k{}^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i\omega_k + \bar{\xi}_k{}^{\mathrm{T}}\bar{\mathscr{P}}_i\bar{\xi}_k - \tilde{x}_k^{\mathrm{T}}\mathscr{P}_i\tilde{x}_k \end{aligned} \tag{31}$$

Recalling to Lemma 1, it yields:

$$\begin{aligned} &\tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i\bar{\mathscr{P}}_i\tilde{B}_i\omega_k + \omega_k^{\mathrm{T}}\tilde{B}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k \\ &\le \tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k + \omega_k^{\mathrm{T}}\tilde{B}^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i\omega_k \end{aligned} \tag{32}$$

$$\tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i\bar{\mathscr{P}}_i\bar{\xi}_k + \bar{\xi}_k^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k \le \tilde{x}_k^{\mathrm{T}}\tilde{A}^{\mathrm{T}}{}_i\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k + \bar{\xi}_k^{\mathrm{T}}\bar{\mathscr{P}}_i\bar{\xi}_k \tag{33}$$

$$\omega_k^{\mathrm{T}}\tilde{B}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\bar{\xi}_k + \bar{\xi}_k^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i\omega_k \le \omega_k^{\mathrm{T}}\tilde{B}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i\omega_k + \bar{\xi}_k^{\mathrm{T}}\bar{\mathscr{P}}_i\bar{\xi}_k \tag{34}$$

Then it derives

$$\begin{aligned} J \le \sum_{k=0}^{T-1}\Big\{&\tilde{x}_k^{\mathrm{T}}\left(\tilde{C}_i^{\mathrm{T}}\tilde{C}_i + 3\tilde{A}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{A}_i - \mathscr{P}_i\right)\tilde{x}_k \\ &+\omega_k^{\mathrm{T}}\left(\tilde{D}_i^{\mathrm{T}}\tilde{D}_i + 3\tilde{B}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i - \gamma^2 I\right)\omega_k \\ &+\tilde{x}_k^{\mathrm{T}}\tilde{C}_i^{\mathrm{T}}\tilde{D}_i\omega_k + \omega_k^{\mathrm{T}}\tilde{D}_i^{\mathrm{T}}\tilde{C}_i\tilde{x}_k + 3\bar{\xi}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\bar{\xi}_i\Big\}. \end{aligned} \tag{35}$$

Using the Schur complement, inequality (35) can be rewritten as

$$J \le \bar{J} = \begin{bmatrix} \aleph - 3\bar{\xi}_k^{\mathrm{T}}\bar{\mathscr{P}}_i \\ * - 3\bar{\mathscr{P}}_i \end{bmatrix} \tag{36}$$

where

$$\begin{aligned} \aleph = &3\tilde{x}_k^{\mathrm{T}}\tilde{A}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{A}_i\tilde{x}_k + \tilde{x}_k^{\mathrm{T}}\tilde{C}_i^{\mathrm{T}}\tilde{C}_i\tilde{x}_k - \tilde{x}_k^{\mathrm{T}}\mathscr{P}_i\tilde{x}_k \\ &+\omega_k^{\mathrm{T}}\tilde{D}_i^{\mathrm{T}}\tilde{D}_i\omega_k + 3\omega_k^{\mathrm{T}}\tilde{B}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i\omega_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k \\ &+\tilde{x}_k^{\mathrm{T}}\tilde{C}_i^{\mathrm{T}}\tilde{D}_i\omega_k + \omega_k^{\mathrm{T}}\tilde{D}_i^{\mathrm{T}}\tilde{C}_i\tilde{x}_k \end{aligned}$$

.

For positive constants a and b with $a - b < 0$, the following formula is established:

$$-2b\bar{\xi}_k^{\mathrm{T}}\bar{\xi}_k + a\bar{\xi}_k^{\mathrm{T}}\bar{\xi}_k < 0 \tag{37}$$

Applying the Schur complement, we can get

$$\begin{bmatrix} -2b\bar{\xi}_k^{\mathrm{T}}\bar{\xi}_k & 0 \\ * & -9a^{-1}\bar{\mathscr{P}}_i{}^2 \end{bmatrix} < \begin{bmatrix} 0 & 3\bar{\xi}_k^{\mathrm{T}}\bar{\mathscr{P}}_i \\ * & 0 \end{bmatrix} \tag{38}$$

Recalling the inequality (36), we have $\bar{J} < 0$, that is

$$\begin{bmatrix} \aleph & 0 \\ * & -3\bar{\mathscr{P}}_i \end{bmatrix} < \begin{bmatrix} 0 & 3\bar{\xi}_k^{\mathrm{T}}\bar{\mathscr{P}}_i \\ * & 0 \end{bmatrix} \tag{39}$$

which is guaranteed by

$$\begin{bmatrix} \aleph & 0 \\ * & -3\bar{\mathscr{P}}_i \end{bmatrix} < \begin{bmatrix} -2b\bar{\xi}_k^{\mathrm{T}}\bar{\xi}_k & 0 \\ * & -9a^{-1}\bar{\mathscr{P}}_i{}^2 \end{bmatrix} \tag{40}$$

Then we have

$$J < \begin{bmatrix} \sum_1 & 0 \\ * & \sum_2 \end{bmatrix} \tag{41}$$

It derives from $J < 0$ that

$$\sum\nolimits_1 = \aleph + 2b\bar{\xi}_k^{\mathrm{T}}\bar{\xi}_k < 0 \tag{42}$$

$$\sum\nolimits_2 = -3\bar{P}_i + 9a^{-1}\bar{\mathscr{P}}_i^2 < 0 \tag{43}$$

Applying the Schur complement, we can obtain inequality (20) from inequality (43). According to inequality (42), we have

$$\hbar_k^{\mathrm{T}}\varpi\hbar_k < 0 \tag{44}$$

where $\hbar_k = \mathrm{col}\left[\tilde{x}_k \quad \omega_k\right]$

$$\varpi = \begin{bmatrix} -\mathscr{P}_i + 3\tilde{A}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{A}_i + \tilde{C}_i^{\mathrm{T}}\tilde{C}_i + 2b\bar{G}^{\mathrm{T}}\bar{G} & \tilde{C}_i^{\mathrm{T}}\tilde{D}_i + 2b\bar{G}^{\mathrm{T}}\bar{L} \\ * & -\gamma^2 I + 3\tilde{B}_i^{\mathrm{T}}\bar{\mathscr{P}}_i\tilde{B}_i + \tilde{D}_i^{\mathrm{T}}\tilde{D}_i + 2b\bar{L}^{\mathrm{T}}\bar{L} \end{bmatrix}$$

Then we have

$$\begin{aligned} &\begin{bmatrix} -\mathscr{P}_i & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\ &-\begin{bmatrix} 3\tilde{A}_i^{\mathrm{T}} \\ 0 \end{bmatrix}(-3\bar{\mathscr{P}}_i{}^{-1})^{-1}\begin{bmatrix} 3\tilde{A}_i^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}} \\ &-\begin{bmatrix} 0 \\ 3\tilde{B}_i^{\mathrm{T}} \end{bmatrix}(-3\bar{\mathscr{P}}_i{}^{-1})^{-1}\begin{bmatrix} 0 \\ 3\tilde{B}_i^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ &-\begin{bmatrix} \tilde{C}_i^{\mathrm{T}} \\ \tilde{D}_i^{\mathrm{T}} \end{bmatrix}(-I)\begin{bmatrix} \tilde{C}_i^{\mathrm{T}} \\ \tilde{D}_i^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ &-\begin{bmatrix} b\bar{G}^{\mathrm{T}} \\ b\bar{L}^{\mathrm{T}} \end{bmatrix}(-\frac{b}{2}I)^{-1}\begin{bmatrix} b\bar{G}^{\mathrm{T}} \\ b\bar{L}^{\mathrm{T}} \end{bmatrix} < 0 \end{aligned} \tag{45}$$

which is equivalent to condition (18) by utilising the Schur complement. Letting $T \to \infty$, we know that condition (29) is still satisfied. This completes the proof. □

Remark 4: In the proof, the Schur complement and the relevant matrix inequality transformations are employed to solve the conic-type linearisation problem in inequality (35). Based on inequality (37), $J < 0$ can be guaranteed by inequality (40). Combing with inequalities (10) and (42), the conic-type non-linear term ξ_k can be expressed as a linear representation. In addition, the stochastic stability of the augmented system (9) and the H_∞ attenuation performance from r_k to ω_k are derived by the LMI method.

Theorem 2: Given a positive scalar γ, the augmented system (9) is stochastically stable and satisfies the given H_∞ performance index if there exist positive-definite symmetric matrices $\mathscr{P}_{i1}$, $\mathscr{P}_{i2}$, $\mathscr{P}_{i3}$, X_{i1}, X_{i2}, Y, matrices $\bar{A}_{fi}$, $\bar{B}_{fi}$ and $\bar{C}_{fi}$ such that the following LMIs hold:

$$\begin{bmatrix} -\mathscr{P}_i & 0 & 3\varphi_i & 0 & \Pi_i & b\bar{G}^{\rm T} \\ * & -\gamma^2 I & 0 & 3\bar{\varphi}_i & \bar{\Pi}_i & b\bar{L}^{\rm T} \\ * & * & -3\Gamma_i & 0 & 0 & 0 \\ * & * & * & -3\Gamma_i & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\frac{b}{2}I \end{bmatrix} < 0 \qquad (46)$$

$$\begin{bmatrix} -\pi_{i1}\mathscr{P}_{11} - \pi_{i2}\mathscr{P}_{21} & -\pi_{i1}\mathscr{P}_{12} - \pi_{i2}\mathscr{P}_{22} & \pi_{i1}\mathscr{P}_{11} + \pi_{i2}\mathscr{P}_{21} & \pi_{i1}\mathscr{P}_{12} + \pi_{i2}\mathscr{P}_{22} \\ * & -\pi_{i1}\mathscr{P}_{13} - \pi_{i2}\mathscr{P}_{23} & \pi_{i1}\mathscr{P}_{12}^{\rm T} + \pi_{i2}\mathscr{P}_{22}^{\rm T} & \pi_{i1}\mathscr{P}_{13} + \pi_{i2}\mathscr{P}_{23} \\ * & * & -\frac{a}{3}I & 0 \\ * & * & * & -\frac{a}{3}I \end{bmatrix} < 0 \qquad (47)$$

where

$$\mathscr{P}_i = \begin{bmatrix} \mathscr{P}_{i1} & \mathscr{P}_{i2} \\ * & \mathscr{P}_{i3} \end{bmatrix}, \quad \varphi_i = \begin{bmatrix} \varphi_{i1} & \varphi_{i2} \\ \bar{A}_{fi}^{\rm T} & \bar{A}_{fi}^{\rm T} \end{bmatrix},$$

$$\bar{\varphi}_i = \begin{bmatrix} B^{\rm T}X_{i1}^{\rm T} & B^{\rm T}X_{i2}^{\rm T} \\ \bar{\varphi}_{i1} & \bar{\varphi}_{i2} \\ \bar{\varphi}_{i3} & \bar{\varphi}_{i4} \end{bmatrix}, \quad \Pi_i = \begin{bmatrix} C_i^{\rm T} \\ -\bar{C}_{fi}^{\rm T} \end{bmatrix},$$

$$\bar{\Pi}_i = \begin{bmatrix} 0 \\ D_i^{\rm T} \\ H_i^{\rm T} \end{bmatrix}, \quad \Gamma_i = \begin{bmatrix} \Gamma_{i1} & \Gamma_i 2 \\ * & \Gamma_{i3} \end{bmatrix},$$

$$\varphi_{i1} = A_i^{\rm T}X_{i1}^{\rm T} + C_i^{\rm T}\bar{B}_{fi}^{\rm T}, \varphi_{i2} = A_i^{\rm T}X_{i2}^{\rm T} + C_i^{\rm T}\bar{B}_{fi}^{\rm T},$$

$$\bar{\varphi}_{i1} = E_i^{\rm T}X_{i1}^{\rm T} + D_i^{\rm T}\bar{B}_{fi}^{\rm T}, \bar{\varphi}_{i2} = E_i^{\rm T}X_{i2}^{\rm T} + D_i^{\rm T}\bar{B}_{fi}^{\rm T},$$

$$\bar{\varphi}_{i3} = F_i^{\rm T}X_{i1}^{\rm T} + H_i^{\rm T}\bar{B}_{fi}^{\rm T}, \bar{\varphi}_{i4} = F_i^{\rm T}X_{i2}^{\rm T} + H_i^{\rm T}\bar{B}_{fi}^{\rm T},$$

$$\Gamma_{i1} = (X_{i1})^* - \sum_{j \in S} \pi_{ij}\mathscr{P}_{j1},$$

$$\Gamma_{i2} = \left(Y_i + X_{i2}^{\rm T}\right) - \sum_{j \in S} \pi_{ij}\mathscr{P}_{j2},$$

$$\Gamma_{i3} = (Y_i)^* - \sum_{j \in S} \pi_{ij}\mathscr{P}_{j3}.$$

Moreover, the FDF parameters are given by

$$A_{fi} = Y^{-1}\bar{A}_{fi}, B_{fi} = Y^{-1}\bar{B}_{fi}, C_{fi} = \bar{C}_{fi}. \qquad (48)$$

Proof: On the basic of Theorem 1, pre- and post-multiplying inequality (18) with matrix

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ * & I & 0 & 0 & 0 & 0 \\ * & * & \Psi_i & 0 & 0 & 0 \\ * & * & * & \Psi_i & 0 & 0 \\ * & * & * & * & I & 0 \\ * & * & * & * & * & I \end{bmatrix} \qquad (49)$$

and its its transpose, respectively, we get

$$\begin{bmatrix} -\mathscr{P}_i & 0 & 3\tilde{A}_i^{\rm T}\Psi_i^{\rm T} & 0 & \tilde{C}_i^{\rm T} & b\bar{G}^{\rm T} \\ * & -\gamma^2 I & 0 & 3\tilde{B}_i^{\rm T}\Psi_i^{\rm T} & \tilde{D}_i^{\rm T} & b\bar{L}^{\rm T} \\ * & * & -3\Psi_i\bar{\mathscr{P}}_i^{-1}\Psi_i^{\rm T} & 0 & 0 & 0 \\ * & * & * & -3\Psi_i\bar{\mathscr{P}}_i^{-1}\Psi_i^{\rm T} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\frac{b}{2}I \end{bmatrix} < 0 \qquad (50)$$

where Ψ_i is non-singular.

Noting $(\bar{\mathscr{P}}_i - \Psi_i)\bar{\mathscr{P}}_i^{-1}(\bar{\mathscr{P}}_i - \Psi_i)^{\rm T} \geq 0$, we have

$$-\Psi_i\bar{\mathscr{P}}_i^{-1\Psi_i}\Psi_i^{\rm T} \leq -\Psi_i - \Psi_i^{\rm T} + \bar{\mathscr{P}}_i = -(\Psi_i)^* + \bar{\mathscr{P}}_i \qquad (51)$$

Since inequality (51) holds, inequality (50) is guaranteed by

$$\begin{bmatrix} -\mathscr{P}_i & 0 & 3\tilde{A}_i^{\rm T}\Psi_i^{\rm T} & 0 & \tilde{C}_i^{\rm T} & b\bar{G}^{\rm T} \\ * & -\gamma^2 I & 0 & 3\tilde{B}_i^{\rm T}\Psi_i^{\rm T} & \tilde{D}_i^{\rm T} & b\bar{L}^{\rm T} \\ * & * & -3(\Psi_i)^* + 3\bar{\mathscr{P}}_i & 0 & 0 & 0 \\ * & * & * & -3(\Psi_i)^* + 3\bar{\mathscr{P}}_i & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\frac{b}{2}I \end{bmatrix} < 0 \qquad (52)$$

where Ψ_i and $\mathscr{P}_i$ are assumed to have the following forms:

$$\Psi_i = \begin{bmatrix} X_{i1} & Y \\ X_{i2} & Y \end{bmatrix}, \quad \mathscr{P}_i = \begin{bmatrix} \mathscr{P}_{i1} & \mathscr{P}_{i2} \\ * & \mathscr{P}_{i3} \end{bmatrix}. \qquad (53)$$

Substituting (53) into (52) and defining $\bar{A}_{fi} = YA_{fi}, \bar{B}_{fi} = YB_{fi}, \bar{C}_{fi} = C_{fi}$, we can get LMI (46) with relation (48). It is concluded that Ψ_i is non-singular according to LMI (46). Combining equality (4) with condition (20), we obtain LMI (47). This completes the proof. □

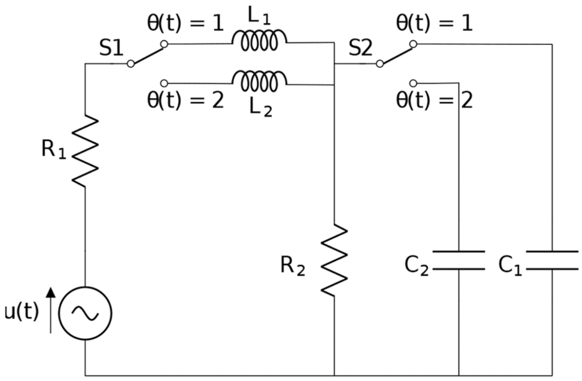

Fig. 1 *Switching R-L-C circuit*

Remark 5: The main results indicate that the stochastic stability of the corresponding augmented system (9) with a desired H_∞ disturbance attenuation level. In the derivation of LMI (46), some complex mathematical transformations are employed to simplify condition (18). At last, the FDF parameters are obtained by using the Matlab LMI tool.

4 Simulation experiments

Consider an $R-L-C$ circuit model with two switches S1 and S2 from [34], as depicted in Fig. 1. The switch $s(t)$ follows a Markov chain and switches at most once in each period T. L is the inductance, C is the capacitance, R is the load resistance, $u(t)$ is the source voltage, $V_c(t)$ represents the capacitor voltage and $i_L(t)$ represents the current through the inductance. Then, we can construct the following $R-L-C$ circuit switching model:

$$\begin{cases} \dfrac{\mathrm{d}V_c(t)}{\mathrm{d}t} = -m(\theta(t))[\dfrac{1}{R_2}V_c(t) + i_L(t)] \\ \dfrac{\mathrm{d}i_L(t)}{\mathrm{d}t} = -n(\theta(t))[-V_c(t) - R_1 i_L(t) + u(t)] \end{cases} \tag{54}$$

where

$$m(\theta(t)) = \begin{cases} \dfrac{1}{C_1} \text{ if } \theta(t) = 1 \\ \dfrac{1}{C_2} \text{ if } \theta(t) = 2 \end{cases}$$

$$n(\theta(t)) = \begin{cases} \dfrac{1}{L_1} \text{ if } \theta(t) = 1 \\ \dfrac{1}{L_2} \text{ if } \theta(t) = 2 \end{cases}$$

We can rewrite model (54) as

$$\dot{x}(t) = \begin{bmatrix} -\dfrac{m(\theta(t))}{R_2} & m(\theta(t)) \\ -n(\theta(t)) & -n(\theta(t))R_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ n(\theta(t)) \end{bmatrix} u(t) \tag{55}$$

where $x(t) = \mathrm{col}[V_c(t)\ i_L(t)]$

$$\mathscr{A}_1 = \begin{bmatrix} -\dfrac{1}{C_1 R_2} & \dfrac{1}{C_1} \\ -\dfrac{1}{L_1} & -\dfrac{R_1}{L_1} \end{bmatrix}, \quad \mathscr{A}_2 = \begin{bmatrix} -\dfrac{1}{C_2 R_2} & \dfrac{1}{C_2} \\ -\dfrac{1}{L_2} & -\dfrac{R_1}{L_2} \end{bmatrix},$$

$$\mathscr{B}_1 = \begin{bmatrix} 0 \\ \dfrac{1}{L_1} \end{bmatrix}, \quad \mathscr{B}_2 = \begin{bmatrix} 0 \\ \dfrac{1}{L_2} \end{bmatrix}.$$

Letting $C_1 = 1.6 \times 10^3\,\mu\mathrm{F}$, $C_2 = 1.0 \times 10^3\,\mu\mathrm{F}$, $L_1 = 0.1\,\mathrm{H}$, $L_2 = 0.1\,\mathrm{H}$, $R_1 = 100\,\Omega$, $R_2 = 1 \times 10^3\,\Omega$, we get

$$\mathscr{A}_1 = \begin{bmatrix} -0.625 & 625 \\ -10 & -1000 \end{bmatrix}, \quad \mathscr{A}_2 = \begin{bmatrix} -1 & 1000 \\ -10 & -1000 \end{bmatrix},$$

$$\mathscr{B}_1 = \mathscr{B}_2 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}.$$

Applying the discretisation method with the sampling period $T_s = 0.1$ s, we have:

$$A_1 = \begin{bmatrix} 0.5038 & 0.3171 \\ -0.0051 & -0.0032 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.4508 \\ 0.0055 \end{bmatrix}.$$

$$A_2 = \begin{bmatrix} 0.3326 & 0.3363 \\ -0.0034 & -0.0034 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.6065 \\ 0.0040 \end{bmatrix}.$$

The other parameters are selected as:

$$C_1 = C_2 = [0.2 \quad 0.1], \quad D_1 = D_2 = [-0.1],$$

$$E_1 = E_2 = [0.1 \quad 0.4], \quad F_1 = \mathrm{col}[0.3 \quad 0.3],$$

$$F_2 = \mathrm{col}[0.1 \quad 0.2], \quad G = [0.01 \quad -0.01],$$

$$G_d = [0.1], \quad H_1 = 2, \quad H_2 = 0.5, \quad L = 0.1.$$

We select the non-linear function as

$$\xi_k = \begin{bmatrix} -0.00059(|x_{k1} + 1| - |x_{k1} - 1|) \\ 0 \end{bmatrix},$$

where x_{kj} is the jth state variable, $j = 1, 2$.

Since the switch $s(t)$ is assumed to be driven by a Markov chain, it is known that $\theta_k \in \{1, 2\}$ is also a Markov chain. Without loss of generality, the transition probability matrix is assumed to be

$$\Pi = \begin{bmatrix} 0.4 & 0.6 \\ 0.25 & 0.75 \end{bmatrix}.$$

To show the simulation results, we set $\gamma = 2.1$, $a = 4$, $b = 5$. By solving LMIs (46) and (47), we can get the FDF parameters as:

$$A_{f1} = \begin{bmatrix} 0.4664 & 0.1522 \\ 0.0570 & -0.0659 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} -0.1606 \\ -0.1405 \end{bmatrix},$$

$$C_{f1} = [-0.1731 \ -0.1071];$$

$$A_{f2} = \begin{bmatrix} 0.3238 & 0.1606 \\ 0.0555 & -0.0577 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -0.6247 \\ -0.1325 \end{bmatrix},$$

$$C_{f2} = [-0.0048 \ -0.1113].$$

The unknown input u_k is given by a step signal with amplitude 1. The external disturbance input d_k shown in Fig. 2 is given by the white noise with power 0.01. The general fault signal is given by

$$f_k = \begin{cases} 1.5, & 10 \le k \le 19 \\ 0, & \text{otherwise} \end{cases}$$

The fault signal and the jump mode are shown in Fig. 3, with the general fault signal (a), the random value (b), the random fault signal (c) and the jump mode (d). Fig. 4 presents the generated residual signal r_k, while Fig. 5 shows the evaluation function of $J_L(r) = \mathscr{E}\{\sum_{l=0}^{k} r^{\mathrm{T}}(l)r(l)\}$ for both the fault case and the fault-free case. Based on the simulation result given in Fig. 5, we know that $J_{\mathrm{th}} = \sup_{f_k=0} \mathscr{E}\{\sum_{l=0}^{35} r^{\mathrm{T}}(l)r(l)\} = 0.24$, which shows $J_L(r) = \mathscr{E}\{\sum_{l=0}^{11} r^{\mathrm{T}}(l)r(l)\} = 0.25 > J_{\mathrm{th}}$. Thus, the random fault signal will be detected after one time step.

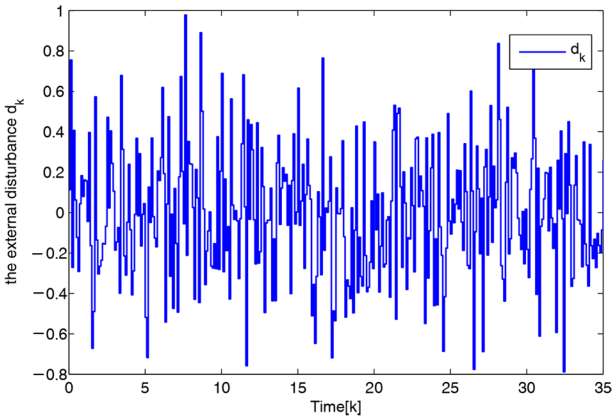

Fig. 2 *Trajectory of the disturbance signal d_k*

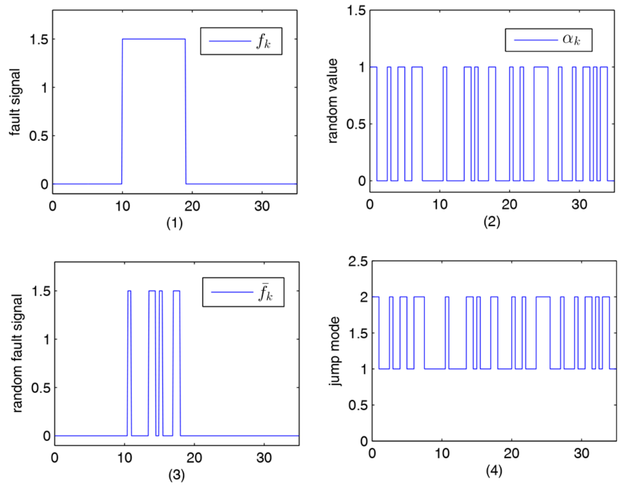

Fig. 3 *Trajectories of the fault signal and the jump mode*

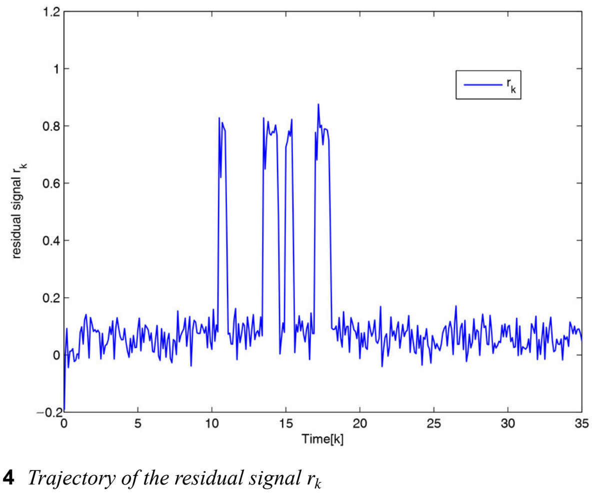

Fig. 4 *Trajectory of the residual signal r_k*

Remark 6: In [16], Wang *et al*. focused on the FD problem for a class of discrete-time linear delay MJSs. The utility and advantage of the established results showed that the fault could be detected in four time steps after its occurrence. Comparing with the existing results, we construct a more complex discrete-time conic-type non-linear MJSs with jump fault signals. From the simulation example in Fig. 5, the random fault signals can be detected in one step after its occurrence. Therefore, the residual can deliver fault alarms quickly when the fault occurs. It indicates that the designed FDF method of this paper can detect the jump fault for this system.

5 Conclusion

In this paper, the robust FDF design problem for a class of discrete-time conic-type non-linear MJSs is investigated. Based on the

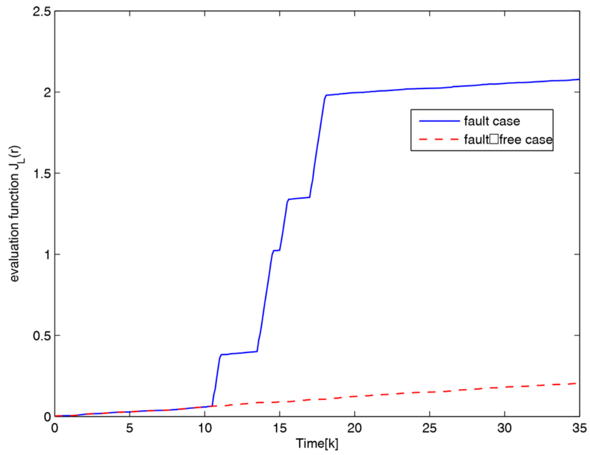

Fig. 5 *Trajectories of the evaluation function $J_L(r)$*

appropriate mode-dependent Lyapunov functional method and LMI techniques, sufficient conditions are established on the existence of FDF such that the augmented MJSs are stochastically stable and satisfy the given H_∞ disturbance attenuation index. A practical example is delivered to demonstrate the feasibility and validity of the main results. It should be noted that this paper introduces a linear inequality in the process of conic linearisation. We have solved the non-linear terms ξ_k by employing appropriate mathematical transformations. However, the amount of calculation for the corresponding LMI has increased significantly. In our future work, we will improve the method of conic linearisation to make it easier. Furthermore, some possible research topics will be extended to the singular non-linear MJSs [35], fuzzy singular MJSs [36] and network-based singular systems [37].

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