PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 109(123) (2021), 35–60 DOI: https://doi.org/10.2298/PIM2123035L

ON STRONGLY REGULAR GRAPHS WITH $m_2 = qm_3$ AND $m_3 = qm_2$ WHERE $q \in \mathbb{Q}$

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Abstract. We say that a regular graph G of order n and degree $r \geqslant 1$ (which is not the complete graph) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where S_k denotes the neighborhood of the vertex k. Let $\lambda_1 = r$, λ_2 and λ_3 be the distinct eigenvalues of a connected strongly regular graph. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3, respectively. We here describe the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{3}{2}, \frac{4}{3}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \frac{6}{5}$.

1. Introduction

Let G be a simple graph of order n with vertex set $V(G) = \{1, 2, \ldots, n\}$. The spectrum of G consists of the eigenvalues $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_n$ of its (0,1) adjacency matrix A and is denoted by $\sigma(G)$. We say that a regular graph G of order n and degree $r \geqslant 1$ (which is not the complete graph K_n) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where $S_k \subseteq V(G)$ denotes the neighborhood of the vertex k. We know that a regular connected graph G is strongly regular if and only if it has exactly three distinct eigenvalues [**1**] (see also [**3**]). Let $\lambda_1 = r$, λ_2 and λ_3 denote the distinct eigenvalues of a connected strongly regular graph G. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3. Further, let $\overline{r} = (n-1) - r$, $\overline{\lambda}_2 = -\lambda_3 - 1$ and $\overline{\lambda}_3 = -\lambda_2 - 1$ denote the distinct eigenvalues of the strongly regular graph $\overline{G}$, where $\overline{G}$ denotes the complement of G. Then $\overline{\tau} = n - 2r - 2 + \theta$ and $\overline{\theta} = n - 2r + \tau$ where $\overline{\tau} = \tau(\overline{G})$ and $\overline{\theta} = \theta(\overline{G})$.

Remark 1.1. (i) if G is a disconnected strongly regular graph of degree r then $G = mK_{r+1}$, where mH denotes the m-fold union of the graph H; (ii) G is a disconnected strongly regular graph if and only if $\theta = 0$.

2010 *Mathematics Subject Classification:* Primary 05C50.
Key words and phrases: strongly regular graph, conference graph, integral graph.
Communicated by Nebojša Ikodinović.

Remark 1.2. (i) a strongly regular graph G of order $n = 4k+1$ and degree $r = 2k$ with $\tau = k-1$ and $\theta = k$ is called a conference graph; (ii) a strongly regular graph is a conference graph if and only if $m_2 = m_3$ and (iii) if $m_2 \neq m_3$ then G is an integral[1] graph.

We have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive integer [**4**]. In the same work we have described the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2,3,4$. Besides, (i) we have described in [**5**] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5,6,7,8$; (ii) we have described in [**6**] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 9,10$ and (iii) we have described in [**7**] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 11,12$. We now proceed to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive rational number. In particular, we here describe the parameters of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{3}{2}, \frac{4}{3}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \frac{6}{5}$, as follows. First,

Proposition 1.1 (Elzinga [**2**]). *Let G be a connected or disconnected strongly regular graph of order n and degree r. Then*

$$r^2 - (\tau - \theta + 1)r - (n-1)\theta = 0. \tag{1.1}$$

Proposition 1.2 (Elzinga [**2**]). *Let G be a connected strongly regular graph of order n and degree r. Then*

$$2r + (\tau - \theta)(m_2 + m_3) + \delta(m_2 - m_3) = 0, \tag{1.2}$$

where $\delta = \lambda_2 - \lambda_3$.

Second, in what follows (x, y) denotes the greatest common divisor of integers $x, y \in \mathbb{N}$, while $x \mid y$ means that x divides y.

Remark 1.3. We note that ($m_2 = qm_3$ and $m_3 = qm_2$) is equivalent to the assertion that ($m_2 = q^{-1}m_3$ and $m_3 = q^{-1}m_2$). In view of this[2] we may assume that $q = \frac{a}{b}$ so that $(a,b) = 1$ and $a > b$.

Using a similar procedure applied in [**4**], we can establish the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for any fixed value $q \in \mathbb{Q}$, where $q = \frac{a}{b}$ so that $(a,b) = 1$ and $a > b$, as follows. First, let $m_3 = p$ and $m_2 = (\frac{a}{b})\,p$, where p is a positive integer. Since $(a,b) = 1$ it follows that $b \mid p$. Replacing p with bp we obtain $m_3 = bp$ and $m_2 = ap$. Since $m_2 + m_3 = n-1$ we obtain $n = (a+b)p + 1$. Next, since $\tau - \theta = \lambda_2 + \lambda_3$ and $\delta = \lambda_2 - \lambda_3$ using (1.2) we obtain $r = p(b|\lambda_3| - a\lambda_2)$. Let $b|\lambda_3| - a\lambda_2 = t$ where[3] $t = 1, 2, \ldots, a+b-1$. Let

[1] We say that a connected or disconnected graph G is integral if its spectrum $\sigma(G)$ consists only of integral values.

[2] It exactly means that ($m_2 = qm_3$ and $m_3 = qm_2$) and ($m_2 = q^{-1}m_3$ and $m_3 = q^{-1}m_2$) are related to the same classes of strongly regular graphs.

[3] We note first that t is a positive integer because $r = pt$. Second, we note that $t \geqslant (a+b)$ is not possible because in that case we have $r = pt \geqslant (a+b)p \geqslant n-1$, a contradiction.

$\lambda_2 = k$ where k is a positive integer. Then (i) $\lambda_3 = -\frac{ak+t}{b}$; (ii) $\tau - \theta = -\frac{(a-b)k+t}{b}$; (iii) $\delta = \frac{(a+b)k+t}{b}$ and (iv) $r = pt$. Since $\delta^2 = (\tau - \theta)^2 + 4(r - \theta)$ (see [**2**]) we obtain (v) $\theta = pt - \frac{ak^2+kt}{b}$. Using (ii), (iv) and (v), we can easily see that (1.1) reduces to

$$(bp+1)t^2 - b\big((a+b)p+1\big)t + a(a+b)k^2 + 2akt = 0. \tag{1.3}$$

Second, let $m_2 = bp$, $m_3 = ap$ and $n = (a+b)p+1$ where $(a, b) = 1$ and $a > b$. Using (1.2) we obtain $r = p(a|\lambda_3| - b\lambda_2)$. Let $a|\lambda_3| - b\lambda_2 = t$ where $t = 1, 2, \dots, a+b-1$. Let $\lambda_3 = -k$ where k is a positive integer. Then (i) $\lambda_2 = \frac{ak-t}{b}$; (ii) $\tau - \theta = \frac{(a-b)k-t}{b}$; (iii) $\delta = \frac{(a+b)k-t}{b}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{ak^2-kt}{b}$. Using (ii), (iv) and (v) we can easily see that (1.1) reduces to

$$(bp+1)t^2 - b\big((a+b)p+1\big)t + a(a+b)k^2 - 2akt = 0. \tag{1.4}$$

Using (1.3) and (1.4), we can obtain for $t = 1, 2, \dots, a+b-1$ the corresponding classes of strongly regular graphs with $m_2 = (\frac{a}{b})m_3$ and $m_3 = (\frac{a}{b})m_2$, respectively. Finally, we arrive at the following two results.

2. Main results

THEOREM 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = ap$ and $m_3 = bp$, where $a, b, p \in \mathbb{N}$ so that $(a, b) = 1$ and $a > b$. Then:*

(1^0) $n = (a+b)p+1$, (2^0) $r = pt$, (3^0) $\tau = \Big(pt - \dfrac{ak^2+kt}{b}\Big) - \dfrac{(a-b)k+t}{b}$,

(4^0) $\theta = pt - \dfrac{ak^2+kt}{b}$, (5^0) $\lambda_2 = k$, (6^0) $\lambda_3 = -\dfrac{ak+t}{b}$, (7^0) $\delta = \dfrac{(a+b)k+t}{b}$,

(8^0) $(bp+1)t^2 - b((a+b)p+1)t + a(a+b)k^2 + 2akt = 0$,

for $k \in \mathbb{N}$ *and* $t = 1, 2, \dots, a+b-1$, *where* $\delta = \lambda_2 - \lambda_3$.

THEOREM 2.2. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = bp$ and $m_3 = ap$, where $a, b, p \in \mathbb{N}$ so that $(a, b) = 1$ and $a > b$. Then:*

(1^0) $n = (a+b)p+1$, (2^0) $r = pt$, (3^0) $\tau = \Big(pt - \dfrac{ak^2-kt}{b}\Big) + \dfrac{(a-b)k-t}{b}$,

(4^0) $\theta = pt - \dfrac{ak^2-kt}{b}$, (5^0) $\lambda_2 = \dfrac{ak-t}{b}$, (6^0) $\lambda_3 = -k$, (7^0) $\delta = \dfrac{(a+b)k-t}{b}$,

(8^0) $(bp+1)t^2 - b((a+b)p+1)t + a(a+b)k^2 - 2akt = 0$,

for $k \in \mathbb{N}$ *and* $t = 1, 2, \dots, a+b-1$, *where* $\delta = \lambda_2 - \lambda_3$.

REMARK 2.1. Since $m_2(\overline{G}) = m_3(G)$ and $m_3(\overline{G}) = m_2(G)$, we note that if $m_2(G) = qm_3(G)$, then $m_3(\overline{G}) = qm_2(\overline{G})$.

REMARK 2.2. In Theorems 2.3, 2.4, 2.5, 2.6, 2.7 and 2.8 the complements of strongly regular graphs appear in pairs in (k^0) and $(\overline{k}^0)$ classes, where k denotes the corresponding number of a class.

REMARK 2.3. $\overline{\alpha K_\beta}$ is a strongly regular graph of order $n = \alpha\beta$ and degree $r = (\alpha-1)\beta$ with $\tau = (\alpha-2)\beta$ and $\theta = (\alpha-1)\beta$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -\beta$ with $m_2 = \alpha(\beta-1)$ and $m_3 = \alpha - 1$.

PROPOSITION 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{3}{2})m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) or (4^0) or $(\overline{5}^0)$ represented in Theorem* 2.3.

PROOF. Let $m_2 = 3p$, $m_3 = 2p$ and $n = 5p+1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 2.1, we have (i) $\lambda_3 = -\frac{3k+t}{2}$; (ii) $\tau - \theta = -\frac{k+t}{2}$; (iii) $\delta = \frac{5k+t}{2}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{3k^2+kt}{2}$, where $t = 1, 2, \ldots, 4$. In this case we can easily see that Theorem 2.1 (8^0) reduces to

$$(2p+1)t^2 - 2(5p+1)t + 15k^2 + 6kt = 0. \tag{2.1}$$

CASE 1 $(t = 1)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{3k+1}{2}$, $\tau - \theta = -\frac{k+1}{2}$, $\delta = \frac{5k+1}{2}$, $r = p$ and $\theta = p - \frac{3k^2+k}{2}$. Using (2.1) we find that $8p+1 = 3k(5k+2)$. Replacing k with $4k-1$ we arrive at $p = 30k^2 - 12k + 1$. So we obtain that G is a strongly regular graph of order $n = 6(5k-1)^2$ and degree $r = 30k^2 - 12k + 1$ with $\tau = 2k(3k-2)$ and $\theta = 2k(3k-1)$.

CASE 2 $(t = 2)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{3k+2}{2}$, $\tau - \theta = -\frac{k+2}{2}$, $\delta = \frac{5k+2}{2}$, $r = 2p$ and $\theta = 2p - \frac{3k^2+2k}{2}$. Using (2.1) we find that $4p = k(5k+4)$. Replacing k with $2k$ we arrive at $p = k(5k+2)$. So we obtain that G is a strongly regular graph of order $n = (5k+1)^2$ and degree $r = 2k(5k+2)$ with $\tau = 4k^2 + k - 1$ and $\theta = 2k(2k+1)$.

CASE 3 $(t = 3)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{3k+3}{2}$, $\tau - \theta = -\frac{k+3}{2}$, $\delta = \frac{5k+3}{2}$, $r = 3p$ and $\theta = 3p - \frac{3k^2+3k}{2}$. Using (2.1) we find that $4p - 1 = k(5k+6)$. Replacing k with $2k-1$ we arrive at $p = k(5k-2)$. So we obtain that G is a strongly regular graph of order $n = (5k-1)^2$ and degree $r = 3k(5k-2)$ with $\tau = 9k^2 - 4k - 1$ and $\theta = 3k(3k-1)$.

CASE 4 $(t = 4)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{3k+4}{2}$, $\tau - \theta = -\frac{k+4}{2}$, $\delta = \frac{5k+4}{2}$, $r = 4p$ and $\theta = 4p - \frac{3k^2+4k}{2}$. Using (2.1) we find that $8p - 8 = 3k(5k+8)$. Replacing k with $4k$ we arrive at $p = 30k^2 + 12k + 1$. So we obtain that G is a strongly regular graph of order $n = 6(5k+1)^2$ and degree $r = 4(30k^2 + 12k + 1)$ with $\tau = 2(3k+1)(16k+1)$ and $\theta = 4(4k+1)(6k+1)$. □

PROPOSITION 2.2. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{3}{2})m_2$. Then G belongs to the class (2^0) or $(\overline{3}^0)$ or $(\overline{4}^0)$ or (5^0) represented in Theorem* 2.3.

PROOF. Let $m_2 = 2p$, $m_3 = 3p$ and $n = 5p+1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 2.2 we have (i) $\lambda_2 = \frac{3k-t}{2}$; (ii) $\tau - \theta = \frac{k-t}{2}$; (iii) $\delta = \frac{5k-t}{2}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{3k^2-kt}{2}$, where $t = 1, 2, \ldots, 4$. In this case we can easily see that Theorem 2.2 (8^0) reduces to

$$(2p+1)t^2 - 2(5p+1)t + 15k^2 - 6kt = 0. \tag{2.2}$$

Case 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{3k-1}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-1}{2}$, $\delta = \frac{5k-1}{2}$, $r = p$ and $\theta = p - \frac{3k^2-k}{2}$. Using (2.2) we find that $8p+1 = 3k(5k-2)$. Replacing k with $4k+1$ we arrive at $p = 30k^2+12k+1$. So we obtain that G is a strongly regular graph of order $n = 6(5k+1)^2$ and degree $r = 30k^2+12k+1$ with $\tau = 2k(3k+2)$ and $\theta = 2k(3k+1)$.

Case 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{3k-2}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-2}{2}$, $\delta = \frac{5k-2}{2}$, $r = 2p$ and $\theta = 2p - \frac{3k^2-2k}{2}$. Using (2.2) we find that $4p = k(5k-4)$. Replacing k with $2k$ we arrive at $p = k(5k-2)$. So we obtain that G is a strongly regular graph of order $n = (5k-1)^2$ and degree $r = 2k(5k-2)$ with $\tau = 4k^2-k-1$ and $\theta = 2k(2k-1)$.

Case 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{3k-3}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-3}{2}$, $\delta = \frac{5k-3}{2}$, $r = 3p$ and $\theta = 3p - \frac{3k^2-3k}{2}$. Using (2.2) we find that $4p-1 = k(5k-6)$. Replacing k with $2k+1$ we arrive at $p = k(5k+2)$. So we obtain that G is a strongly regular graph of order $n = (5k+1)^2$ and degree $r = 3k(5k+2)$ with $\tau = 9k^2+4k-1$ and $\theta = 3k(3k+1)$.

Case 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{3k-4}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-4}{2}$, $\delta = \frac{5k-4}{2}$, $r = 4p$ and $\theta = 4p - \frac{3k^2-4k}{2}$. Using (2.2) we find that $8p-8 = 3k(5k-8)$. Replacing k with $4k$ we arrive at $p = 30k^2-12k+1$. So we obtain that G is a strongly regular graph of order $n = 6(5k-1)^2$ and degree $r = 4(30k^2-12k+1)$ with $\tau = 2(3k-1)(16k-1)$ and $\theta = 4(4k-1)(6k-1)$. □

Remark 2.4. We note that $\overline{3K_2}$ is a strongly regular graph with $m_2 = (\frac{3}{2})m_3$. It is obtained from the class Theorem 2.3 ($\overline{5}^0$) for $k = 0$.

Theorem 2.3. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{3}{2})m_3$ or $m_3 = (\frac{3}{2})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is the strongly regular graph $\overline{3K_2}$ of order $n = 6$ and degree $r = 4$ with $\tau = 2$ and $\theta = 4$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -2$ with $m_2 = 3$ and $m_3 = 2$,*

(2^0) *G is a strongly regular graph of order $n = (5k-1)^2$ and degree $r = 2k(5k-2)$ with $\tau = 4k^2-k-1$ and $\theta = 2k(2k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -2k$ with $m_2 = 2k(5k-2)$ and $m_3 = 3k(5k-2)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (5k-1)^2$ and degree $r = 3k(5k-2)$ with $\tau = 9k^2-4k-1$ and $\theta = 3k(3k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k-1$ and $\lambda_3 = -3k$ with $m_2 = 3k(5k-2)$ and $m_3 = 2k(5k-2)$;*

(3^0) *G is a strongly regular graph of order $n = (5k+1)^2$ and degree $r = 2k(5k+2)$ with $\tau = 4k^2+k-1$ and $\theta = 2k(2k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = -(3k+1)$ with $m_2 = 3k(5k+2)$ and $m_3 = 2k(5k+2)$;*

($\overline{3}^0$) *G is a strongly regular graph of order $n = (5k+1)^2$ and degree $r = 3k(5k+2)$ with $\tau = 9k^2+4k-1$ and $\theta = 3k(3k+1)$, where $k \in \mathbb{N}$. Its* *eigenvalues are* $\lambda_2 = 3k$ *and* $\lambda_3 = -(2k+1)$ *with* $m_2 = 2k(5k+2)$ *and* $m_3 = 3k(5k+2)$*;*

(4^0) G *is a strongly regular graph of order* $n = 6(5k-1)^2$ *and degree* $r = 30k^2 - 12k + 1$ *with* $\tau = 2k(3k-2)$ *and* $\theta = 2k(3k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k-1$ *and* $\lambda_3 = -(6k-1)$ *with* $m_2 = 3(30k^2-12k+1)$ *and* $m_3 = 2(30k^2 - 12k + 1)$*;*

($\overline{4}^0$) G *is a strongly regular graph of order* $n = 6(5k-1)^2$ *and degree* $r = 4(30k^2 - 12k + 1)$ *with* $\tau = 2(3k-1)(16k-1)$ *and* $\theta = 4(4k-1)(6k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 6k-2$ *and* $\lambda_3 = -4k$ *with* $m_2 = 2(30k^2 - 12k + 1)$ *and* $m_3 = 3(30k^2 - 12k + 1)$*;*

(5^0) G *is a strongly regular graph of order* $n = 6(5k+1)^2$ *and degree* $r = 30k^2 + 12k + 1$ *with* $\tau = 2k(3k+2)$ *and* $\theta = 2k(3k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 6k+1$ *and* $\lambda_3 = -(4k+1)$ *with* $m_2 = 2(30k^2+12k+1)$ *and* $m_3 = 3(30k^2 + 12k + 1)$*;*

($\overline{5}^0$) G *is a strongly regular graph of order* $n = 6(5k+1)^2$ *and degree* $r = 4(30k^2 + 12k + 1)$ *with* $\tau = 2(3k+1)(16k+1)$ *and* $\theta = 4(4k+1)(6k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k$ *and* $\lambda_3 = -(6k+2)$ *with* $m_2 = 3(30k^2 + 12k + 1)$ *and* $m_3 = 2(30k^2 + 12k + 1)$*.*

PROOF. First, according to Remark 2.3 we have $2\alpha(\beta - 1) = 3(\alpha - 1)$, from which we find that $\alpha = 3$, $\beta = 2$. In view of this we obtain the strongly regular graph represented in Theorem 2.3 (1^0). Next, according to Proposition 2.1 it turns out that G belongs to the class ($\overline{2}^0$) or (3^0) or (4^0) or ($\overline{5}^0$) if $m_2 = (\frac{3}{2})m_3$. According to Proposition 2.2 it turns out that G belongs to the class (2^0) or ($\overline{3}^0$) or ($\overline{4}^0$) or (5^0) if $m_3 = (\frac{3}{2})m_2$. □

PROPOSITION 2.3. *Let* G *be a connected strongly regular graph of order* n *and degree* r *with* $m_2 = (\frac{4}{3})m_3$*. Then* G *belongs to the class* ($\overline{2}^0$) *or* (3^0) *or* (4^0) *or* ($\overline{5}^0$) *or* ($\overline{6}^0$) *or* (7^0) *represented in Theorem* 2.4*.*

PROOF. Let $m_2 = 4p$, $m_3 = 3p$ and $n = 7p+1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 2.1 we have (i) $\lambda_3 = -\frac{4k+t}{3}$; (ii) $\tau - \theta = -\frac{k+t}{3}$; (iii) $\delta = \frac{7k+t}{3}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{4k^2+kt}{3}$, where $t = 1, 2, \ldots, 6$. In this case we can easily see that Theorem 2.1 (8^0) reduces to

$$(3p+1)t^2 - 3(7p+1)t + 28k^2 + 8kt = 0. \tag{2.3}$$

CASE 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{4k+1}{3}$, $\tau - \theta = -\frac{k+1}{3}$, $\delta = \frac{7k+1}{3}$, $r = p$ and $\theta = p - \frac{4k^2+k}{3}$. Using (2.3) we find that $9p + 1 = 2k(7k+2)$. Replacing k with $3k-1$ we arrive at $p = 14k^2 - 8k + 1$. So we obtain that G is a strongly regular graph of order $n = 2(7k-2)^2$ and degree $r = 14k^2 - 8k + 1$ with $\tau = 2k(k-1)$ and $\theta = k(2k-1)$.

CASE 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{4k+2}{3}$, $\tau - \theta = -\frac{k+2}{3}$, $\delta = \frac{7k+2}{3}$, $r = 2p$ and $\theta = 2p - \frac{4k^2+2k}{3}$. Using (2.3) we find that $15p+1 = 2k(7k+4)$. Replacing k with $15k+4$ we arrive at $p = 210k^2+120k+17$.

So we obtain that G is a strongly regular graph of order $n = 30(7k+2)^2$ and degree $r = 2(210k^2 + 120k + 17)$ with $\tau = 120k^2 + 65k + 8$ and $\theta = 10(3k + 1)(4k + 1)$.

Case 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{4k+3}{3}$, $\tau - \theta = -\frac{k+3}{3}$, $\delta = \frac{7k+3}{3}$, $r = 3p$ and $\theta = 3p - \frac{4k^2+3k}{3}$. Using (2.3) we find that $9p = k(7k + 6)$. Replacing k with $3k$ we arrive at $p = k(7k + 2)$. So we obtain that G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 3k(7k+2)$ with $\tau = 9k^2 + 2k - 1$ and $\theta = 3k(3k + 1)$.

Case 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{4k+4}{3}$, $\tau - \theta = -\frac{k+4}{3}$, $\delta = \frac{7k+4}{3}$, $r = 4p$ and $\theta = 4p - \frac{4k^2+4k}{3}$. Using (2.3) we find that $9p - 1 = k(7k + 8)$. Replacing k with $3k - 1$ we arrive at $p = k(7k - 2)$. So we obtain that G is a strongly regular graph of order $n = (7k - 1)^2$ and degree $r = 4k(7k - 2)$ with $\tau = 16k^2 - 5k - 1$ and $\theta = 4k(4k - 1)$.

Case 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{4k+5}{3}$, $\tau - \theta = -\frac{k+5}{3}$, $\delta = \frac{7k+5}{3}$, $r = 5p$ and $\theta = 5p - \frac{4k^2+5k}{3}$. Using (2.3) we find that $15p - 5 = 2k(7k + 10)$. Replacing k with $15k - 5$ we arrive at $p = 210k^2 - 120k + 17$. So we obtain that G is a strongly regular graph of order $n = 30(7k - 2)^2$ and degree $r = 5(210k^2 - 120k + 17)$ with $\tau = 10(75k^2 - 43k + 6)$ and $\theta = 5(10k - 3)(15k - 4)$.

Case 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{4k+6}{3}$, $\tau - \theta = -\frac{k+6}{3}$, $\delta = \frac{7k+6}{3}$, $r = 6p$ and $\theta = 6p - \frac{4k^2+6k}{3}$. Using (2.3) we find that $9p - 9 = 2k(7k + 12)$. Replacing k with $3k$ we arrive at $p = 14k^2 + 8k + 1$. So we obtain that G is a strongly regular graph of order $n = 2(7k+2)^2$ and degree $r = 6(14k^2 + 8k + 1)$ with $\tau = (8k + 1)(9k + 4)$ and $\theta = 6(3k + 1)(4k + 1)$. □

Proposition 2.4. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{4}{3})m_2$. Then G belongs to the class* (2^0) *or* $(\overline{3}^0)$ *or* $(\overline{4}^0)$ *or* (5^0) *or* (6^0) *or* $(\overline{7}^0)$ *represented in Theorem* 2.4.

Proof. Let $m_2 = 3p$, $m_3 = 4p$ and $n = 7p+1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 2.2 we have (i) $\lambda_2 = \frac{4k-t}{3}$; (ii) $\tau - \theta = \frac{k-t}{3}$; (iii) $\delta = \frac{7k-t}{3}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{4k^2-kt}{3}$, where $t = 1, 2, \ldots, 6$. In this case we can easily see that Theorem 2.2 (8^0) reduces to

$$(3p + 1)t^2 - 3(7p + 1)t + 28k^2 - 8kt = 0. \tag{2.4}$$

Case 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{4k-1}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-1}{3}$, $\delta = \frac{7k-1}{3}$, $r = p$ and $\theta = p - \frac{4k^2-k}{3}$. Using (2.4) we find that $9p + 1 = 2k(7k - 2)$. Replacing k with $3k + 1$ we arrive at $p = 14k^2 + 8k + 1$. So we obtain that G is a strongly regular graph of order $n = 2(7k + 2)^2$ and degree $r = 14k^2 + 8k + 1$ with $\tau = 2k(k + 1)$ and $\theta = k(2k + 1)$.

Case 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{4k-2}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-2}{3}$, $\delta = \frac{7k-2}{3}$, $r = 2p$ and $\theta = 2p - \frac{4k^2-2k}{3}$. Using (2.4) we find that $15p+1 = 2k(7k-4)$. Replacing k with $15k-4$ we arrive at $p = 210k^2-120k+17$.

So we obtain that G is a strongly regular graph of order $n = 30(7k-2)^2$ and degree $r = 2(210k^2 - 120k + 17)$ with $\tau = 120k^2 - 65k + 8$ and $\theta = 10(3k-1)(4k-1)$.

CASE 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{4k-3}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-3}{3}$, $\delta = \frac{7k-3}{3}$, $r = 3p$ and $\theta = 3p - \frac{4k^2-3k}{3}$. Using (2.4) we find that $9p = k(7k-6)$. Replacing k with $3k$ we arrive at $p = k(7k-2)$. So we obtain that G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = 3k(7k-2)$ with $\tau = 9k^2 - 2k - 1$ and $\theta = 3k(3k-1)$.

CASE 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{4k-4}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-4}{3}$, $\delta = \frac{7k-4}{3}$, $r = 4p$ and $\theta = 4p - \frac{4k^2-4k}{3}$. Using (2.4) we find that $9p - 1 = k(7k-8)$. Replacing k with $3k+1$ we arrive at $p = k(7k+2)$. So we obtain that G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 4k(7k+2)$ with $\tau = 16k^2 + 5k - 1$ and $\theta = 4k(4k+1)$.

CASE 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{4k-5}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-5}{3}$, $\delta = \frac{7k-5}{3}$, $r = 5p$ and $\theta = 5p - \frac{4k^2-5k}{3}$. Using (2.4) we find that $15p - 5 = 2k(7k-10)$. Replacing k with $15k+5$ we arrive at $p = 210k^2 + 120k + 17$. So we obtain that G is a strongly regular graph of order $n = 30(7k+2)^2$ and degree $r = 5(210k^2 + 120k + 17)$ with $\tau = 10(75k^2 + 43k + 6)$ and $\theta = 5(10k+3)(15k+4)$.

CASE 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{4k-6}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-6}{3}$, $\delta = \frac{7k-6}{3}$, $r = 6p$ and $\theta = 6p - \frac{4k^2-6k}{3}$. Using (2.4) we find that $9p - 9 = 2k(7k-12)$. Replacing k with $3k$ we arrive at $p = 14k^2 - 8k + 1$. So we obtain that G is a strongly regular graph of order $n = 2(7k-2)^2$ and degree $r = 6(14k^2 - 8k + 1)$ with $\tau = (8k-1)(9k-4)$ and $\theta = 6(3k-1)(4k-1)$. □

REMARK 2.5. We note that $\overline{4K_2}$ is a strongly regular graph with $m_2 = (\frac{4}{3})m_3$. It is obtained from the class Theorem 2.4 ($\overline{5}^0$) for $k = 0$.

THEOREM 2.4. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{4}{3})m_3$ or $m_3 = (\frac{4}{3})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is the strongly regular graph $\overline{4K_2}$ of order $n = 8$ and degree $r = 6$ with $\tau = 4$ and $\theta = 6$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -2$ with $m_2 = 4$ and $m_3 = 3$;*

(2^0) *G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = 3k(7k-2)$ with $\tau = 9k^2 - 2k - 1$ and $\theta = 3k(3k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k-1$ and $\lambda_3 = -3k$ with $m_2 = 3k(7k-2)$ and $m_3 = 4k(7k-2)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = 4k(7k-2)$ with $\tau = 16k^2 - 5k - 1$ and $\theta = 4k(4k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -4k$ with $m_2 = 4k(7k-2)$ and $m_3 = 3k(7k-2)$;*

(3^0) *G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 3k(7k+2)$ with $\tau = 9k^2 + 2k - 1$ and $\theta = 3k(3k+1)$, where $k \in \mathbb{N}$. Its* *eigenvalues are* $\lambda_2 = 3k$ *and* $\lambda_3 = -(4k+1)$ *with* $m_2 = 4k(7k+2)$ *and* $m_3 = 3k(7k+2)$*;*

($\overline{3}^0$) *G is a strongly regular graph of order* $n = (7k+1)^2$ *and degree* $r = 4k(7k+2)$ *with* $\tau = 16k^2+5k-1$ *and* $\theta = 4k(4k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k$ *and* $\lambda_3 = -(3k+1)$ *with* $m_2 = 3k(7k+2)$ *and* $m_3 = 4k(7k+2)$*;*

(4^0) *G is a strongly regular graph of order* $n = 2(7k-2)^2$ *and degree* $r = 14k^2-8k+1$ *with* $\tau = 2k(k-1)$ *and* $\theta = k(2k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 3k-1$ *and* $\lambda_3 = -(4k-1)$ *with* $m_2 = 4(14k^2-8k+1)$ *and* $m_3 = 3(14k^2-8k+1)$*;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 2(7k-2)^2$ *and degree* $r = 6(14k^2-8k+1)$ *with* $\tau = (8k-1)(9k-4)$ *and* $\theta = 6(3k-1)(4k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k-2$ *and* $\lambda_3 = -3k$ *with* $m_2 = 3(14k^2-8k+1)$ *and* $m_3 = 4(14k^2-8k+1)$*;*

(5^0) *G is a strongly regular graph of order* $n = 2(7k+2)^2$ *and degree* $r = 14k^2+8k+1$ *with* $\tau = 2k(k+1)$ *and* $\theta = k(2k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k+1$ *and* $\lambda_3 = -(3k+1)$ *with* $m_2 = 3(14k^2+8k+1)$ *and* $m_3 = 4(14k^2+8k+1)$*;*

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 2(7k+2)^2$ *and degree* $r = 6(14k^2+8k+1)$ *with* $\tau = (8k+1)(9k+4)$ *and* $\theta = 6(3k+1)(4k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 3k$ *and* $\lambda_3 = -(4k+2)$ *with* $m_2 = 4(14k^2+8k+1)$ *and* $m_3 = 3(14k^2+8k+1)$*;*

(6^0) *G is a strongly regular graph of order* $n = 30(7k-2)^2$ *and degree* $r = 2(210k^2-120k+17)$ *with* $\tau = 120k^2-65k+8$ *and* $\theta = 10(3k-1)(4k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 20k-6$ *and* $\lambda_3 = -(15k-4)$ *with* $m_2 = 3(210k^2-120k+17)$ *and* $m_3 = 4(210k^2-120k+17)$*;*

($\overline{6}^0$) *G is a strongly regular graph of order* $n = 30(7k-2)^2$ *and degree* $r = 5(210k^2-120k+17)$ *with* $\tau = 10(75k^2-43k+6)$ *and* $\theta = 5(10k-3)(15k-4)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 15k-5$ *and* $\lambda_3 = -(20k-5)$ *with* $m_2 = 4(210k^2-120k+17)$ *and* $m_3 = 3(210k^2-120k+17)$*;*

(7^0) *G is a strongly regular graph of order* $n = 30(7k+2)^2$ *and degree* $r = 2(210k^2+120k+17)$ *with* $\tau = 120k^2+65k+8$ *and* $\theta = 10(3k+1)(4k+1)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 15k+4$ *and* $\lambda_3 = -(20k+6)$ *with* $m_2 = 4(210k^2+120k+17)$ *and* $m_3 = 3(210k^2+120k+17)$*;*

($\overline{7}^0$) *G is a strongly regular graph of order* $n = 30(7k+2)^2$ *and degree* $r = 5(210k^2+120k+17)$ *with* $\tau = 10(75k^2+43k+6)$ *and* $\theta = 5(10k+3)(15k+4)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 20k+5$ *and* $\lambda_3 = -(15k+5)$ *with* $m_2 = 3(210k^2+120k+17)$ *and* $m_3 = 4(210k^2+120k+17)$*.*

Proof. First, according to Remark 2.3 we have $3\alpha(\beta-1) = 4(\alpha-1)$, from which we find that $\alpha = 4$, $\beta = 2$. In view of this, we obtain the strongly regular graph represented in Theorem 2.4 (1^0). Next, according to Proposition 2.3 it turns out that G belongs to the class ($\overline{2}^0$) or (3^0) or (4^0) or ($\overline{5}^0$) or ($\overline{6}^0$) or (7^0) if

$m_2 = (\frac{4}{3})m_3$. According to Proposition 2.4 it turns out that G belongs to the class (2^0) or $(\overline{3}^0)$ or $(\overline{4}^0)$ or (5^0) or (6^0) or $(\overline{7}^0)$ if $m_3 = (\frac{4}{3})m_2$. □

PROPOSITION 2.5. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{5}{2})m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) or (4^0) or $(\overline{5}^0)$ or $(\overline{6}^0)$ or (7^0) represented in Theorem* 2.5.

PROOF. Let $m_2 = 5p$, $m_3 = 2p$ and $n = 7p+1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 2.1 we have (i) $\lambda_3 = -\frac{5k+t}{2}$; (ii) $\tau - \theta = -\frac{3k+t}{2}$; (iii) $\delta = \frac{7k+t}{2}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{5k^2+kt}{2}$, where $t = 1, 2, \ldots, 6$. In this case we can easily see that Theorem 2.1 (8^0) reduces to

$$(2p+1)t^2 - 2(7p+1)t + 35k^2 + 10kt = 0. \tag{2.5}$$

CASE 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+1}{2}$, $\tau - \theta = -\frac{3k+1}{2}$, $\delta = \frac{7k+1}{2}$, $r = p$ and $\theta = p - \frac{5k^2+k}{2}$. Using (2.5) we find that $12p+1 = 5k(7k+2)$. Replacing k with $6k-1$ we arrive at $p = 105k^2 - 30k + 2$. So we obtain that G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 105k^2 - 30k + 2$ with $\tau = 15k^2 - 12k + 1$ and $\theta = 3k(5k-1)$.

CASE 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+2}{2}$, $\tau - \theta = -\frac{3k+2}{2}$, $\delta = \frac{7k+2}{2}$, $r = 2p$ and $\theta = 2p - \frac{5k^2+2k}{2}$. Using (2.5) we find that $4p = k(7k+4)$. Replacing k with $2k$ we arrive at $p = k(7k+2)$. So we obtain that G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 2k(7k+2)$ with $\tau = 4k^2 - k - 1$ and $\theta = 2k(2k+1)$.

CASE 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+3}{2}$, $\tau - \theta = -\frac{3k+3}{2}$, $\delta = \frac{7k+3}{2}$, $r = 3p$ and $\theta = 3p - \frac{5k^2+3k}{2}$. Using (2.5) we find that $24p-3 = 5k(7k+6)$. Replacing k with $12k+3$ we arrive at $p = 210k^2+120k+17$. So we obtain that G is a strongly regular graph of order $n = 30(7k+2)^2$ and degree $r = 3(210k^2 + 120k + 17)$ with $\tau = 18(3k+1)(5k+1)$ and $\theta = 6(3k+1)(15k+4)$.

CASE 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+4}{2}$, $\tau - \theta = -\frac{3k+4}{2}$, $\delta = \frac{7k+4}{2}$, $r = 4p$ and $\theta = 4p - \frac{5k^2+4k}{2}$. Using (2.5) we find that $24p-8 = 5k(7k+8)$. Replacing k with $12k-4$ we arrive at $p = 210k^2-120k+17$. So we obtain that G is a strongly regular graph of order $n = 30(7k-2)^2$ and degree $r = 4(210k^2-120k+17)$ with $\tau = 2(240k^2-141k+20)$ and $\theta = 12(4k-1)(10k-3)$.

CASE 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+5}{2}$, $\tau - \theta = -\frac{3k+5}{2}$, $\delta = \frac{7k+5}{2}$, $r = 5p$ and $\theta = 5p - \frac{5k^2+5k}{2}$. Using (2.5) we find that $4p-3 = k(7k+10)$. Replacing k with $2k-1$ we arrive at $p = k(7k-2)$. So we obtain that G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = 5k(7k-2)$ with $\tau = 25k^2 - 8k - 1$ and $\theta = 5k(5k-1)$.

CASE 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+6}{2}$, $\tau - \theta = -\frac{3k+6}{2}$, $\delta = \frac{7k+6}{2}$, $r = 6p$ and $\theta = 6p - \frac{5k^2+6k}{2}$. Using (2.5) we find that $12p-24 = 5k(7k+12)$. Replacing k with $6k$ we arrive at $p = 105k^2+30k+2$. So we obtain that G is a strongly regular graph of order $n = 15(7k+1)^2$ and degree $r = 6(105k^2+30k+2)$ with $\tau = 9(5k+1)(12k+1)$ and $\theta = 6(6k+1)(15k+2)$. □

PROPOSITION 2.6. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{5}{2})m_2$. Then G belongs to the class* (2^0) *or* $(\overline{3}^0)$ *or* $(\overline{4}^0)$ *or* (5^0) *or* (6^0) *or* $(\overline{7}^0)$ *represented in Theorem* 2.5.

PROOF. Let $m_2 = 2p$, $m_3 = 5p$ and $n = 7p+1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 2.2 we have (i) $\lambda_2 = \frac{5k-t}{2}$; (ii) $\tau-\theta = \frac{3k-t}{2}$; (iii) $\delta = \frac{7k-t}{2}$; (iv) $r = pt$ and (v) $\theta = pt-\frac{5k^2-kt}{2}$, where $t = 1, 2, \ldots, 6$. In this case we can easily see that Theorem 2.2 (8^0) reduces to

$$(2p+1)t^2 - 2(7p+1)t + 35k^2 - 10kt = 0. \tag{2.6}$$

CASE 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-1}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{3k-1}{2}$, $\delta = \frac{7k-1}{2}$, $r = p$ and $\theta = p - \frac{5k^2-k}{2}$. Using (2.6) we find that $12p+1 = 5k(7k-2)$. Replacing k with $6k+1$ we arrive at $p = 105k^2+30k+2$. So we obtain that G is a strongly regular graph of order $n = 15(7k+1)^2$ and degree $r = 105k^2 + 30k + 2$ with $\tau = 15k^2 + 12k + 1$ and $\theta = 3k(5k + 1)$.

CASE 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-2}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{3k-2}{2}$, $\delta = \frac{7k-2}{2}$, $r = 2p$ and $\theta = 2p - \frac{5k^2-2k}{2}$. Using (2.6) we find that $4p = k(7k - 4)$. Replacing k with $2k$ we arrive at $p = k(7k - 2)$. So we obtain that G is a strongly regular graph of order $n = (7k - 1)^2$ and degree $r = 2k(7k - 2)$ with $\tau = 4k^2 + k - 1$ and $\theta = 2k(2k - 1)$.

CASE 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-3}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{3k-3}{2}$, $\delta = \frac{7k-3}{2}$, $r = 3p$ and $\theta = 3p - \frac{5k^2-3k}{2}$. Using (2.6) we find that $24p-3 = 5k(7k-6)$. Replacing k with $12k-3$ we arrive at $p = 210k^2-120k+17$. So we obtain that G is a strongly regular graph of order $n = 30(7k-2)^2$ and degree $r = 3(210k^2 - 120k + 17)$ with $\tau = 18(3k - 1)(5k - 1)$ and $\theta = 6(3k - 1)(15k - 4)$.

CASE 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-4}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{3k-4}{2}$, $\delta = \frac{7k-4}{2}$, $r = 4p$ and $\theta = 4p - \frac{5k^2-4k}{2}$. Using (2.6) we find that $24p-8 = 5k(7k-8)$. Replacing k with $12k+4$ we arrive at $p = 210k^2+120k+17$. So we obtain that G is a strongly regular graph of order $n = 30(7k+2)^2$ and degree $r = 4(210k^2+120k+17)$ with $\tau = 2(240k^2+141k+20)$ and $\theta = 12(4k+1)(10k+3)$.

CASE 5. ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-5}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{3k-5}{2}$, $\delta = \frac{7k-5}{2}$, $r = 5p$ and $\theta = 5p - \frac{5k^2-5k}{2}$. Using (2.6) we find that $4p - 3 = k(7k - 10)$. Replacing k with $2k + 1$ we arrive at $p = k(7k + 2)$. So we obtain that G is a strongly regular graph of order $n = (7k + 1)^2$ and degree $r = 5k(7k + 2)$ with $\tau = 25k^2 + 8k - 1$ and $\theta = 5k(5k + 1)$.

CASE 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-6}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{3k-6}{2}$, $\delta = \frac{7k-6}{2}$, $r = 6p$ and $\theta = 6p - \frac{5k^2-6k}{2}$. Using (2.6) we find that $12p-24 = 5k(7k-12)$. Replacing k with $6k$ we arrive at $p = 105k^2-30k+2$. So we obtain that G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 6(105k^2-30k+2)$ with $\tau = 9(5k-1)(12k-1)$ and $\theta = 6(6k-1)(15k-2)$. □

REMARK 2.6. We note that $\overline{5K_3}$ is a strongly regular graph with $m_2 = (\frac{5}{2})m_3$. It is obtained from the class Theorem 2.5 $(\overline{5}^0)$ for $k = 0$.

THEOREM 2.5. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{5}{2})m_3$ or $m_3 = (\frac{5}{2})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is the strongly regular graph $\overline{5K_3}$ of order $n = 15$ and degree $r = 12$ with $\tau = 9$ and $\theta = 12$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -3$ with $m_2 = 10$ and $m_3 = 4$;*

(2^0) *G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = 2k(7k-2)$ with $\tau = 4k^2 + k - 1$ and $\theta = 2k(2k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k - 1$ and $\lambda_3 = -2k$ with $m_2 = 2k(7k-2)$ and $m_3 = 5k(7k-2)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = 5k(7k-2)$ with $\tau = 25k^2 - 8k - 1$ and $\theta = 5k(5k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k - 1$ and $\lambda_3 = -5k$ with $m_2 = 5k(7k-2)$ and $m_3 = 2k(7k-2)$;*

(3^0) *G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 2k(7k+2)$ with $\tau = 4k^2 - k - 1$ and $\theta = 2k(2k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = -(5k+1)$ with $m_2 = 5k(7k+2)$ and $m_3 = 2k(7k+2)$;*

($\overline{3}^0$) *G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 5k(7k+2)$ with $\tau = 25k^2 + 8k - 1$ and $\theta = 5k(5k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(2k+1)$ with $m_2 = 2k(7k+2)$ and $m_3 = 5k(7k+2)$;*

(4^0) *G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 105k^2 - 30k + 2$ with $\tau = 15k^2 - 12k + 1$ and $\theta = 3k(5k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k - 1$ and $\lambda_3 = -(15k-2)$ with $m_2 = 5(105k^2 - 30k + 2)$ and $m_3 = 2(105k^2 - 30k + 2)$;*

($\overline{4}^0$) *G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 6(105k^2 - 30k + 2)$ with $\tau = 9(5k-1)(12k-1)$ and $\theta = 6(6k-1)(15k-2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 15k - 3$ and $\lambda_3 = -6k$ with $m_2 = 2(105k^2 - 30k + 2)$ and $m_3 = 5(105k^2 - 30k + 2)$;*

(5^0) *G is a strongly regular graph of order $n = 15(7k+1)^2$ and degree $r = 105k^2 + 30k + 2$ with $\tau = 15k^2 + 12k + 1$ and $\theta = 3k(5k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 15k + 2$ and $\lambda_3 = -(6k+1)$ with $m_2 = 2(105k^2 + 30k + 2)$ and $m_3 = 5(105k^2 + 30k + 2)$;*

($\overline{5}^0$) *G is a strongly regular graph of order $n = 15(7k+1)^2$ and degree $r = 6(105k^2 + 30k + 2)$ with $\tau = 9(5k+1)(12k+1)$ and $\theta = 6(6k+1)(15k+2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k$ and $\lambda_3 = -(15k+3)$ with $m_2 = 5(105k^2 + 30k + 2)$ and $m_3 = 2(105k^2 + 30k + 2)$;*

(6^0) *G is a strongly regular graph of order $n = 30(7k-2)^2$ and degree $r = 3(210k^2 - 120k + 17)$ with $\tau = 18(3k-1)(5k-1)$ and $\theta = 6(3k-1)(15k-4)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k - 9$ and $\lambda_3 = -(12k-3)$ with $m_2 = 2(210k^2 - 120k + 17)$ and $m_3 = 5(210k^2 - 120k + 17)$;*

($\overline{6}^0$) *G is a strongly regular graph of order $n = 30(7k-2)^2$ and degree $r = 4(210k^2 - 120k + 17)$ with $\tau = 2(240k^2 - 141k + 20)$ and $\theta = 12(4k-1)(10k-$*

3), *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k - 4$ *and* $\lambda_3 = -(30k - 8)$ *with* $m_2 = 5(210k^2 - 120k + 17)$ *and* $m_3 = 2(210k^2 - 120k + 17)$*;*

(7^0) G *is a strongly regular graph of order* $n = 30(7k + 2)^2$ *and degree* $r = 3(210k^2+120k+17)$ *with* $\tau = 18(3k+1)(5k+1)$ *and* $\theta = 6(3k+1)(15k+4)$, *where* $k \geqslant 0$. *Its eigenvalues are* $\lambda_2 = 12k + 3$ *and* $\lambda_3 = -(30k + 9)$ *with* $m_2 = 5(210k^2 + 120k + 17)$ *and* $m_3 = 2(210k^2 + 120k + 17)$*;*

($\overline{7}^0$) G *is a strongly regular graph of order* $n = 30(7k + 2)^2$ *and degree* $r = 4(210k^2+120k+17)$ *with* $\tau = 2(240k^2+141k+20)$ *and* $\theta = 12(4k+1)(10k+3)$, *where* $k \geqslant 0$. *Its eigenvalues are* $\lambda_2 = 30k + 8$ *and* $\lambda_3 = -(12k + 4)$ *with* $m_2 = 2(210k^2 + 120k + 17)$ *and* $m_3 = 5(210k^2 + 120k + 17)$.

PROOF. First, according to Remark 2.3 we have $2\alpha(\beta - 1) = 5(\alpha - 1)$, from which we find that $\alpha = 5$, $\beta = 3$. In view of this we obtain the strongly regular graph represented in Theorem 2.5 (1^0). Next, according to Proposition 2.5 it turns out that G belongs to the class ($\overline{2}^0$) or (3^0) or (4^0) or ($\overline{5}^0$) or ($\overline{6}^0$) or (7^0) if $m_2 = (\frac{5}{2})m_3$. According to Proposition 2.6 it turns out that G belongs to the class (2^0) or ($\overline{3}^0$) or ($\overline{4}^0$) or (5^0) or (6^0) or ($\overline{7}^0$) if $m_3 = (\frac{5}{2})m_2$. □

PROPOSITION 2.7. *Let* G *be a connected strongly regular graph of order* n *and degree* r *with* $m_2 = (\frac{5}{3})m_3$. *Then* G *belongs to the class* ($\overline{1}^0$) *or* (2^0) *or* (3^0) *or* ($\overline{4}^0$) *represented in Theorem* 2.6.

PROOF. Let $m_2 = 5p$, $m_3 = 3p$ and $n = 8p+1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 2.1 we have (i) $\lambda_3 = -\frac{5k+t}{3}$; (ii) $\tau - \theta = -\frac{2k+t}{3}$; (iii) $\delta = \frac{8k+t}{3}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{5k^2+kt}{3}$, where $t = 1, 2, \ldots, 7$. In this case we can easily see that Theorem 2.1 (8^0) reduces to

$$(3p + 1)t^2 - 3(8p + 1)t + 40k^2 + 10kt = 0. \tag{2.7}$$

CASE 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+1}{3}$, $\tau-\theta = -\frac{2k+1}{3}$, $\delta = \frac{8k+1}{3}$, $r = p$ and $\theta = p-\frac{5k^2+k}{3}$. Using (2.7) we find that $21p+2 = 10k(4k+1)$. Replacing k with $21k-8$ we arrive at $p = 840k^2-630k+118$. So we obtain that G is a strongly regular graph of order $n = 105(8k-3)^2$ and degree $r = 840k^2 - 630k + 118$ with $\tau = 105k^2 - 91k + 19$ and $\theta = 7(3k - 1)(5k - 2)$.

CASE 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+2}{3}$, $\tau - \theta = -\frac{2k+2}{3}$, $\delta = \frac{8k+2}{3}$, $r = 2p$ and $\theta = 2p - \frac{5k^2+2k}{3}$. Using (2.7) we find that $18p + 1 = 10k(2k + 1)$, a contradiction because $2 \nmid 18p + 1$.

CASE 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+3}{3}$, $\tau - \theta = -\frac{2k+3}{3}$, $\delta = \frac{8k+3}{3}$, $r = 3p$ and $\theta = 3p - \frac{5k^2+3k}{3}$. Using (2.7) we find that $9p = 2k(4k + 3)$. Replacing k with $3k$ we arrive at $p = 2k(4k + 1)$. So we obtain that G is a strongly regular graph of order $n = (8k + 1)^2$ and degree $r = 6k(4k + 1)$ with $\tau = 9k^2 + k - 1$ and $\theta = 3k(3k + 1)$.

CASE 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+4}{3}$, $\tau - \theta = -\frac{2k+4}{3}$, $\delta = \frac{8k+4}{3}$, $r = 4p$ and $\theta = 4p - \frac{5k^2+4k}{3}$. Using (2.7) we find that $12p - 1 = 10k(k + 1)$, a contradiction because $2 \nmid 12p - 1$.

Case 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+5}{3}$, $\tau - \theta = -\frac{2k+5}{3}$, $\delta = \frac{8k+5}{3}$, $r = 5p$ and $\theta = 5p - \frac{5k^2+5k}{3}$. Using (2.7) we find that $9p - 2 = 2k(4k + 5)$. Replacing k with $3k - 1$ we arrive at $p = 2k(4k - 1)$. So we obtain that G is a strongly regular graph of order $n = (8k - 1)^2$ and degree $r = 10k(4k - 1)$ with $\tau = 25k^2 - 7k - 1$ and $\theta = 5k(5k - 1)$.

Case 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+6}{3}$, $\tau - \theta = -\frac{2k+6}{3}$, $\delta = \frac{8k+6}{3}$, $r = 6p$ and $\theta = 6p - \frac{5k^2+6k}{3}$. Using (2.7) we find that $18p - 9 = 10k(2k + 3)$, a contradiction because $2 \nmid 18p - 9$.

Case 7 ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+7}{3}$, $\tau - \theta = -\frac{2k+7}{3}$, $\delta = \frac{8k+7}{3}$, $r = 7p$ and $\theta = 7p - \frac{5k^2+7k}{3}$. Using (2.7) we find that $21p - 28 = 10k(4k + 7)$. Replacing k with $21k + 7$ we arrive at $p = 840k^2 + 630k + 118$. So we obtain that G is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 7(840k^2+630k+118)$ with $\tau = 7(735k^2+551k+103)$ and $\theta = 7(21k + 8)(35k + 13)$. □

Proposition 2.8. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{5}{3})m_2$. Then G belongs to the class* (1^0) *or* $(\overline{2}^0)$ *or* $(\overline{3}^0)$ *or* (4^0) *represented in Theorem* 2.6*.*

Proof. Let $m_2 = 3p$, $m_3 = 5p$ and $n = 8p+1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 2.2 we have (i) $\lambda_2 = \frac{5k-t}{3}$; (ii) $\tau-\theta = \frac{2k-t}{3}$; (iii) $\delta = \frac{8k-t}{3}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{5k^2-kt}{3}$, where $t = 1, 2, \ldots, 7$. In this case we can easily see that Theorem 2.2 (8^0) reduces to

$$(3p + 1)t^2 - 3(8p + 1)t + 40k^2 - 10kt = 0. \tag{2.8}$$

Case 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-1}{3}$ and $\lambda_3 = -k$, $\tau-\theta = \frac{2k-1}{3}$, $\delta = \frac{8k-1}{3}$, $r = p$ and $\theta = p - \frac{5k^2-k}{3}$. Using (2.8) we find that $21p+2 = 10k(4k-1)$. Replacing k with $21k+8$ we arrive at $p = 840k^2+630k+118$. So we obtain that G is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 840k^2 + 630k + 118$ with $\tau = 105k^2 + 91k + 19$ and $\theta = 7(3k + 1)(5k + 2)$.

Case 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-2}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{2k-2}{3}$, $\delta = \frac{8k-2}{3}$, $r = 2p$ and $\theta = 2p - \frac{5k^2-2k}{3}$. Using (2.8) we find that $18p + 1 = 10k(2k - 1)$, a contradiction because $2 \nmid 18p + 1$.

Case 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-3}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{2k-3}{3}$, $\delta = \frac{8k-3}{3}$, $r = 3p$ and $\theta = 3p - \frac{5k^2-3k}{3}$. Using (2.8) we find that $9p = 2k(4k - 3)$. Replacing k with $3k$ we arrive at $p = 2k(4k - 1)$. So we obtain that G is a strongly regular graph of order $n = (8k - 1)^2$ and degree $r = 6k(4k - 1)$ with $\tau = 9k^2 - k - 1$ and $\theta = 3k(3k - 1)$.

Case 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-4}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{2k-4}{3}$, $\delta = \frac{8k-4}{3}$, $r = 4p$ and $\theta = 4p - \frac{5k^2-4k}{3}$. Using (2.8) we find that $12p - 1 = 10k(k - 1)$, a contradiction because $2 \nmid 12p - 1$.

Case 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-5}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{2k-5}{3}$, $\delta = \frac{8k-5}{3}$, $r = 5p$ and $\theta = 5p - \frac{5k^2-5k}{3}$. Using (2.8) we find that $9p-2 = 2k(4k-5)$. Replacing k with $3k+1$ we arrive at $p = 2k(4k+1)$. So we obtain that G is a strongly regular graph of order $n = (8k+1)^2$ and degree $r = 10k(4k+1)$ with $\tau = 25k^2+7k-1$ and $\theta = 5k(5k+1)$.

Case 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-6}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{2k-6}{3}$, $\delta = \frac{8k-6}{3}$, $r = 6p$ and $\theta = 6p - \frac{5k^2-6k}{3}$. Using (2.8) we find that $18p-9 = 10k(2k-3)$, a contradiction because $2 \nmid 18p-9$.

Case 7 ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-7}{3}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{2k-7}{3}$, $\delta = \frac{8k-7}{3}$, $r = 7p$ and $\theta = 7p - \frac{5k^2-7k}{3}$. Using (2.8) we find that $21p-28 = 10k(4k-7)$. Replacing k with $21k-7$ we arrive at $p = 840k^2-630k+118$. So we obtain that G is a strongly regular graph of order $n = 105(8k-3)^2$ and degree $r = 7(840k^2-630k+118)$ with $\tau = 7(735k^2-551k+103)$ and $\theta = 7(21k-8)(35k-13)$. □

Theorem 2.6. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{5}{3})m_3$ or $m_3 = (\frac{5}{3})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 6k(4k-1)$ with $\tau = 9k^2-k-1$ and $\theta = 3k(3k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k-1$ and $\lambda_3 = -3k$ with $m_2 = 6k(4k-1)$ and $m_3 = 10k(4k-1)$;*

($\overline{1}^0$) *G is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 10k(4k-1)$ with $\tau = 25k^2-7k-1$ and $\theta = 5k(5k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -5k$ with $m_2 = 10k(4k-1)$ and $m_3 = 6k(4k-1)$;*

(2^0) *G is a strongly regular graph of order $n = (8k+1)^2$ and degree $r = 6k(4k+1)$ with $\tau = 9k^2+k-1$ and $\theta = 3k(3k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(5k+1)$ with $m_2 = 10k(4k+1)$ and $m_3 = 6k(4k+1)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (8k+1)^2$ and degree $r = 10k(4k+1)$ with $\tau = 25k^2+7k-1$ and $\theta = 5k(5k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(3k+1)$ with $m_2 = 6k(4k+1)$ and $m_3 = 10k(4k+1)$;*

(3^0) *G is a strongly regular graph of order $n = 105(8k-3)^2$ and degree $r = 840k^2-630k+118$ with $\tau = 105k^2-91k+19$ and $\theta = 7(3k-1)(5k-2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 21k-8$ and $\lambda_3 = -(35k-13)$ with $m_2 = 5(840k^2-630k+118)$ and $m_3 = 3(840k^2-630k+118)$;*

($\overline{3}^0$) *G is a strongly regular graph of order $n = 105(8k-3)^2$ and degree $r = 7(840k^2-630k+118)$ with $\tau = 7(735k^2-551k+103)$ and $\theta = 7(21k-8)(35k-13)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 35k-14$ and $\lambda_3 = -(21k-7)$ with $m_2 = 3(840k^2-630k+118)$ and $m_3 = 5(840k^2-630k+118)$;*

(4^0) *G is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 840k^2+630k+118$ with $\tau = 105k^2+91k+19$ and $\theta = 7(3k+1)(5k+2)$,* *where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 35k + 13$ *and* $\lambda_3 = -(21k + 8)$ *with* $m_2 = 3(840k^2 + 630k + 118)$ *and* $m_3 = 5(840k^2 + 630k + 118)$*;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 105(8k + 3)^2$ *and degree* $r = 7(840k^2 + 630k + 118)$ *with* $\tau = 7(735k^2 + 551k + 103)$ *and* $\theta = 7(21k + 8)(35k + 13)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 21k + 7$ *and* $\lambda_3 = -(35k + 14)$ *with* $m_2 = 5(840k^2 + 630k + 118)$ *and* $m_3 = 3(840k^2 + 630k + 118)$*.*

Proof. First, according to Remark 2.3 we have $3\alpha(\beta - 1) = 5(\alpha - 1)$, from which we find no integral solution for α and β. Next, according to Proposition 2.7 it turns out that G belongs to the class ($\overline{1}^0$) or (2^0) or (3^0) or ($\overline{4}^0$) if $m_2 = (\frac{5}{3})m_3$. According to Proposition 2.8 it turns out that G belongs to the class (1^0) or ($\overline{2}^0$) or ($\overline{3}^0$) or (4^0) if $m_3 = (\frac{5}{3})m_2$. □

Proposition 2.9. *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = (\frac{5}{4})m_3$*. Then G belongs to the class* ($\overline{2}^0$) *or* (3^0) *or* (4^0) *or* ($\overline{5}^0$) *or* ($\overline{6}^0$) *or* (7^0) *or* ($\overline{8}^0$) *or* (9^0) *represented in Theorem* 2.7*.*

Proof. Let $m_2 = 5p$, $m_3 = 4p$ and $n = 9p + 1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 2.1 we have (i) $\lambda_3 = -\frac{5k+t}{4}$; (ii) $\tau - \theta = -\frac{k+t}{4}$; (iii) $\delta = \frac{9k+t}{4}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{5k^2+kt}{4}$, where $t = 1, 2, \ldots, 8$. In this case we can easily see that Theorem 2.1 (8^0) reduces to

$$(4p + 1)t^2 - 4(9p + 1)t + 45k^2 + 10kt = 0. \tag{2.9}$$

Case 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+1}{4}$, $\tau - \theta = -\frac{k+1}{4}$, $\delta = \frac{9k+1}{4}$, $r = p$ and $\theta = p - \frac{5k^2+k}{4}$. Using (2.9) we find that $32p + 3 = 5k(9k + 2)$. Replacing k with $8k - 1$ we arrive at $p = 90k^2 - 20k + 1$. So we obtain that G is a strongly regular graph of order $n = 10(9k - 1)^2$ and degree $r = 90k^2 - 20k + 1$ with $\tau = 2k(5k - 2)$ and $\theta = 2k(5k - 1)$.

Case 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+2}{4}$, $\tau - \theta = -\frac{k+2}{4}$, $\delta = \frac{9k+2}{4}$, $r = 2p$ and $\theta = 2p - \frac{5k^2+2k}{4}$. Using (2.9) we find that $56p+4 = 5k(9k+4)$. Replacing k with $28k+6$ we arrive at $p = 630k^2+280k+31$. So we obtain that G is a strongly regular graph of order $n = 70(9k+2)^2$ and degree $r = 2(630k^2 + 280k + 31)$ with $\tau = 280k^2 + 119k + 12$ and $\theta = 14(4k + 1)(5k + 1)$.

Case 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+3}{4}$, $\tau - \theta = -\frac{k+3}{4}$, $\delta = \frac{9k+3}{4}$, $r = 3p$ and $\theta = 3p - \frac{5k^2+3k}{4}$. Using (2.9) we find that $24p+1 = 5k(3k+2)$. Replacing k with $12k+1$ we arrive at $p = 90k^2+20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 3(90k^2 + 20k + 1)$ with $\tau = 18k(5k + 1)$ and $\theta = (6k + 1)(15k + 1)$.

Case 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+4}{4}$, $\tau - \theta = -\frac{k+4}{4}$, $\delta = \frac{9k+4}{4}$, $r = 4p$ and $\theta = 4p - \frac{5k^2+4k}{4}$. Using (2.9) we find that $16p = k(9k+8)$. Replacing k with $4k$ we arrive at $p = k(9k+2)$. So we obtain that G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 4k(9k+2)$ with $\tau = 16k^2 + 3k - 1$ and $\theta = 4k(4k + 1)$.

Case 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+5}{4}$, $\tau - \theta = -\frac{k+5}{4}$, $\delta = \frac{9k+5}{4}$, $r = 5p$ and $\theta = 5p - \frac{5k^2+5k}{4}$. Using (2.9) we find that $16p - 1 = k(9k + 10)$. Replacing k with $4k - 1$ we arrive at $p = k(9k - 2)$. So we obtain that G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 5k(9k - 2)$ with $\tau = 25k^2 - 6k - 1$ and $\theta = 5k(5k - 1)$.

Case 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+6}{4}$, $\tau - \theta = -\frac{k+6}{4}$, $\delta = \frac{9k+6}{4}$, $r = 6p$ and $\theta = 6p - \frac{5k^2+6k}{4}$. Using (2.9) we find that $24p-4 = 5k(3k+4)$. Replacing k with $12k-2$ we arrive at $p = 90k^2-20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k-1)^2$ and degree $r = 6(90k^2 - 20k + 1)$ with $\tau = 3(120k^2 - 27k + 1)$ and $\theta = 2(12k - 1)(15k - 2)$.

Case 7 ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+7}{4}$, $\tau - \theta = -\frac{k+7}{4}$, $\delta = \frac{9k+7}{4}$, $r = 7p$ and $\theta = 7p - \frac{5k^2+7k}{4}$. Using (2.9) we find that $56p - 21 = 5k(9k + 14)$. Replacing k with $28k - 7$ we arrive at $p = 630k^2 - 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k-2)^2$ and degree $r = 7(630k^2-280k+31)$ with $\tau = 14(5k-1)(49k-12)$ and $\theta = 7(14k - 3)(35k - 8)$.

Case 8. ($t = 8$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{5k+8}{4}$, $\tau - \theta = -\frac{k+8}{4}$, $\delta = \frac{9k+8}{4}$, $r = 8p$ and $\theta = 8p - \frac{5k^2+8k}{4}$. Using (2.9) we find that $32p-32 = 5k(9k+16)$. Replacing k with $8k$ we arrive at $p = 90k^2+20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 8(90k^2 + 20k + 1)$ with $\tau = 2(320k^2 + 71k + 3)$ and $\theta = 8(8k + 1)(10k + 1)$. □

Proposition 2.10. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{5}{4})m_2$. Then G belongs to the class (2^0) or $(\overline{3}^0)$ or $(\overline{4}^0)$ or (5^0) or (6^0) or $(\overline{7}^0)$ or (8^0) or $(\overline{9}^0)$ represented in Theorem 2.7.*

Proof. Let $m_2 = 4p$, $m_3 = 5p$ and $n = 9p+1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 2.2 we have (i) $\lambda_2 = \frac{5k-t}{4}$; (ii) $\tau-\theta = \frac{k-t}{4}$; (iii) $\delta = \frac{9k-t}{4}$; (iv) $r = pt$ and (v) $\theta = pt-\frac{5k^2-kt}{4}$, where $t = 1, 2, \dots, 8$. In this case we can easily see that Theorem 2.2 (8^0) reduces to

$$(4p + 1)t^2 - 4(9p + 1)t + 45k^2 - 10kt = 0. \tag{2.10}$$

Case 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-1}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-1}{4}$, $\delta = \frac{9k-1}{4}$, $r = p$ and $\theta = p - \frac{5k^2-k}{4}$. Using (2.10) we find that $32p+3 = 5k(9k-2)$. Replacing k with $8k+1$ we arrive at $p = 90k^2+20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 90k^2 + 20k + 1$ with $\tau = 2k(5k + 2)$ and $\theta = 2k(5k + 1)$.

Case 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-2}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-2}{4}$, $\delta = \frac{9k-2}{4}$, $r = 2p$ and $\theta = 2p - \frac{5k^2-2k}{4}$. Using (2.10) we find that $56p+4 = 5k(9k-4)$. Replacing k with $28k-6$ we arrive at $p = 630k^2-280k+31$. So we obtain that G is a strongly regular graph of order $n = 70(9k-2)^2$ and degree $r = 2(630k^2 - 280k + 31)$ with $\tau = 280k^2 - 119k + 12$ and $\theta = 14(4k - 1)(5k - 1)$.

Case 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-3}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-3}{4}$, $\delta = \frac{9k-3}{4}$, $r = 3p$ and $\theta = 3p - \frac{5k^2-3k}{4}$. Using (2.10) we find that $24p+1 = 5k(3k-2)$. Replacing k with $12k-1$ we arrive at $p = 90k^2-20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k-1)^2$ and degree $r = 3(90k^2 - 20k + 1)$ with $\tau = 18k(5k - 1)$ and $\theta = (6k - 1)(15k - 1)$.

Case 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-4}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-4}{4}$, $\delta = \frac{9k-4}{4}$, $r = 4p$ and $\theta = 4p - \frac{5k^2-4k}{4}$. Using (2.10) we find that $16p = k(9k-8)$. Replacing k with $4k$ we arrive at $p = k(9k-2)$. So we obtain that G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 4k(9k-2)$ with $\tau = 16k^2 - 3k - 1$ and $\theta = 4k(4k - 1)$.

Case 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-5}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-5}{4}$, $\delta = \frac{9k-5}{4}$, $r = 5p$ and $\theta = 5p - \frac{5k^2-5k}{4}$. Using (2.10) we find that $16p - 1 = k(9k - 10)$. Replacing k with $4k + 1$ we arrive at $p = k(9k + 2)$. So we obtain that G is a strongly regular graph of order $n = (9k + 1)^2$ and degree $r = 5k(9k + 2)$ with $\tau = 25k^2 + 6k - 1$ and $\theta = 5k(5k + 1)$.

Case 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-6}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-6}{4}$, $\delta = \frac{9k-6}{4}$, $r = 6p$ and $\theta = 6p - \frac{5k^2-6k}{4}$. Using (2.10) we find that $24p-4 = 5k(3k-4)$. Replacing k with $12k+2$ we arrive at $p = 90k^2+20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 6(90k^2 + 20k + 1)$ with $\tau = 3(120k^2 + 27k + 1)$ and $\theta = 2(12k + 1)(15k + 2)$.

Case 7 ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-7}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-7}{4}$, $\delta = \frac{9k-7}{4}$, $r = 7p$ and $\theta = 7p - \frac{5k^2-7k}{4}$. Using (2.10) we find that $56p - 21 = 5k(9k - 14)$. Replacing k with $28k + 7$ we arrive at $p = 630k^2 + 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k+2)^2$ and degree $r = 7(630k^2+280k+31)$ with $\tau = 14(5k+1)(49k+12)$ and $\theta = 7(14k + 3)(35k + 8)$.

Case 8 ($t = 8$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{5k-8}{4}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-8}{4}$, $\delta = \frac{9k-8}{4}$, $r = 8p$ and $\theta = 8p - \frac{5k^2-8k}{4}$. Using (2.10) we find that $32p - 32 = 5k(9k - 16)$. Replacing k with $8k$ we arrive at $p = 90k^2 - 20k + 1$. So we obtain that G is a strongly regular graph of order $n = 10(9k-1)^2$ and degree $r = 8(90k^2 - 20k + 1)$ with $\tau = 2(320k^2 - 71k + 3)$ and $\theta = 8(8k - 1)(10k - 1)$. □

Remark 2.7. We note that $\overline{5K_2}$ is a strongly regular graph with $m_2 = (\frac{5}{4})m_3$. It is obtained from class Theorem 2.7 ($\overline{6}^0$) for $k = 0$.

Theorem 2.7. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{5}{4})m_3$ or $m_3 = (\frac{5}{4})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is the strongly regular graph $\overline{5K_2}$ of order $n = 10$ and degree $r = 8$ with $\tau = 6$ and $\theta = 8$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -2$ with $m_2 = 5$ and $m_3 = 4$;*

(2^0) *G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 4k(9k - 2)$ with $\tau = 16k^2 - 3k - 1$ and $\theta = 4k(4k - 1)$, where $k \in \mathbb{N}$. Its* *eigenvalues are* $\lambda_2 = 5k-1$ *and* $\lambda_3 = -4k$ *with* $m_2 = 4k(9k-2)$ *and* $m_3 = 5k(9k-2)$*;*

($\overline{2}^0$) G *is a strongly regular graph of order* $n = (9k-1)^2$ *and degree* $r = 5k(9k-2)$ *with* $\tau = 25k^2-6k-1$ *and* $\theta = 5k(5k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k-1$ *and* $\lambda_3 = -5k$ *with* $m_2 = 5k(9k-2)$ *and* $m_3 = 4k(9k-2)$*;*

(3^0) G *is a strongly regular graph of order* $n = (9k+1)^2$ *and degree* $r = 4k(9k+2)$ *with* $\tau = 16k^2+3k-1$ *and* $\theta = 4k(4k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k$ *and* $\lambda_3 = -(5k+1)$ *with* $m_2 = 5k(9k+2)$ *and* $m_3 = 4k(9k+2)$*;*

($\overline{3}^0$) G *is a strongly regular graph of order* $n = (9k+1)^2$ *and degree* $r = 5k(9k+2)$ *with* $\tau = 25k^2+6k-1$ *and* $\theta = 5k(5k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 5k$ *and* $\lambda_3 = -(4k+1)$ *with* $m_2 = 4k(9k+2)$ *and* $m_3 = 5k(9k+2)$*;*

(4^0) G *is a strongly regular graph of order* $n = 10(9k-1)^2$ *and degree* $r = 90k^2-20k+1$ *with* $\tau = 2k(5k-2)$ *and* $\theta = 2k(5k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 8k-1$ *and* $\lambda_3 = -(10k-1)$ *with* $m_2 = 5(90k^2-20k+1)$ *and* $m_3 = 4(90k^2-20k+1)$*;*

($\overline{4}^0$) G *is a strongly regular graph of order* $n = 10(9k-1)^2$ *and degree* $r = 8(90k^2-20k+1)$ *with* $\tau = 2(320k^2-71k+3)$ *and* $\theta = 8(8k-1)(10k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 10k-2$ *and* $\lambda_3 = -8k$ *with* $m_2 = 4(90k^2-20k+1)$ *and* $m_3 = 5(90k^2-20k+1)$*;*

(5^0) G *is a strongly regular graph of order* $n = 10(9k-1)^2$ *and degree* $r = 3(90k^2-20k+1)$ *with* $\tau = 18k(5k-1)$ *and* $\theta = (6k-1)(15k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 15k-2$ *and* $\lambda_3 = -(12k-1)$ *with* $m_2 = 4(90k^2-20k+1)$ *and* $m_3 = 5(90k^2-20k+1)$*;*

($\overline{5}^0$) G *is a strongly regular graph of order* $n = 10(9k-1)^2$ *and degree* $r = 6(90k^2-20k+1)$ *with* $\tau = 3(120k^2-27k+1)$ *and* $\theta = 2(12k-1)(15k-2)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 12k-2$ *and* $\lambda_3 = -(15k-1)$ *with* $m_2 = 5(90k^2-20k+1)$ *and* $m_3 = 4(90k^2-20k+1)$*;*

(6^0) G *is a strongly regular graph of order* $n = 10(9k+1)^2$ *and degree* $r = 90k^2+20k+1$ *with* $\tau = 2k(5k+2)$ *and* $\theta = 2k(5k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 10k+1$ *and* $\lambda_3 = -(8k+1)$ *with* $m_2 = 4(90k^2+20k+1)$ *and* $m_3 = 5(90k^2+20k+1)$*;*

($\overline{6}^0$) G *is a strongly regular graph of order* $n = 10(9k+1)^2$ *and degree* $r = 8(90k^2+20k+1)$ *with* $\tau = 2(320k^2+71k+3)$ *and* $\theta = 8(8k+1)(10k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 8k$ *and* $\lambda_3 = -(10k+2)$ *with* $m_2 = 5(90k^2+20k+1)$ *and* $m_3 = 4(90k^2+20k+1)$*;*

(7^0) G *is a strongly regular graph of order* $n = 10(9k+1)^2$ *and degree* $r = 3(90k^2+20k+1)$ *with* $\tau = 18k(5k+1)$ *and* $\theta = (6k+1)(15k+1)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 12k+1$ *and* $\lambda_3 = -(15k+2)$ *with* $m_2 = 5(90k^2+20k+1)$ *and* $m_3 = 4(90k^2+20k+1)$*;*

($\overline{7}^0$) G *is a strongly regular graph of order* $n = 10(9k+1)^2$ *and degree* $r = 6(90k^2+20k+1)$ *with* $\tau = 3(120k^2+27k+1)$ *and* $\theta = 2(12k+1)(15k+2)$*,* *where $k \geqslant 0$. Its eigenvalues are $\lambda_2 = 15k + 1$ and $\lambda_3 = -(12k + 2)$ with $m_2 = 4(90k^2 + 20k + 1)$ and $m_3 = 5(90k^2 + 20k + 1)$;*

(8^0) *G is a strongly regular graph of order $n = 70(9k - 2)^2$ and degree $r = 2(630k^2-280k+31)$ with $\tau = 280k^2-119k+12$ and $\theta = 14(4k-1)(5k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 35k - 8$ and $\lambda_3 = -(28k - 6)$ with $m_2 = 4(630k^2 - 280k + 31)$ and $m_3 = 5(630k^2 - 280k + 31)$;*

$(\overline{8}^0)$ *G is a strongly regular graph of order $n = 70(9k - 2)^2$ and degree $r = 7(630k^2-280k+31)$ with $\tau = 14(5k-1)(49k-12)$ and $\theta = 7(14k-3)(35k-8)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 28k - 7$ and $\lambda_3 = -(35k - 7)$ with $m_2 = 5(630k^2 - 280k + 31)$ and $m_3 = 4(630k^2 - 280k + 31)$;*

(9^0) *G is a strongly regular graph of order $n = 70(9k + 2)^2$ and degree $r = 2(630k^2+280k+31)$ with $\tau = 280k^2+119k+12$ and $\theta = 14(4k+1)(5k+1)$, where $k \geqslant 0$. Its eigenvalues are $\lambda_2 = 28k + 6$ and $\lambda_3 = -(35k + 8)$ with $m_2 = 5(630k^2 + 280k + 31)$ and $m_3 = 4(630k^2 + 280k + 31)$;*

$(\overline{9}^0)$ *G is a strongly regular graph of order $n = 70(9k + 2)^2$ and degree $r = 7(630k^2+280k+31)$ with $\tau = 14(5k+1)(49k+12)$ and $\theta = 7(14k+3)(35k+8)$, where $k \geqslant 0$. Its eigenvalues are $\lambda_2 = 35k + 7$ and $\lambda_3 = -(28k + 7)$ with $m_2 = 4(630k^2 + 280k + 31)$ and $m_3 = 5(630k^2 + 280k + 31)$.*

Proof. First, according to Remark 2.3 we have $4\alpha(\beta - 1) = 5(\alpha - 1)$, from which we find that $\alpha = 5$, $\beta = 2$. In view of this we obtain the strongly regular graph represented in Theorem 2.7 (1^0). Next, according to Proposition 2.9 it turns out that G belongs to the class $(\overline{2}^0)$ or (3^0) or (4^0) or $(\overline{5}^0)$ or $(\overline{6}^0)$ or (7^0) or $(\overline{8}^0)$ or (9^0) if $m_2 = (\frac{5}{4})m_3$. According to Proposition 2.10 it turns out that G belongs to the class (2^0) or $(\overline{3}^0)$ or $(\overline{4}^0)$ or (5^0) or (6^0) or $(\overline{7}^0)$ or (8^0) or $(\overline{9}^0)$ if $m_3 = (\frac{5}{4})m_2$. □

Proposition 2.11. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{6}{5})m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) or (4^0) or $(\overline{5}^0)$ or (6^0) or $(\overline{7}^0)$ or (8^0) or $(\overline{9}^0)$ or $(\overline{10}^0)$ or (11^0) represented in Theorem 2.8.*

Proof. Let $m_2 = 6p$, $m_3 = 5p$ and $n = 11p+1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 2.1 we have (i) $\lambda_3 = -\frac{6k+t}{5}$; (ii) $\tau - \theta = -\frac{k+t}{5}$; (iii) $\delta = \frac{11k+t}{5}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{6k^2+kt}{5}$, where $t = 1, 2, \ldots, 10$. In this case we can easily see that Theorem 2.1 (8^0) reduces to

$$(5p + 1)t^2 - 5(11p + 1)t + 66k^2 + 12kt = 0. \tag{2.11}$$

Case 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+1}{5}$, $\tau - \theta = -\frac{k+1}{5}$, $\delta = \frac{11k+1}{5}$, $r = p$ and $\theta = p - \frac{6k^2+k}{5}$. Using (2.11) we find that $25p+2 = 3k(11k+2)$. Replacing k with $5k-1$ we arrive at $p = 33k^2-12k+1$. So we obtain that G is a strongly regular graph of order $n = 3(11k-2)^2$ and degree $r = 33k^2 - 12k + 1$ with $\tau = k(3k - 2)$ and $\theta = k(3k - 1)$.

Case 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+2}{5}$, $\tau-\theta = -\frac{k+2}{5}$, $\delta = \frac{11k+2}{5}$, $r = 2p$ and $\theta = 2p-\frac{6k^2+2k}{5}$. Using (2.11) we find that $15p+1 = k(11k+4)$. Replacing k with $15k-7$ we arrive at $p = 165k^2-150k+34$.

So we obtain that G is a strongly regular graph of order $n = 15(11k-5)^2$ and degree $r = 2(165k^2 - 150k + 34)$ with $\tau = 60k^2 - 57k + 13$ and $\theta = 6(2k-1)(5k-2)$.

Case 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+3}{5}$, $\tau - \theta = -\frac{k+3}{5}$, $\delta = \frac{11k+3}{5}$, $r = 3p$ and $\theta = 3p - \frac{6k^2+3k}{5}$. Using (2.11) we find that $20p+1 = k(11k+6)$. Replacing k with $10k-3$ we arrive at $p = 55k^2 - 30k + 4$. So we obtain that G is a strongly regular graph of order $n = 5(11k-3)^2$ and degree $r = 3(55k^2 - 30k + 4)$ with $\tau = 45k^2 - 26k + 3$ and $\theta = 3(3k-1)(5k-1)$.

Case 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+4}{5}$, $\tau - \theta = -\frac{k+4}{5}$, $\delta = \frac{11k+4}{5}$, $r = 4p$ and $\theta = 4p - \frac{6k^2+4k}{5}$. Using (2.11) we find that $70p + 2 = 3k(11k + 8)$. Replacing k with $70k + 6$ we arrive at $p = 2310k^2 + 420k + 19$. So we obtain that G is a strongly regular graph of order $n = 210(11k+1)^2$ and degree $r = 4(2310k^2+420k+19)$ with $\tau = 2(1680k^2+301k+13)$ and $\theta = 28(10k+1)(12k+1)$.

Case 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+5}{5}$, $\tau - \theta = -\frac{k+5}{5}$, $\delta = \frac{11k+5}{5}$, $r = 5p$ and $\theta = 5p - \frac{6k^2+5k}{5}$. Using (2.11) we find that $25p = k(11k + 10)$. Replacing k with $5k$ we arrive at $p = k(11k + 2)$. So we obtain that G is a strongly regular graph of order $n = (11k + 1)^2$ and degree $r = 5k(11k + 2)$ with $\tau = 25k^2 + 4k - 1$ and $\theta = 5k(5k + 1)$.

Case 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+6}{5}$, $\tau - \theta = -\frac{k+6}{5}$, $\delta = \frac{11k+6}{5}$, $r = 6p$ and $\theta = 6p - \frac{6k^2+6k}{5}$. Using (2.11) we find that $25p - 1 = k(11k + 12)$. Replacing k with $5k - 1$ we arrive at $p = k(11k - 2)$. So we obtain that G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = 6k(11k - 2)$ with $\tau = 36k^2 - 7k - 1$ and $\theta = 6k(6k - 1)$.

Case 7 ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+7}{5}$, $\tau - \theta = -\frac{k+7}{5}$, $\delta = \frac{11k+7}{5}$, $r = 7p$ and $\theta = 7p - \frac{6k^2+7k}{5}$. Using (2.11) we find that $70p - 7 = 3k(11k + 14)$. Replacing k with $70k - 7$ we arrive at $p = 2310k^2 - 420k + 19$. So we obtain that G is a strongly regular graph of order $n = 210(11k-1)^2$ and degree $r = 7(2310k^2-420k+19)$ with $\tau = 14(735k^2-134k+6)$ and $\theta = 14(21k - 2)(35k - 3)$.

Case 8 ($t = 8$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+8}{5}$, $\tau - \theta = -\frac{k+8}{5}$, $\delta = \frac{11k+8}{5}$, $r = 8p$ and $\theta = 8p - \frac{6k^2+8k}{5}$. Using (2.11) we find that $20p-4 = k(11k+16)$. Replacing k with $10k+2$ we arrive at $p = 55k^2+30k+4$. So we obtain that G is a strongly regular graph of order $n = 5(11k+3)^2$ and degree $r = 8(55k^2 + 30k + 4)$ with $\tau = 2(5k + 1)(32k + 11)$ and $\theta = 8(4k + 1)(10k + 3)$.

Case 9 ($t = 9$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+9}{5}$, $\tau - \theta = -\frac{k+9}{5}$, $\delta = \frac{11k+9}{5}$, $r = 9p$ and $\theta = 9p - \frac{6k^2+9k}{5}$. Using (2.11) we find that $15p - 6 = k(11k + 18)$. Replacing k with $15k + 6$ we arrive at $p = 165k^2 + 150k + 34$. So we obtain that G is a strongly regular graph of order $n = 15(11k+5)^2$ and degree $r = 9(165k^2+150k+34)$ with $\tau = 3(405k^2+368k+83)$ and $\theta = 9(9k + 4)(15k + 7)$.

Case 10 ($t = 10$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{6k+10}{5}$, $\tau - \theta = -\frac{k+10}{5}$, $\delta = \frac{11k+10}{5}$, $r = 10p$ and $\theta = 10p - \frac{6k^2+10k}{5}$. Using

(2.11) we find that $25p - 25 = 3k(11k + 20)$. Replacing k with $5k$ we arrive at $p = 33k^2 + 12k + 1$. So we obtain that G is a strongly regular graph of order $n = 3(11k + 2)^2$ and degree $r = 10(33k^2 + 12k + 1)$ with $\tau = 300k^2 + 109k + 8$ and $\theta = 10(5k + 1)(6k + 1)$. □

Proposition 2.12. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{6}{5})m_2$. Then G belongs to the class* (2^0) *or* $(\overline{3}^0)$ *or* $(\overline{4}^0)$ *or* (5^0) *or* $(\overline{6}^0)$ *or* (7^0) *or* $(\overline{8}^0)$ *or* (9^0) *or* (10^0) *or* $(\overline{11}^0)$ *represented in Theorem* 2.8.

Proof. Let $m_2 = 5p$, $m_3 = 6p$ and $n = 11p + 1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 2.2 we have (i) $\lambda_2 = \frac{6k-t}{5}$; (ii) $\tau - \theta = \frac{k-t}{5}$; (iii) $\delta = \frac{11k-t}{5}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{6k^2-kt}{5}$, where $t = 1, 2, \ldots, 10$. In this case we can easily see that Theorem 2.2 (8^0) reduces to

$$(5p + 1)t^2 - 5(11p + 1)t + 66k^2 - 12kt = 0. \tag{2.12}$$

Case 1 ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-1}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-1}{5}$, $\delta = \frac{11k-1}{5}$, $r = p$ and $\theta = p - \frac{6k^2-k}{5}$. Using (2.12) we find that $25p+2 = 3k(11k-2)$. Replacing k with $5k+1$ we arrive at $p = 33k^2+12k+1$. So we obtain that G is a strongly regular graph of order $n = 3(11k+2)^2$ and degree $r = 33k^2 + 12k + 1$ with $\tau = k(3k + 2)$ and $\theta = k(3k + 1)$.

Case 2 ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-2}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-2}{5}$, $\delta = \frac{11k-2}{5}$, $r = 2p$ and $\theta = 2p - \frac{6k^2-2k}{5}$. Using (2.12) we find that $15p+1 = k(11k-4)$. Replacing k with $15k+7$ we arrive at $p = 165k^2+150k+34$. So we obtain that G is a strongly regular graph of order $n = 15(11k+5)^2$ and degree $r = 2(165k^2 + 150k + 34)$ with $\tau = 60k^2 + 57k + 13$ and $\theta = 6(2k + 1)(5k + 2)$.

Case 3 ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-3}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-3}{5}$, $\delta = \frac{11k-3}{5}$, $r = 3p$ and $\theta = 3p - \frac{6k^2-3k}{5}$. Using (2.12) we find that $20p+1 = k(11k-6)$. Replacing k with $10k+3$ we arrive at $p = 55k^2+30k+4$. So we obtain that G is a strongly regular graph of order $n = 5(11k+3)^2$ and degree $r = 3(55k^2 + 30k + 4)$ with $\tau = 45k^2 + 26k + 3$ and $\theta = 3(3k + 1)(5k + 1)$.

Case 4 ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-4}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-4}{5}$, $\delta = \frac{11k-4}{5}$, $r = 4p$ and $\theta = 4p - \frac{6k^2-4k}{5}$. Using (2.12) we find that $70p + 2 = 3k(11k - 8)$. Replacing k with $70k - 6$ we arrive at $p = 2310k^2 - 420k + 19$. So we obtain that G is a strongly regular graph of order $n = 210(11k-1)^2$ and degree $r = 4(2310k^2-420k+19)$ with $\tau = 2(1680k^2-301k+13)$ and $\theta = 28(10k - 1)(12k - 1)$.

Case 5 ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-5}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-5}{5}$, $\delta = \frac{11k-5}{5}$, $r = 5p$ and $\theta = 5p - \frac{6k^2-5k}{5}$. Using (2.12) we find that $25p = k(11k - 10)$. Replacing k with $5k$ we arrive at $p = k(11k - 2)$. So we obtain that G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = 5k(11k - 2)$ with $\tau = 25k^2 - 4k - 1$ and $\theta = 5k(5k - 1)$.

Case 6 ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-6}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-6}{5}$, $\delta = \frac{11k-6}{5}$, $r = 6p$ and $\theta = 6p - \frac{6k^2-6k}{5}$. Using (2.12) we find that $25p - 1 = k(11k - 12)$. Replacing k with $5k + 1$ we arrive at $p = k(11k + 2)$. So we obtain that G is a strongly regular graph of order $n = (11k + 1)^2$ and degree $r = 6k(11k + 2)$ with $\tau = 36k^2 + 7k - 1$ and $\theta = 6k(6k + 1)$.

Case 7 ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-7}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-7}{5}$, $\delta = \frac{11k-7}{5}$, $r = 7p$ and $\theta = 7p - \frac{6k^2-7k}{5}$. Using (2.12) we find that $70p - 7 = 3k(11k - 14)$. Replacing k with $70k + 7$ we arrive at $p = 2310k^2 + 420k + 19$. So we obtain that G is a strongly regular graph of order $n = 210(11k+1)^2$ and degree $r = 7(2310k^2+420k+19)$ with $\tau = 14(735k^2+134k+6)$ and $\theta = 14(21k + 2)(35k + 3)$.

Case 8 ($t = 8$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-8}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-8}{5}$, $\delta = \frac{11k-8}{5}$, $r = 8p$ and $\theta = 8p - \frac{6k^2-8k}{5}$. Using (2.12) we find that $20p - 4 = k(11k - 16)$. Replacing k with $10k - 2$ we arrive at $p = 55k^2 - 30k + 4$. So we obtain that G is a strongly regular graph of order $n = 5(11k - 3)^2$ and degree $r = 8(55k^2 - 30k + 4)$ with $\tau = 2(5k - 1)(32k - 11)$ and $\theta = 8(4k - 1)(10k - 3)$.

Case 9 ($t = 9$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-9}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-9}{5}$, $\delta = \frac{11k-9}{5}$, $r = 9p$ and $\theta = 9p - \frac{6k^2-9k}{5}$. Using (2.12) we find that $15p - 6 = k(11k - 18)$. Replacing k with $15k - 6$ we arrive at $p = 165k^2 - 150k + 34$. So we obtain that G is a strongly regular graph of order $n = 15(11k - 5)^2$ and degree $r = 9(165k^2 - 150k + 34)$ with $\tau = 3(405k^2 - 368k + 83)$ and $\theta = 9(9k - 4)(15k - 7)$.

Case 10 ($t = 10$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = \frac{6k-10}{5}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{k-10}{5}$, $\delta = \frac{11k-10}{5}$, $r = 10p$ and $\theta = 10p - \frac{6k^2-10k}{5}$. Using (2.12) we find that $25p - 25 = 3k(11k - 20)$. Replacing k with $5k$ we arrive at $p = 33k^2 - 12k + 1$. So we obtain that G is a strongly regular graph of order $n = 3(11k - 2)^2$ and degree $r = 10(33k^2 - 12k + 1)$ with $\tau = 300k^2 - 109k + 8$ and $\theta = 10(5k - 1)(6k - 1)$. □

Remark 2.8. We note that $\overline{6K_2}$ is a strongly regular graph with $m_2 = (\frac{6}{5})m_3$. It is obtained from the class Theorem 2.8 ($\overline{5}^0$) for $k = 0$.

Theorem 2.8. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{6}{5})m_3$ or $m_3 = (\frac{6}{5})m_2$. Then G is one of the following strongly regular graphs:*

- (1^0) *G is the strongly regular graph $\overline{6K_2}$ of order $n = 12$ and degree $r = 10$ with $\tau = 8$ and $\theta = 10$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -2$ with $m_2 = 6$ and $m_3 = 5$;*
- (2^0) *G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = 5k(11k - 2)$ with $\tau = 25k^2 - 4k - 1$ and $\theta = 5k(5k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k - 1$ and $\lambda_3 = -5k$ with $m_2 = 5k(11k - 2)$ and $m_3 = 6k(11k - 2)$;*
- ($\overline{2}^0$) *G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = 6k(11k - 2)$ with $\tau = 36k^2 - 7k - 1$ and $\theta = 6k(6k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k - 1$ and $\lambda_3 = -6k$ with $m_2 = 6k(11k - 2)$ and $m_3 = 5k(11k - 2)$;*

(3^0) *G is a strongly regular graph of order* $n = (11k+1)^2$ *and degree* $r = 5k(11k+2)$ *with* $\tau = 25k^2+4k-1$ *and* $\theta = 5k(5k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 5k$ *and* $\lambda_3 = -(6k+1)$ *with* $m_2 = 6k(11k+2)$ *and* $m_3 = 5k(11k+2)$;

($\overline{3}^0$) *G is a strongly regular graph of order* $n = (11k+1)^2$ *and degree* $r = 6k(11k+2)$ *with* $\tau = 36k^2+7k-1$ *and* $\theta = 6k(6k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 6k$ *and* $\lambda_3 = -(5k+1)$ *with* $m_2 = 5k(11k+2)$ *and* $m_3 = 6k(11k+2)$;

(4^0) *G is a strongly regular graph of order* $n = 3(11k-2)^2$ *and degree* $r = 33k^2-12k+1$ *with* $\tau = k(3k-2)$ *and* $\theta = k(3k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 5k-1$ *and* $\lambda_3 = -(6k-1)$ *with* $m_2 = 6(33k^2-12k+1)$ *and* $m_3 = 5(33k^2-12k+1)$;

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 3(11k-2)^2$ *and degree* $r = 10(33k^2-12k+1)$ *with* $\tau = 300k^2-109k+8$ *and* $\theta = 10(5k-1)(6k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 6k-2$ *and* $\lambda_3 = -5k$ *with* $m_2 = 5(33k^2-12k+1)$ *and* $m_3 = 6(33k^2-12k+1)$;

(5^0) *G is a strongly regular graph of order* $n = 3(11k+2)^2$ *and degree* $r = 33k^2+12k+1$ *with* $\tau = k(3k+2)$ *and* $\theta = k(3k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 6k+1$ *and* $\lambda_3 = -(5k+1)$ *with* $m_2 = 5(33k^2+12k+1)$ *and* $m_3 = 6(33k^2+12k+1)$;

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 3(11k+2)^2$ *and degree* $r = 10(33k^2+12k+1)$ *with* $\tau = 300k^2+109k+8$ *and* $\theta = 10(5k+1)(6k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 5k$ *and* $\lambda_3 = -(6k+2)$ *with* $m_2 = 6(33k^2+12k+1)$ *and* $m_3 = 5(33k^2+12k+1)$;

(6^0) *G is a strongly regular graph of order* $n = 5(11k-3)^2$ *and degree* $r = 3(55k^2-30k+4)$ *with* $\tau = 45k^2-26k+3$ *and* $\theta = 3(3k-1)(5k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 10k-3$ *and* $\lambda_3 = -(12k-3)$ *with* $m_2 = 6(55k^2-30k+4)$ *and* $m_3 = 5(55k^2-30k+4)$;

($\overline{6}^0$) *G is a strongly regular graph of order* $n = 5(11k-3)^2$ *and degree* $r = 8(55k^2-30k+4)$ *with* $\tau = 2(5k-1)(32k-11)$ *and* $\theta = 8(4k-1)(10k-3)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k-4$ *and* $\lambda_3 = -(10k-2)$ *with* $m_2 = 5(55k^2-30k+4)$ *and* $m_3 = 6(55k^2-30k+4)$;

(7^0) *G is a strongly regular graph of order* $n = 5(11k+3)^2$ *and degree* $r = 3(55k^2+30k+4)$ *with* $\tau = 45k^2+26k+3$ *and* $\theta = 3(3k+1)(5k+1)$, *where* $k \geqslant 0$. *Its eigenvalues are* $\lambda_2 = 12k+3$ *and* $\lambda_3 = -(10k+3)$ *with* $m_2 = 5(55k^2+30k+4)$ *and* $m_3 = 6(55k^2+30k+4)$;

($\overline{7}^0$) *G is a strongly regular graph of order* $n = 5(11k+3)^2$ *and degree* $r = 8(55k^2+30k+4)$ *with* $\tau = 2(5k+1)(32k+11)$ *and* $\theta = 8(4k+1)(10k+3)$, *where* $k \geqslant 0$. *Its eigenvalues are* $\lambda_2 = 10k+2$ *and* $\lambda_3 = -(12k+4)$ *with* $m_2 = 6(55k^2+30k+4)$ *and* $m_3 = 5(55k^2+30k+4)$;

(8^0) *G is a strongly regular graph of order* $n = 15(11k-5)^2$ *and degree* $r = 2(165k^2-150k+34)$ *with* $\tau = 60k^2-57k+13$ *and* $\theta = 6(2k-1)(5k-2)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 15k-7$ *and* $\lambda_3 = -(18k-8)$ *with* $m_2 = 6(165k^2-150k+34)$ *and* $m_3 = 5(165k^2-150k+34)$;

($\overline{8}^0$) *G is a strongly regular graph of order* $n = 15(11k-5)^2$ *and degree* $r = 9(165k^2-150k+34)$ *with* $\tau = 3(405k^2-368k+83)$ *and* $\theta = 9(9k-4)(15k-7)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 18k-9$ *and* $\lambda_3 = -(15k-6)$ *with* $m_2 = 5(165k^2-150k+34)$ *and* $m_3 = 6(165k^2-150k+34)$*;*

(9^0) *G is a strongly regular graph of order* $n = 15(11k+5)^2$ *and degree* $r = 2(165k^2+150k+34)$ *with* $\tau = 60k^2+57k+13$ *and* $\theta = 6(2k+1)(5k+2)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 18k+8$ *and* $\lambda_3 = -(15k+7)$ *with* $m_2 = 5(165k^2+150k+34)$ *and* $m_3 = 6(165k^2+150k+34)$*;*

($\overline{9}^0$) *G is a strongly regular graph of order* $n = 15(11k+5)^2$ *and degree* $r = 9(165k^2+150k+34)$ *with* $\tau = 3(405k^2+368k+83)$ *and* $\theta = 9(9k+4)(15k+7)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 15k+6$ *and* $\lambda_3 = -(18k+9)$ *with* $m_2 = 6(165k^2+150k+34)$ *and* $m_3 = 5(165k^2+150k+34)$*;*

(10^0) *G is a strongly regular graph of order* $n = 210(11k-1)^2$ *and degree* $r = 4(2310k^2-420k+19)$ *with* $\tau = 2(1680k^2-301k+13)$ *and* $\theta = 28(10k-1)(12k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 84k-8$ *and* $\lambda_3 = -(70k-6)$ *with* $m_2 = 5(2310k^2-420k+19)$ *and* $m_3 = 6(2310k^2-420k+19)$*;*

($\overline{10}^0$) *G is a strongly regular graph of order* $n = 210(11k-1)^2$ *and degree* $r = 7(2310k^2-420k+19)$ *with* $\tau = 14(735k^2-134k+6)$ *and* $\theta = 14(21k-2)(35k-3)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 70k-7$ *and* $\lambda_3 = -(84k-7)$ *with* $m_2 = 6(2310k^2-420k+19)$ *and* $m_3 = 5(2310k^2-420k+19)$*;*

(11^0) *G is a strongly regular graph of order* $n = 210(11k+1)^2$ *and degree* $r = 4(2310k^2+420k+19)$ *with* $\tau = 2(1680k^2+301k+13)$ *and* $\theta = 28(10k+1)(12k+1)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 70k+6$ *and* $\lambda_3 = -(84k+8)$ *with* $m_2 = 6(2310k^2+420k+19)$ *and* $m_3 = 5(2310k^2+420k+19)$*;*

($\overline{11}^0$) *G is a strongly regular graph of order* $n = 210(11k+1)^2$ *and degree* $r = 7(2310k^2+420k+19)$ *with* $\tau = 14(735k^2+134k+6)$ *and* $\theta = 14(21k+2)(35k+3)$*, where* $k \geqslant 0$*. Its eigenvalues are* $\lambda_2 = 84k+7$ *and* $\lambda_3 = -(70k+7)$ *with* $m_2 = 5(2310k^2+420k+19)$ *and* $m_3 = 6(2310k^2+420k+19)$*.*

PROOF. First, according to Remark 2.3 we have $5\alpha(\beta-1) = 6(\alpha-1)$, from which we find that $\alpha = 6$, $\beta = 2$. In view of this we obtain the strongly regular graph represented in Theorem 2.8 (1^0). Next, according to Proposition 2.11 it turns out that G belongs to the class ($\overline{2}^0$) or (3^0) or (4^0) or ($\overline{5}^0$) or (6^0) or ($\overline{7}^0$) or (8^0) or ($\overline{9}^0$) or ($\overline{10}^0$) or (11^0) if $m_2 = (\frac{6}{5})m_3$. According to Proposition 2.12 it turns out that G belongs to the class (2^0) or ($\overline{3}^0$) or ($\overline{4}^0$) or (5^0) or ($\overline{6}^0$) or (7^0) or ($\overline{8}^0$) or (9^0) or (10^0) or ($\overline{11}^0$) if $m_3 = (\frac{6}{5})m_2$. □

3. Concluding remarks

Using Theorems 2.1 and 2.2 it is possible to describe the parameters n, r, τ and θ for any connected strongly regular graph by using only one parameter k. In the forthcoming paper we shall describe the parameters n, r, τ and θ for strongly[4] regular graphs[5] with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$.

Acknowledgments. The author is very grateful to the referees for their valuable comments and suggestions.

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(Received 15 12 2019)
(Revised 04 10 2020)

[4]All the results in this paper are verified by using the computer program `srgpar.exe`, written by the author in the programming language `Borland C++Builder 5.5`.

[5]One can use the web page https://www.win.tue.nl/~{}aeb/graphs/srg/srgtab.html that contains the parameters of strongly regular graphs from 5 up to 1300 vertices.