

Application of GWO Algorithm for Closed Path Generation in Optimal Synthesis of Planar Mechanisms

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The problem of optimal synthesis of four-bar linkage and adjustable slider crank mechanism for generating a closed path was considered in this paper. Two cases were considered. In the first case, the goal is to optimize the path given by a set of predefined points. In the second case, a multi-criteria optimization problem is considered, i.e. the path and adjustable length of slider were optimized. The grey wolf algorithm was applied in the process of optimal synthesis. The proposed algorithm has been tested on appropriate numerical examples from the literature to demonstrate its efficiency.

Key words: Optimal synthesis, Adjustable slider crank mechanism, Four-bar linkage, Grey wolf optimizer

1. INTRODUCTION

Mechanisms are studied through two stages. The first stage involves the process of analysis, while the second stage involves the process of synthesis or design of mechanisms. Optimal synthesis involves the design of the mechanism using the optimization process [1]. In other words, optimal synthesis means the generation of the best mechanism through the repeated procedure of analysis [2]. For the purpose of optimal synthesis of the mechanism, it is first necessary to perform a detailed analysis of the mechanism in order to define project variables, objective function and constraints. Further more, the problem of optimal synthesis of adjustable planar mechanisms as a path generator will be discussed. This problem has been discussed in references [3,4,5].

2. FORMULATION OF THE PROBLEM OF SYNTHESIS OF THE FOUR-BAR LINKAGE

2.1. Position analysis

The subject of analysis is an adjustable four-bar linkage whose parameters are shown in Figure 1.

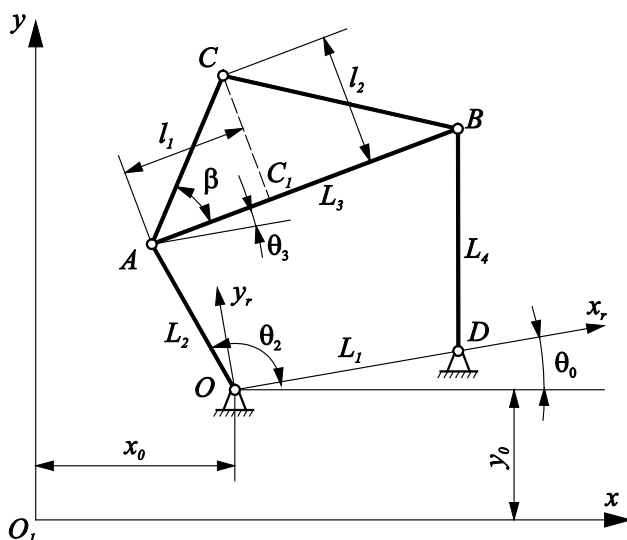


Figure 1: Geometry of the four-bar linkage

The lengths of the mechanism links are indicated by L_i . For the purpose of further analysis, two coordinate

systems were introduced - the global coordinate system xO_1y and the local (relative) coordinate system x_rO_yr . Point C indicates the point of the coupler that must pass through the preset points on the path.

The analysis of the four-bar linkage is performed using equations known in the literature. Thus, on the basis of Freudenstein equation, the angles θ_2 and θ_3 are determined, while the position of point C with respect to the global coordinate system xO_1y is defined by equation (1):

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{bmatrix} \begin{bmatrix} C_{xr} \\ C_{yr} \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (1)$$

where:

$$C_{xr} = L_2 \cdot \cos \theta_2 + l_1 \cdot \cos \theta_3 - l_2 \cdot \sin \theta_3$$

$$C_{yr} = L_2 \cdot \sin \theta_2 + l_1 \cdot \sin \theta_3 + l_2 \cdot \cos \theta_3$$

x_0, y_0 - the coordinates of the point O with respect to the global coordinate system xO_1y .

2.2. Design parameters

In the examples of synthesis of the considered mechanism as a path generator with prescribed time, nine design variables are optimized: $L_1, L_2, L_3, L_4, l_1, l_2, x_0, y_0$ and θ_0 . In the case of the synthesis of the mechanism as a path generator without prescribed time, the input angles of the crank θ_2^i ($i = 1, \dots, N$) corresponding to predefined points on the path are also optimized [6]. In general, the vector of design variables can be defined as follows:

$$\mathbf{X} = [L_1, L_2, L_3, L_4, l_1, l_2, x_0, y_0, \theta_0, \theta_2^1, \theta_2^2, \dots, \theta_2^N] \quad (2)$$

where N is the number of given points. For each design variable it is necessary to define the lower x_j^{lb} and upper x_j^{ub} bounds, while the NP indicates the number of design variables.

$$x_j \in [x_j^{lb}, x_j^{ub}], \forall x_j \in \mathbf{X}, j = 1, \dots, NP \quad (3)$$

2.3. Objective function and constraints

The objective function has two parts. The first part of the objective function defines the error of the deviation of the sum of the squares between the set of given and the set of real points described by the point C of the coupler during the

motion. The second part of the objective function considers constraints. When defining an optimization problem, two

$$\min \left\{ \sum_{i=1}^N \left[(C_{xd}^i - C_x^i)^2 + (C_{yd}^i - C_y^i)^2 \right] + M_1 h_1(\mathbf{X}) + M_2 h_2(\mathbf{X}) \right\}$$

where:

N – the number of required set points,

(C_{xd}^i, C_{yd}^i) – the coordinates of set points with respect to the global coordinate system,

(C_x^i, C_y^i) – the coordinates generated by the point C of coupler (real points),

$h_1(\mathbf{X})$ – refers to the conditions of a Grashof,

$h_2(\mathbf{X})$ – refers to the input angle of the crank

$\theta_2^i (i = 1, \dots, N)$,

M_1, M_2 – penalty functions that penalize the objective function when constraints are not satisfied.

3. FORMULATION OF THE PROBLEM OF SYNTHESIS OF ADJUSTABLE SLIDER CRANK MECHANISM

3.1. Position analysis

The subject of analysis is an adjustable (slider crank) mechanism whose parameters are shown in Figure 2. The lengths of the links are indicated by $L_i (i = 2, 3, 4)$, while the angles defining the position of the corresponding link with respect to the x -axis are indicated by $\theta_i (i = 2, 3, 4)$. The mechanism is placed in the xOy plane.

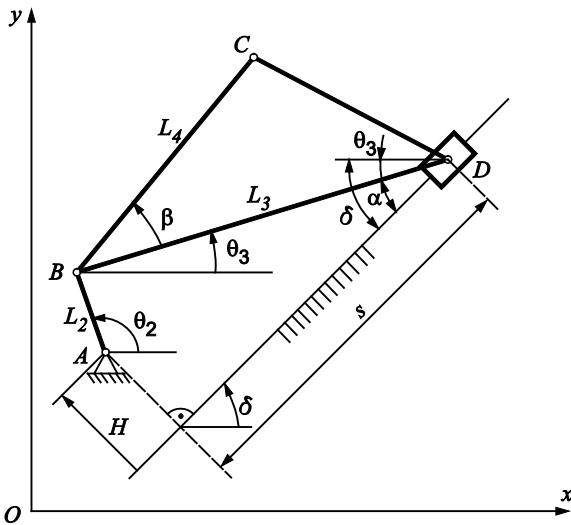


Figure 2: Adjustable slider crank mechanism

$$\min \left\{ \sum_{i=1}^N \left[(C_{xd}^i - C_x^i)^2 + (C_{yd}^i - C_y^i)^2 \right] + M_1 h_1(\mathbf{X}) + M_2 h_2(\mathbf{X}) \right\} \quad (14)$$

The previously defined objective function is subject to the constraint given in the form of inequality:

$$g_1 = |H + L_2 \sin(\theta_2 - \delta)| - L_3 \sin \alpha \leq 0 \quad (15)$$

where α is the angle that defines the position of the link BC of the adjustable mechanism relative to the direction of movement of slider C (see Figure 2).

$$\alpha = \delta - \theta_3 \quad (16)$$

constraints that contain penal functions are imposed, so that:

At the outset, it is necessary to define the following sizes [3]:

$$R_{\max} = L_2 + L_4 \quad (5)$$

$$R_{\min} = |L_2 - L_4| \quad (6)$$

where R_{\max} indicates the longest distance from point A to point C of the coupler, and R_{\min} is the shortest distance between these two points (when the driving member AB and the coupler BC are collinear).

The position of point C with respect to the coordinate system xOy is defined as follows:

$$x_C = x_A + L_2 \cos \theta_2 + L_4 \cos(\theta_3 + \beta) \quad (7)$$

$$y_C = y_A + L_2 \sin \theta_2 + L_4 \sin(\theta_3 + \beta) \quad (8)$$

The angle θ_2 defines the position of the driving link ($0 \leq \theta_2 \leq 2\pi$), while the angle θ_3 is determined by the relation:

$$\theta_3 = \delta + \arcsin \left[-\frac{H}{L_3} - \frac{L_2}{L_3} \sin(\theta_2 - \delta) \right] \quad (9)$$

In general, the size H (see Figure 4.2) is defined by the relation:

$$H = -L_2 \sin(\theta_2 - \delta) - L_3 \sin(\theta_3 - \delta) \quad (10)$$

Adjustable value s is determined as follows:

$$s = \sqrt{L_2^2 + L_3^2 - H^2 + 2L_2L_3 \cos(\theta_2 - \theta_3)} \quad (11)$$

3.2. Defining the design parameters and objective functions

Case 1 – Path optimization

In this case, nine design variables are optimized, so the vector of project variables \mathbf{X} is defined as follows:

$$\mathbf{X} = \{L_2, L_3, L_4, \beta, \delta, H, x_A, y_A, \theta_2\} \quad (12)$$

For each design variable $x_j (j = 1, \dots, 9)$ the lower x_j^{lb} and upper x_j^{ub} bounds must be defined:

$$x_j \in [x_j^{lb}, x_j^{ub}], \forall x_j \in \mathbf{X}, j = 1, \dots, 9 \quad (13)$$

The objective function is defined by the following relation:

Case 2 – Optimization of path and adjustable length s

In this case, applying the multi-criteria optimization procedure the two objective functions will be simultaneously minimize. The optimization problem is defined as follows:

$$\min \{f_1(\mathbf{X}), f_2(\mathbf{X})\} \quad (17)$$

Path optimization is achieved by defining an objective function $f_1(\mathbf{X})$ as follows:

$$\min \left\{ \sum_{i=1}^N \left[(C_{xd}^i - C_x^i)^2 + (C_{yd}^i - C_y^i)^2 \right] + M_1 h_1(\mathbf{X}) + M_2 h_2(\mathbf{X}) \right\} \quad (18)$$

Optimization of adjustable length is achieved by defining an objective function $f_2(\mathbf{X})$ as follows:

$$f_2(\mathbf{X}) = \min(\Delta s) \quad (19)$$

where $\Delta s = abs(s_{\max} - s_{\min})$. The vector of design variables is defined by relation (12), that is, on the same way as in *Case I*.

4. THE GREY WOLF ALGORITHM

The grey wolf algorithm belongs to the class of biologically inspired algorithms created by Mirjalili [6]. The algorithm mimics the life of these species of wolves in nature, ie. the principles by which they function, namely strict hierarchy and group hunting. Since they belong to the family of the beast, they are considered as predators that are at the top of the food chain. Grey wolves live in a pack of 5-12 individuals on average. There is a strict social hierarchy in the pack. The role of leader belongs to alphas that can be both males and females. Alpha makes all the important decisions and commands the pack. In other words, alpha is the first level in the hierarchy of grey wolves. The next level is the beta wolves who are tasked with assisting alphas in making decisions as well as taking care of discipline in the pack. Delta represent the third level in the grey wolf hierarchy, ie. they are subordinate to alpha and beta wolves. Delta wolves are guardians, scouts, but also old wolves. The lowest ranked grey wolves are called omega wolves. They are subordinate to all the above categories of wolves. Although it often seems that omega wolves are not of particular importance, they have an essential role in maintaining the structure of domination and often take care of the young.

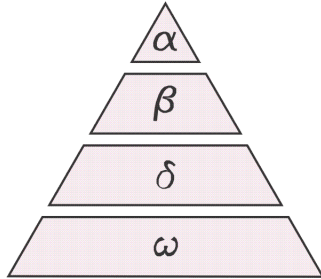


Figure 3: Schematic representation of the grey wolf hierarchy

Another important feature of grey wolves is their hunting behavior ie. the hunting mechanism. There are three basic strategies that these predators use in hunting [6]:

1. Tracking, pursuing and approaching prey
2. The encirclement and harassment prey until it calms down
3. Attack on prey

The application of the grey wolf algorithm (GWO) to solve various optimization problems can be seen in references [7-9].

4.1. The mathematical model

In order to mathematically model the behavior of grey wolves, it is necessary to divide the initial population in the GWO algorithm into four groups: α, β, δ and ω . In the GWO algorithm hunting ie. the search for the optimal solution is

led by the first three best solutions that are considered as α, β and δ wolves, while ω wolves follow them.

The main stage in group hunting is the surrounding of prey and this hunting strategy can be modeled by the following equations:

$$\mathbf{D} = |\mathbf{C} \cdot \mathbf{X}_p(t) - \mathbf{X}(t)| \quad (20)$$

$$\mathbf{X}(t+1) = \mathbf{X}_p(t) - \mathbf{A} \cdot \mathbf{D} \quad (21)$$

where t denotes current iteration, \mathbf{X}_p is the position of prey, \mathbf{X} is vector of grey wolf position, while the vectors \mathbf{A} i \mathbf{C} can be calculated using the following expressions:

$$\mathbf{A} = 2\mathbf{a} \cdot \mathbf{r}_1 - \mathbf{a} \quad (22)$$

$$\mathbf{C} = 2 \cdot \mathbf{r}_2 \quad (23)$$

where $\mathbf{r}_1, \mathbf{r}_2$ are random vectors from range $[0, 1]$, while the vector \mathbf{a} decreases linearly from 2 to 0 during iterations.

Namely, the GWO algorithm starts from the assumption that positions of α, β and δ wolves determine the position of prey. The first three best solutions (positions) are considered as positions of α, β and δ wolves, while other agents in the search (omega wolves) change their positions with respect to α, β and δ wolves.

Changing the position of omega wolves can be represented by the following equations:

$$\mathbf{D}_\alpha = |\mathbf{C}_1 \cdot \mathbf{X}_\alpha - \mathbf{X}|;$$

$$\mathbf{D}_\beta = |\mathbf{C}_2 \cdot \mathbf{X}_\beta - \mathbf{X}|; \quad (24)$$

$$\mathbf{D}_\delta = |\mathbf{C}_3 \cdot \mathbf{X}_\delta - \mathbf{X}|;$$

$$\mathbf{X}_1 = |\mathbf{X}_\alpha - \mathbf{A}_1 \cdot \mathbf{D}_\alpha|;$$

$$\mathbf{X}_2 = |\mathbf{X}_\beta - \mathbf{A}_2 \cdot \mathbf{D}_\beta|; \quad (25)$$

$$\mathbf{X}_3 = |\mathbf{X}_\delta - \mathbf{A}_3 \cdot \mathbf{D}_\delta|;$$

$$\mathbf{X}(t+1) = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3} \quad (26)$$

The search process in the GWO algorithm begins by generating an initial population, ie. by forming a pack. In a further iterative procedure, α, β and δ wolves also assess the position of the prey, with each potential solution implying approaching the prey. In this sense, when $|\mathbf{A}| > 1$ potential solutions tend to stray away from prey, respectively when $|\mathbf{A}| < 1$ potential solutions are approaching prey.

The pseudo code of the GWO algorithm is given below.

-
- 1: Defining the number of agents (wolves) N and the maximum number of iterations $\max iter$
 - 2: Initialization of initial population X_i ($i = 1, 2, \dots, n$)
 - 3: Initialization of vectors $\mathbf{a}, \mathbf{A}, \mathbf{C}$
 - 4: Calculating the fitness value of each agent
 - 5: $X_\alpha =$ the best agent
 - 6: $X_\beta =$ the second best agent
 - 7: $X_\delta =$ the third best agent
 - 8: **while** ($t < \max iter$)
 - 9: **for** %% each search agent

10: *update position of current search agent based on equation (26)*
 11: **end for**
 12: *Calculating of vectors a, A, C*
 13: *Calculating fitness value of each agent*
 14: *Finding new $X_\alpha, X_\beta, X_\delta$*
 15: $t = t + 1$
 16: *Sorting the population based on fitness value*
 17: **for** $i := (n / 2) + 1, n$
 18: *Update position of i -th wolf based on equation (27)*
 19: **end for**
 20: **end while**
 21: *otherwise print X_α*
 22: *Postprocessing of results*

$$\{C_d^i\} = \left\{ \begin{array}{lll} (0.5, 1.1); & (0.4, 1.1); & (0.3, 1.1); \\ (0.02, 0.6); & (0, 0.5); & (0, 0.4); \\ (0.2, 0.3); & (0.3, 0.4); & (0.4, 0.5); \end{array} \right. \quad (28)$$

The input angle of the curve is determined using the following relation:

$$\{\theta_2^i\} = \{\theta_2^1, \theta_2^1 + 20 \cdot i\}, i = 1, \dots, 17 \quad (29)$$

For each design variable, boundaries are defined:

$$\begin{aligned} 0 &\leq L_1, L_2, L_3, L_4 \leq 50; \\ -50 &\leq l_1, l_2, x_o, y_o \leq 50; \\ 0 &\leq \theta_0, \theta_2 \leq 2\pi \end{aligned} \quad (30)$$

The parameters of the GWO algorithm used in the optimization process are: $\max\ iter = 50$ (maximum number

5. NUMERICAL EXAMPLE

5.1. Example 1 - Optimal synthesis of four-bar linkage

The problem of synthesis of four-bar linkage as a path generator without prescribed time is considered. The point S of the coupler should pass through a set of eighteen predefined points.

Based on relation (2), the design variables for the considered problem are defined as follows:

$$\mathbf{X} = [L_1, L_2, L_3, L_4, l_1, l_2, x_o, y_o, \theta_0, \theta_2^1] \quad (27)$$

The coordinates of the desired (predefined) points on the path are:

of iteration), while the number of the agents is 30 (SearchAgents_no=30).

Using the grey wolf algorithm in the optimization procedure one obtains a mechanism whose design parameters are given in Table 1. For comparison, the table shows the results of other authors [10,11,12,13,14] who solved the same problem by applying different optimization algorithms. Thus, the aim is to demonstrate the effectiveness of the GWO algorithm in the example of the four-bar linkage, discussed in the literature.

Table 1: Comparative view of design parameters obtained using different optimization algorithms

	Kunjur, Krishnamurty [10] (GA)	Ortiz et al. [13] (IOA)	Cabrera et al. [11] (GA)	Cabrera et al. [12] (MUMSA)	Bulatović et al.[14] (MKH)	GWO
	0.274853	0.245216	0.237803	0.297057	0.42180	0.41970
	1.180253	6.38294	4.828954	3.913095	0.87821	0.98857
	2.138209	2.620532	2.056456	0.849372	0.58013	0.58240
	1.879660	4.040435	3.057878	4.453772	1.00429	1.10427
	-0.833592	1.139106	0.767038	1.6610626	0.35907	0.40047
	-0.378770	1.866109	1.850828	2.7387359	0.38081	0.44529
	1.132062	1.891805	1.776808	2.806964	0.26886	0.28691
	0.663433	-0.761339	-0.641991	4.853543	0.17715	0.09855
	4.354224	1.187751	1.002168	-1.309243	0.29294	0.33948
	2.558625	0.000000	0.226186	4.853543	0.88595	0.84827
error	0.043	0.0349	0.0337	0.0196	0.00911	0.00908

Figure 4 shows the best mechanism obtained by applying the grey wolf algorithm (GWO) as well as the path it generates.

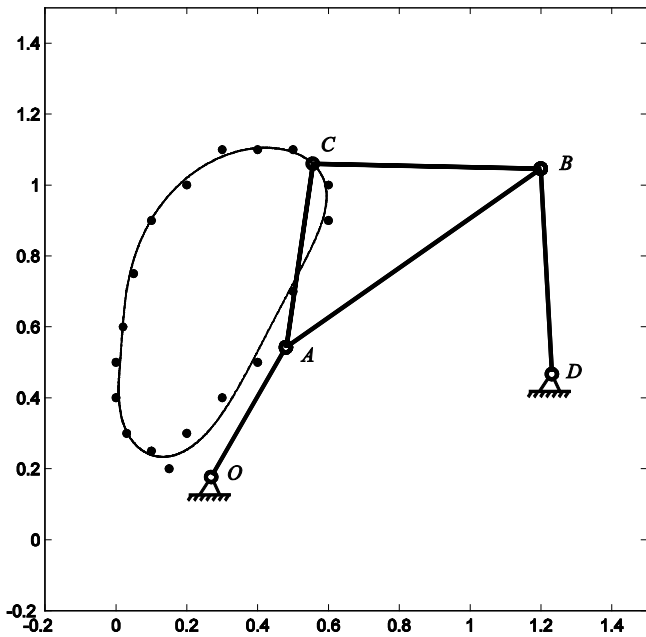


Figure 4: The best mechanism in Example 1

Figure 5 shows, in parallel, the paths described by point C of a given mechanism, which were obtained using various optimization algorithms (Bulatovic et al. [14] - MKH, Cabrera et al. [12] - MUMSA algorithm, Ortiz et al. [13] - IOA).

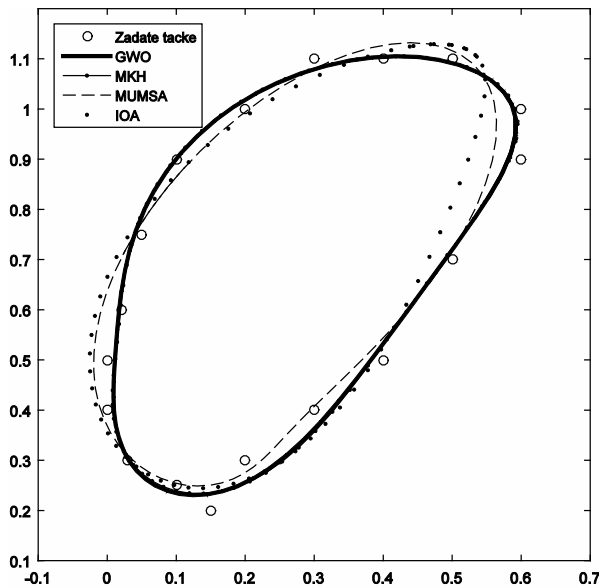


Figure 5: Coupling curves

5.2 Example 2 – Optimal synthesis of adjustable (slider crank) mechanism

As previously stated, two cases will be considered in the example of optimal synthesis of the slider crank mechanism. In the first case, the goal is to optimize the path, ie. synthesis of adjustable mechanism as a path generator. In

the second case, the goal is to perform simultaneous optimization of the path and adjustable length s .

Case 1 – Path optimization

Initially, it is necessary to define a vector of design variables:

$$\mathbf{X} = \{L_2, L_3, L_4, \beta, \delta, H, x_A, y_A, \theta_2\} \quad (31)$$

The coordinates of the desired (preset) points in the path are the same as in Example 1, ie. they are defined by relation (4.28). Namely, the goal is to perform the synthesis of various types of planar mechanisms which generate the same trajectory. Here, the path generator (which is given by the same set of points as in Example 1) is the adjustable mechanism shown in Fig. 2.

For each design variable, boundaries are defined:

$$\begin{aligned} 0 &\leq L_2, L_3, L_4 \leq 50; \\ -50 &\leq H, x_A, y_A \leq 50; \\ 0 &\leq \theta_2, \beta, \delta \leq 2\pi \end{aligned} \quad (32)$$

The parameters of the GWO algorithm used in the optimization process are the same as in the previous case. Using the grey wolf algorithm, an adjustable mechanism is obtained and its design parameters are shown in Table 2.

Table 2 Optimal values of design parameters for Case 1

Design parameters	Optimal values GWO
L_2	0.32563
L_3	0.50213
L_4	0.36236
β	-1.14240
δ	-1.42252
H	0.15411
x_A	0.55131
y_A	0.73173
θ_2	0.53607
Δs	0.52426
error	0.00994

As there are no references in the available literature to consider the path optimization of this type of planar mechanisms, it is not possible to provide comparative results.

Figure 6 shows the best mechanism obtained by applying the grey wolf algorithm (GWO) as well as the path it generates.

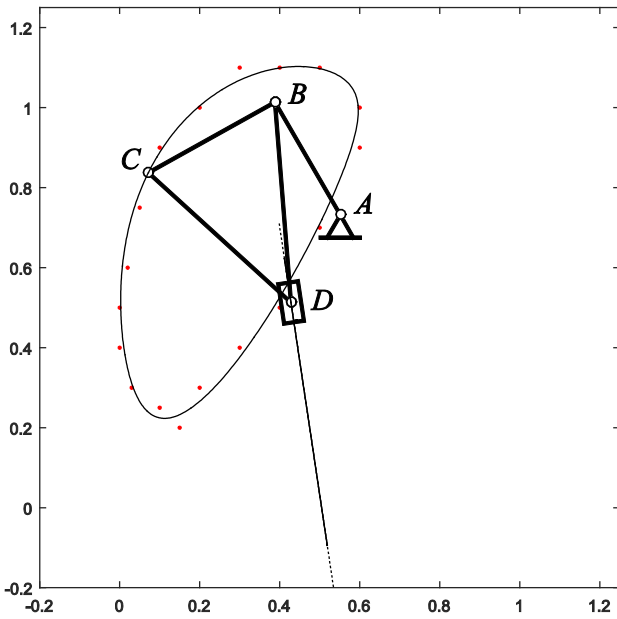


Figure 6: The best mechanism in Example 2 – Case 1

Figure 7 shows the path described by point M of the considered adjustable mechanism, and the same is obtained using the grey wolf algorithm.

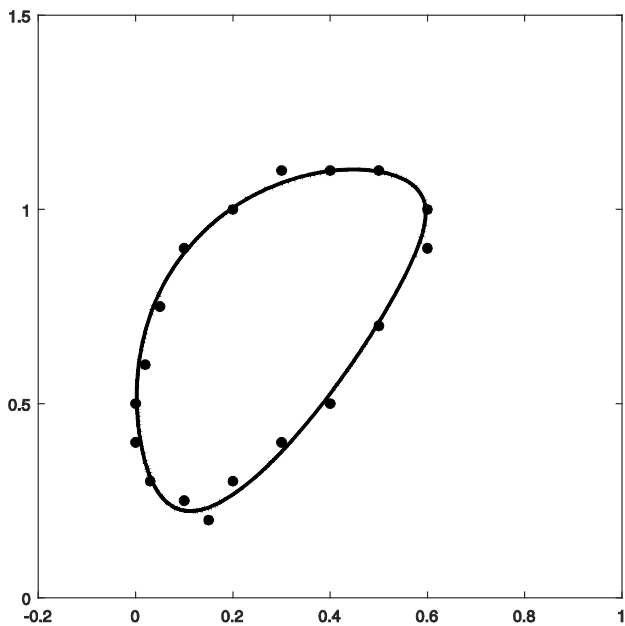


Figure 7: Coupling curve

Case 2 – Optimization of path and adjustable length s

The vector of design variables in this case is defined in the same way as in Case 1, by using relation (31). The coordinates of the desired points in the path are the same as in the previous examples, since the goal is to generate the same path using two types of planar mechanisms. Unlike Case 1, where only path optimization is considered, simultaneous optimization of the path and adjustable length is performed here. Using the grey wolf algorithm in the multicriteria optimization process, a number of adjustable mechanisms (solutions) with different values of design

parameters are generated. Table 4.3 shows the design parameters for the four best solutions (mechanisms) obtained in the optimization process.

Table 3: Optimal values of design parameters for the four best solutions in Case 2

Design variab.	Opt. values (Example1)	Opt. values (Example2)	Opt. values (Example3)	Opt. values (Example4)
L_2	0.46357	0.33887	0.46319	0.30143
L_3	1.11772	0.84714	1.03291	1.50697
L_4	0.61417	0.51169	0.56197	1.02869
β	6.81167	8.34521	0.53718	2.39888
δ	-5.27894	-2.53315	1.00504	3.61733
H	0.14973	0.43825	0.15378	0.90908
x_A	0.14271	0.01185	0.14909	-0.36041
y_A	0.08941	1.08347	0.14390	1.47563
θ_2	-5.22872	1.27817	1.04741	1.29291
$f_{1\min}$	0.03292	0.03782	0.03305	0.03602
$f_{2\min}$	0.48965	0.48626	0.49073	0.48052

Figures 8 - 11 show the mechanisms (with the parameters of Examples 1, 2, 3 and 4, respectively) obtained by the multicriteria optimization procedure and the application of the grey wolf algorithm (GWO). The same figures show the paths generated by the adjustable mechanisms obtained in Examples 1, 2, 3 and 4.

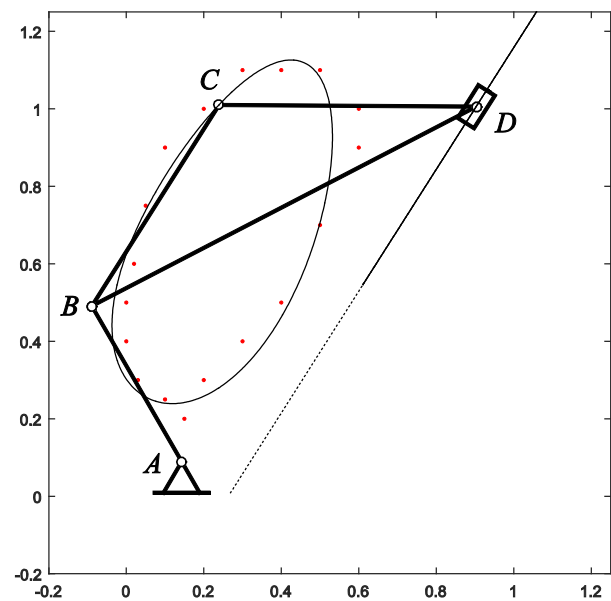


Figure 8: Mechanism and its path – Example 1

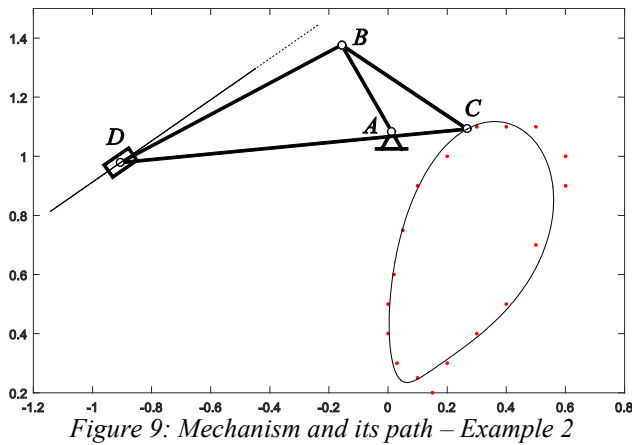


Figure 9: Mechanism and its path – Example 2

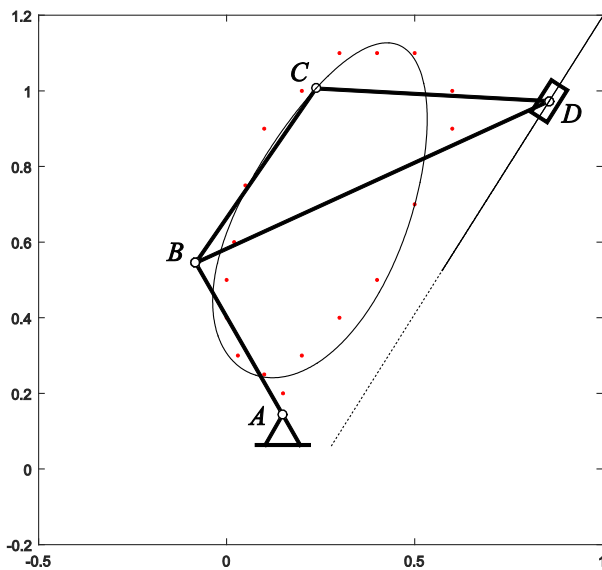


Figure 10: Mechanism and its path – Example 3

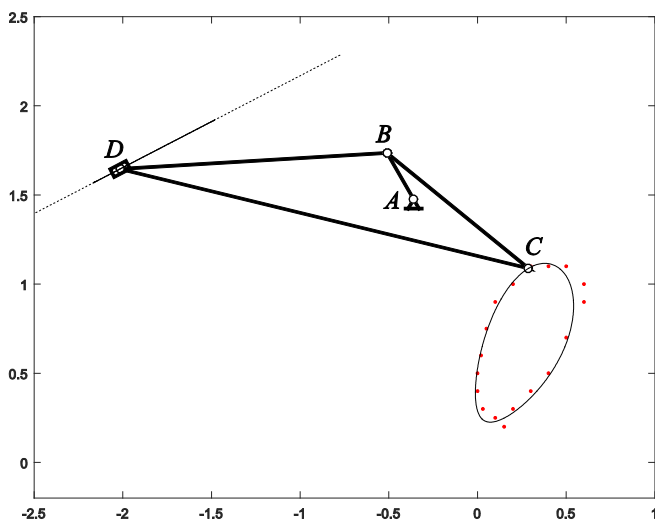


Figure 11: Mechanism and its path – Example 4

6. CONCLUSION

In this paper, the problem of optimal synthesis of planar mechanisms as a path generator is discussed. To solve the problem of optimal synthesis, the grey wolf algorithm

was applied. There is no research available in the literature in which the problem of optimal synthesis of mechanisms was solved by applying the GWO algorithm. Testing the efficiency of the grey wolf algorithm was performed by the example of optimal synthesis of a four-bar linkage as a path generator (Example 1). The results obtained by applying the GWO algorithm are better than the results in [10,11,12,13], while they are approximate to results in [14] (see Table 1). Then, two cases of optimal synthesis of an adjustable slider crank mechanism were considered (Example 2). Firstly, in Case 1, it was performed an optimal synthesis of path which is defined by the same set of points as in Example 1. The obtained results are excellent, the path is almost identical to that one generated by the four-bar linkage (Figure 7). The deviations between actual and desired path are minimal. In Case 2, multi-criteria optimization of the path and adjustable length s was performed. However, the results obtained in this case are not satisfactory. Namely, the magnitude Δs is slightly reduced compared to Case 1 (single-criteria optimization), while there is a drastic deviation of the actual from the desired (given) path (see Figures 8 - 11).

Based on the above, the conclusion is that the grey wolf algorithm provides excellent results in the case of single-criteria optimization. However, the application of the GWO algorithm did not give the expected good results in the multi-objective optimization. In this sense, certain modifications should be made to the standard GWO algorithm to improve its efficiency in the MOO process.

Finally, it should be noted that there are no references in the available literature in which the problem of multi-criteria optimization of the trajectory and stroke of the slider of the adjustable curved piston mechanism has been discussed. This has made some contribution to the study of the problem of optimal synthesis of tunable plane mechanisms.

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