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Prilog analizi krutosti ramova utovarno-istovarnih kolica pretovarnih mosnih dizalica

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U radu je izvršena analiza zavisnosti krutosti nosećih ramova utovarno-istovarnih kolica pretovarnih mosnih dizalica od promene geometrijskih parametara ramova.

Formiran je proračunski model ramova sa poprečnim ukrućenjima koji dodatno ukrućuju oslonu površinu ramova za vezu sa aksijalno-radijalnim ležajem.

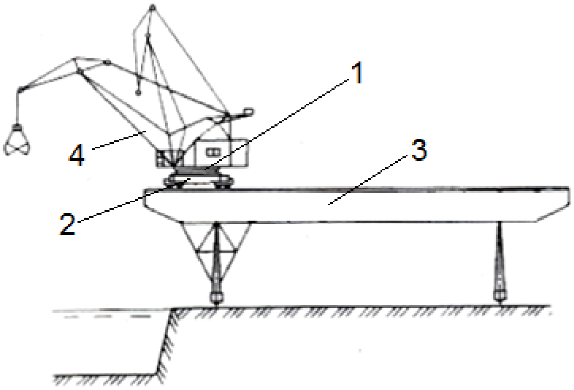
Uspostavljeni su odnosi karakterističnih geometrijskih veličina podužnih i poprečnih elemenata pri kojima se ostvaruje potrebna krutost ramova utovarno-istovarnih kolica.

Takođe, uvedene su promenljive i konstante geometrijske veličine, kao i konstantne vrednosti odgovarajućih koeficijenata, čijim se variranjem pružaju mogućnosti formiranja optimalne konstrukcije ramova utovarno-istovarnih kolica.

Ključne reči: Geometrija nosača, Odnosi momenata inercije, Poprečna ukrućenja, Parametar debljine limova, Dijagramske zavisnosti

1. UVOD

Pojavom mašina sa utovarno-istovarnim kolicima i transportnom trakom dolazi do novih koncepcijskih rešenja transportnih mašina u obliku pretovarnih mosnih dizalica, koje u znatnouj meri povećavaju kapacitet ostvarenih radova. Primena konstrukcijskih rešenja sa trakom pripada grupi sa manje složenim postrojenjima ali i sa znatno smanjenim prostorom opsluživanja, jer se u procesu transporta vrši i premeštanje pretovarne mosne dizalice. Taj nedostatak se može otkloniti primenom transportnih mašina kod kojih se sa gornje strane dizalica ugrađuju uzovarno-istovarna kolica sa okretnom platformom. Takva rešenja spadaju u red savremenijih i sa njima se mogu ostvariti znatno veći kapaciteti transporta. Kod njih (sl. 1) utovarno-istovarna kolica 2 imaju platformu sa strelom 4. Između platforme i kolica ugrađen je aksijalno-radijalni ležaj 1.



Slika 1: Pretovarna dizalica sa utovarno-istovarnim kolicima

Kolica 2 se kreću po gornjem pojasu glavnih nosača 3.

Opterećenja u vertikalnom pravcu se sa okretne platforme, preko aksijalno-radijalnog ležaja 1, koji se oslanja na ram utovarno-istovarnih kolica, prenose na glavne podužne nosače mosta 3 i dalje preko nogu na dizalične staze. Vidi se da od pouzdanosti veze okretne platforme sa ramom utovarno-istovarnih kolica zavisi funkcionalna sposobnost pretovarnih mosnih dizalica. Dakle u cilju ostvarenja pouzdanosti veze okretne platforme i rama utovarno-istovarnih kolica posredstvom radijalno-aksijalnog ležaja, potrebno je uspostaviti zavisnosti momenta inercije poprečnih preseka grednih nosača sandučastog tipa tako da bude ispunjen uslov krutosti rama utovarno-istovarnih kolica koji obezbeđuje traženu horizontalnost površine na kojoj leži aksijalno-radijalni ležaj.

Zato je neophodno formirati proračunski model rama koji će da pruži mogućnost variranja promenljivih koeficijenata koji zavise od:

- geometrijske identifikacije rama utovarno-istovarnih kolica,
- odnosa momenata inercije poprečnog preseka segmenata nosećeg rama utovarno-istovarnih kolica,
- dužina samog segmenta podužnih nosača nosećeg rama utovarno-istovarnih kolica (koeficijent w)
- dužine prvog segmenta poprečnog nosača prema dužini srednjeg segmenta poprečnog nosača (koeficijent ψ),
- odnosa momenta inercije početnog dela poprečnog nosača prema momentu inercije podužnog nosača (φ),
- odnosa momenta inercije kratkog poprečnog nosača prema momentu inercije preseka poprečnog nosača (koeficijent β).

Pored promenljivih koeficijenata, uvode se i konstantni koeficijenti koji se odnose na debljinu vertikalnih (t_1) i horizontalnih limova (t_2), razmak poprečnih nosača (l_2), razmak srednjih podužnih nosača (L_2) i zazor između oslonaca noseće konstrukcije pretovarnog mosta i dizalične staze (Δ). Ovaj zazor uslovljava pojavu zazora u jednom od oslonaca rama utovarno-istovarnih kolica (na primer kod oslonca C). Konstantni koeficijenti koji predstavljaju odnos debljina horizontalnih i vertikalnih limova označeni su sa (λ).

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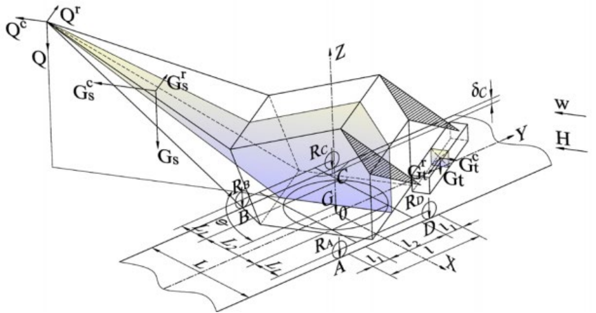
2. FORMIRANJE MODELA RAMA UTOVARNO - ISTOVARNIH KOLICA

Celokupni sistem pretovarne mosne dizalice se može razdvojiti u dve celine i to:

- prvu, koju čini noseći ram utovarno-istovarnih kolica na koji se oslanja okretna platforma sa strelom;
- drugu, koju čini noseća struktura pretovarnog mosta, koja prima sva opterećenja od prve celine i dalje ih prenosi na dizalične staze.

Šematski prikaz noseće konstrukcije utovarno-istovarnih kolica sa osloncima u tačkama ABCD dat je na slici 2.



Slika 2: Šema noseće konstrukcije utovarno-istovarnih kolica

Modelski prikaz konstrukcije rama utovarno-istovarnih kolica dat je na slici 3. Ako se noseća konstrukcija rama utovarno-istovarnih kolica posmatra krutom, onda će oslonci ABCD ležati u ravni π. Zbog elastičnih deformacija elemenata rama, oslonci ABCD iz ravni π prećiće u A'B'C'D' koji su sada u ravni π_1.

Ako se pretpostavi da se oslonac C spustio za veličinu δ_c, onda će u osloncima na dijagonali BD doći do porasta reakcije R_B i R_D [1], odnosno:

$$R = R_K \pm c\delta \tag{1}$$

gde su:

- R_K - pritisci u osloncima kada svi oslonci leže u jednoj ravni (π),
- c - krutost rama utovarno-istovarnih kolica.

Znak (-) u izrazu (1) se odnosi na dijagonalne oslonce sa zazorom, a znak (+) na druga dva oslonca.

Određivanje dopunske sile u osloncu se može izvršiti korišćenjem kanonskih jednačina:

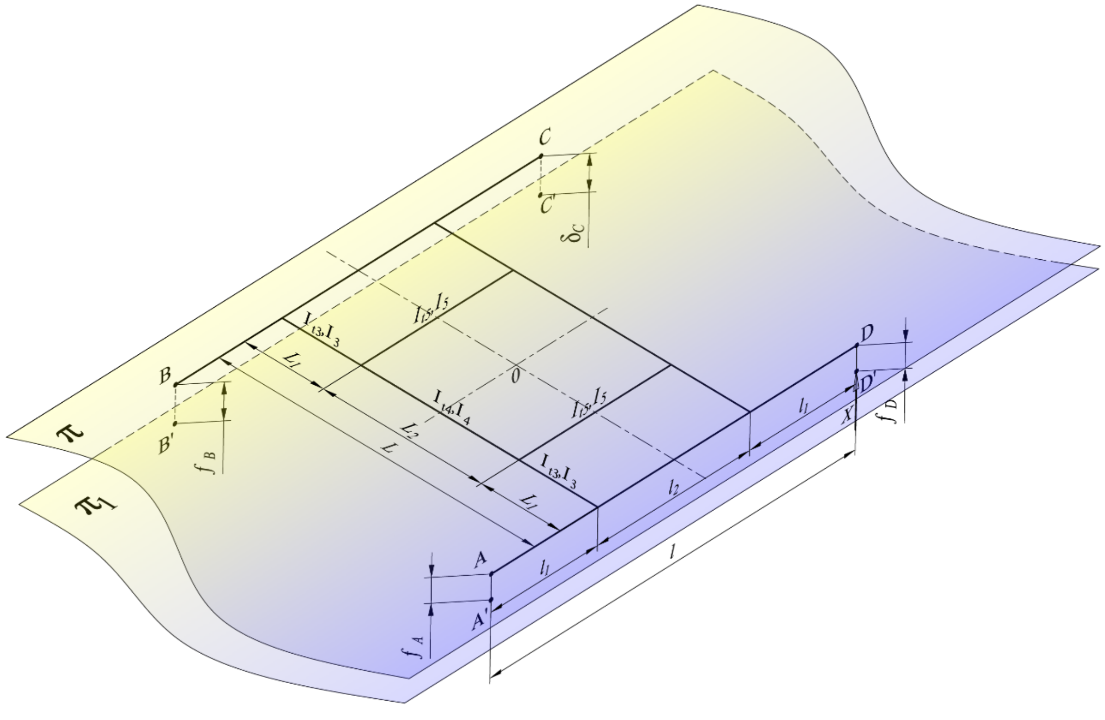
$$A\vec{X} = \vec{\delta}_c \tag{2}$$

gde su:

- A - kvadratna matrica sastavljena od uticajnih koeficijenata $a_{ik}\left(i,k = 1,2,3,\ldots,7\right)$
- $\overrightarrow{X}$ - vektor kolona nepoznatih sila i momenata $X_i\left(i = 1,2,3,...,7\right)$
- $\vec{\delta}_c$ - vektor kolona pomeranja u vertikalnom pravcu $\delta_i\left(i = 1,2,3,...,7\right)$.

Za rešavanje sistema jednačina (2), neophodno je odrediti članove kvadratne matrice, odnosno uticajne koeficijente. Njih je moguće odrediti korišćenjem Maksel-Morovog integrala [2,3]. Za razmatrani slučaj proračuna, opšti izraz za Maksvel-Morov integral će imati sledeći oblik:

$$a_{ik} = \int_l \frac{M_{zk} \cdot M_{zi}}{GI_t} dx + \int_l \frac{M_{xk} \cdot M_{xi}}{EI_x} dx \tag{3}$$



Slika 3: Model rama utovarno-istovarnih kolica

Formiranjem momentnih dijagrama od jednačina sila i momenata i integraljenjem po celoj konturi, dobijaju se uticajni koeficijenti a_{ik}.

Za razmatrani slučaj oni iznose:

$$a_{11}=\frac{4\cdot l_1^3}{3\cdot E\cdot I_1}+\frac{2\cdot l_1\cdot l_2^2\cdot\left(1+\frac{l_1}{l_2}+\frac{l_2}{3\cdot l_1}\right)}{E\cdot I_2}+\frac{2\cdot L_1\cdot l_2}{G\cdot I_{t3}}+\frac{L_2\cdot l_2}{G\cdot I_{t4}}$$

$$a_{22}=\frac{l_2^3}{3\cdot E\cdot I_2}+\frac{L_1^3}{6\cdot E\cdot I_3}+\frac{L_1^2\cdot l_2}{4\cdot G\cdot I_{t5}}+\frac{l_2^3}{3\cdot E\cdot I_5}$$

$$a_{33}=\frac{l_2}{E\cdot I_2}+\frac{l_2}{E\cdot I_5}+\frac{2\cdot L_1}{G\cdot I_{t3}}$$

$$a_{44}=\frac{2\cdot l_2^3}{3\cdot E\cdot I_5}+\frac{L_2^3}{6\cdot E\cdot I_4}+\frac{L_2^2\cdot l_2}{2\cdot G\cdot I_{t5}}$$

$$a_{55}=\frac{2\cdot l_2}{E\cdot I_5}+\frac{2\cdot L_2}{G\cdot I_{t5}}$$

$$a_{66}=\frac{l_2^3}{3\cdot E\cdot I_2}+\frac{l_2^3}{3\cdot E\cdot I_5}+\frac{L_1^3}{6\cdot E\cdot I_3}+\frac{L_1^2\cdot l_2}{4\cdot G\cdot I_{t2}}+\frac{L_1^2\cdot l_2}{4\cdot G\cdot I_{t5}}$$

$$a_{77}=\frac{l_2}{E\cdot I_2}+\frac{l_2}{E\cdot I_5}+\frac{2\cdot L_1}{G\cdot I_{t3}}$$

$$a_{12}=\frac{l_2^2\cdot(3\cdot l_1+2\cdot l_2)}{6\cdot E\cdot I_2}$$

$$a_{13}=-\frac{L_1\cdot l_2}{G\cdot I_{t3}} \tag{4}$$

$$a_{14}=0$$

$$a_{15}=-\frac{L_2\cdot l_2}{G\cdot I_{t4}}$$

$$a_{16}=\frac{l_2^2\cdot(3\cdot l_1+2\cdot l_2)}{6\cdot E\cdot I_2}$$

$$a_{17}=-\frac{l_2\cdot(2\cdot l_1+l_2)}{2\cdot E\cdot I_2}-\frac{L_1\cdot l_2}{G\cdot I_{t3}}$$

$$a_{23}=-\frac{l_2^2}{2\cdot E\cdot I_2}-\frac{l_2^2}{2\cdot E\cdot I_5}$$

$$a_{24}=-\frac{l_2^3}{3\cdot E\cdot I_5}+\frac{L_1\cdot l_2\cdot L_2}{4\cdot G\cdot I_{t5}}\quad a_{25}=\frac{l_2^2}{2\cdot E\cdot I_5}\quad a_{26}=a_{27}=0$$

$$a_{34}=\frac{l_2^2}{2\cdot E\cdot I_5}\quad a_{35}=\frac{l_2}{E\cdot I_5}\quad a_{36}=a_{37}=0$$

$$a_{45}=-\frac{l_2^2}{E\cdot I_5}\quad a_{46}=-\frac{l_2^3}{3\cdot E\cdot I_5}+\frac{L_1\cdot l_2\cdot L_2}{4\cdot G\cdot I_{t5}}\quad a_{47}=\frac{l_2^2}{2\cdot E\cdot I_5}$$

$$a_{56}=\frac{l_2^2}{2\cdot E\cdot I_5}\quad a_{57}=-\frac{l_2}{E\cdot I_5}\quad a_{67}=-\frac{l_2^2}{2\cdot E\cdot I_5}-\frac{l_2^2}{1\cdot E\cdot I_2}$$

Kako su poprečni preseci elemenata noseće konstrukcije rama pretovarno-utovarnih kolica sandučastog tipa, to za njih važi odnos [1]:

$$\frac{E\cdot I_x}{G\cdot I_t}=k\sqrt{3} \tag{5}$$

gde je k koeficijent odnosa visine h i širine b poprečnog preseka sandučastog nosača. Njegova vrednost se kreće u granicama od 1 do 3.
Kako pored koeficijenta k na odnos savojne i torzione krutosti utiče i koeficijent λ, koji predstavlja odnos debljina pojasnih i vertikalnih limova, to se izraz (5) transformiše i dobija sledeći vid:

$$\frac{E\cdot I_x}{G\cdot I_t}=1{,}6\cdot k\cdot\lambda^{0{,}35} \tag{6}$$

Treba napomenuti da se vrednosti koeficijenta λ kreću u granicama od 1,0 do 1,5. Optimalna vrednost koeficijenta λ iznosi $\lambda=1{,}3$.

Unošenjem zavisnosti (6) u kanonske jednačine (2), mogu se proračunati vrednosti dopunskih sila pri pojavi zazora δ_c ispod oslonca C.

3. KRUTOST NOSEĆE STRUKTURE RAMOVA UTOVARNO-ISTOVARNIH KOLICA PRETOVARNIH MOSNIH DIZALICA

Odnos dužina segmenata l_1 i l_2 (sl. 3) podužnih nosača noseće strukture ramova utovarno-istovarnih kolica definisan je koeficijentom:

$$w=\frac{l_1}{l_2} \tag{7}$$

Odnos dužina prvog segmenta L_1 prema dužini srednjeg segmenta L_2 definisan je koeficijentom:

$$\psi=\frac{L_1}{L_2} \tag{8}$$

Koeficijentom φ definiše se odnos momenata inercije početnog dela nosača I_3 prema momentu inercije podužnog nosača I_2, odnosno:

$$\varphi=\frac{I_3}{I_2} \tag{9}$$

Na kraju uvodi se i koeficijent β kojim se definiše odnos momenata inercije kratkog nosača I_4 prema momentu inercije podužnog nosača I_2:

$$\beta=\frac{I_4}{I_2} \tag{10}$$

Istraživanje promene krutosti noseće konstrukcije ramova utovarno-istovarnih kolica sprovedeno je za sledeće vrednosti promenljivih koeficijenata:

$$w=0{,}4;0{,}6;0{,}8;1{,}0$$
$$\psi=0{,}4;0{,}6;0{,}8;1{,}0$$
$$\varphi=1{,}0;1{,}5;2{,}0$$
$$\beta=1{,}0;1{,}5;2{,}0$$

Variranjem koeficijenata β, w, φ i ψ, pri $\lambda = 1,3$, iz kanonskih jednačina se mogu odrediti i vrednosti dopunskih sila u ramu istovarno-utovarnih kolica, kada se pojavi zazor δ_c ispod oslonca C (sl. 3). Onako kako se menjaju vrednosti dopunskih sila X_1 u osloncu D_1 u istom odnosu dolazi i do promene krutosti rama utovarno-istovarnih kolica pri različitim koeficijentima (11) proračunatih vrednosti za dopunske sile u osloncu D, koje su date u tabelama T₁, T₂, T₃ i T₄.

Tabela 1: $T_1 - X_1 = f(\beta, w)$

β	w	X_1(kN)
1,0	0,4	93,74
	0,6	84,02
	0,8	74,29
	1,0	64,57
1,5	0,4	120,00
	0,6	106,68
	0,8	93,37
	1,0	80,05
2,0	0,4	142,92
	0,6	126,30
	0,8	109,69
	1,0	93,07

Tabela 2: $T_2 - X_1 = f(\psi, \varphi)$

ψ	φ	X_1(kN)
0,4	1,0	72,58
	1,5	77,79
	2,0	83,00
0,6	1,0	66,72
	1,5	72,82
	2,0	78,91
0,8	1,0	60,90
	1,5	67,91
	2,0	74,92
1,0	1,0	56,26
	1,5	63,64
	2,0	71,02

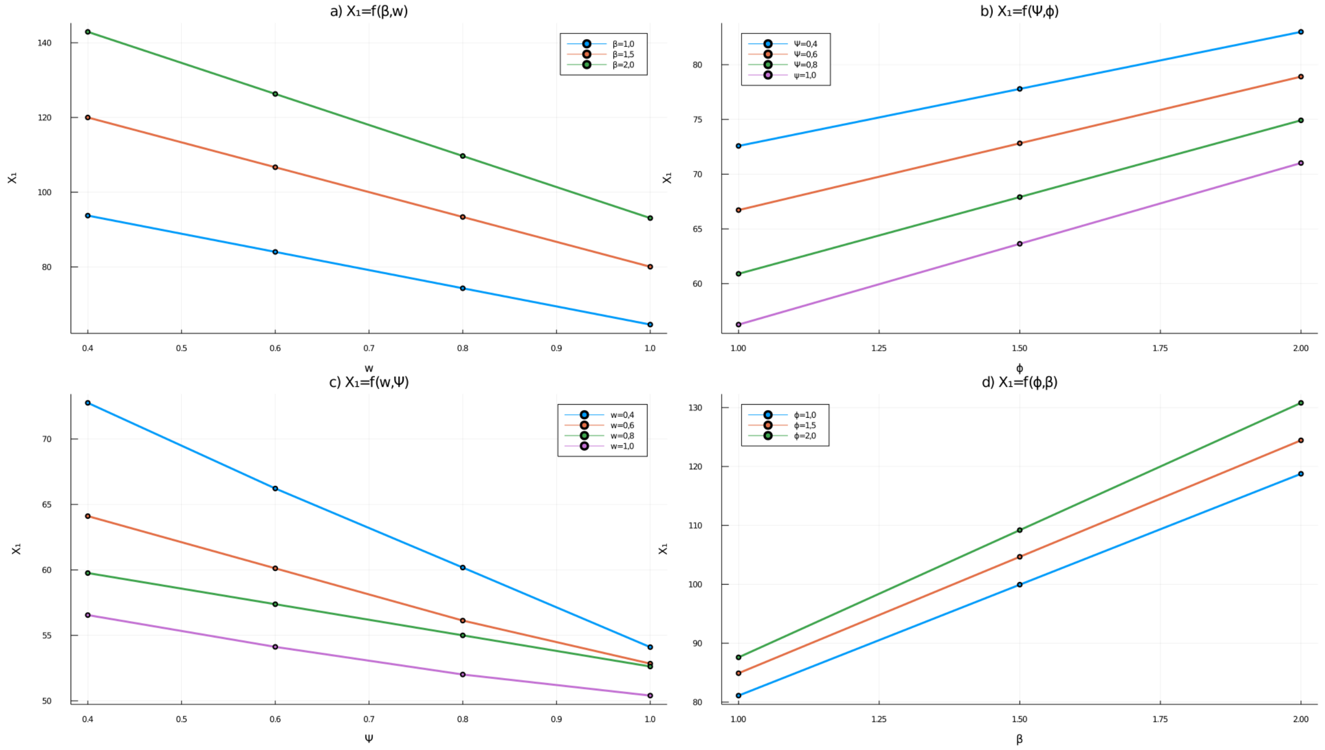
Tabela 3: $T_3 - X_1 = f(w, \psi)$

w	ψ	X_1(kN)
0,4	0,4	72,76
	0,6	66,22
	0,8	60,18
	1,0	54,10
0,6	0,4	64,11
	0,6	60,12
	0,8	56,13
	1,0	52,84
0,8	0,4	59,76
	0,6	57,38
	0,8	55,00
	1,0	52,62
1,0	0,4	56,55
	0,6	54,12
	0,8	52,01
	1,0	50,40

Tabela 4: $T_4 - X_1 = f(\varphi, \beta)$

β	w	X_1(kN)
1,0	1,0	81,11
	1,5	99,94
	2,0	118,76
1,5	1,0	84,91
	1,5	104,67
	2,0	124,44
2,0	1,0	87,60
	1,5	109,20
	2,0	130,80

Dijagramski prikaz promene krutosti ramova utovarno-istovarnih kolica u funkciji promene koeficijenata *β, w, φ* i *ψ* pri $\lambda = 1,3$ dat je na slici 4.



Slika 4: Zavisnost krutosti ramova utovarno-istovarnih kolica u funkciji promene koeficijenata β, w, φ i ψ, pri $\lambda = 1,3$

4. ZAKLJUČAK

Analizirajući promene krutosti ramova utovarno-istovarnih kolica pretovarnih mosnih dizalica za optimalne vrednosti odnosa debljina pojasnih i vertikalnih limova sandučastih nosača $\lambda = 1,3$, mogu se formirati sledeći zaključci:

a) Sa porastom foeficijenta β raste i krutost ramova utovarno-istovarnih kolica. Uočava se, takođe, da sa porastom koeficijenta β raste i nagib linija koje predstavljaju zakon promene krutosti.

b) Sa porastom koeficijenta w dolazi do opadanja krutosti ramova utovarno-istovarnih kolica pretovarnih mosnih dizalica, pri čemu se koeficijent nagiba linija koje predstavljaju zakon promene krutosti smanjuje.

c) Sa porastom koeficijenta ψ dolazi do opadanja krutosti ramova utovarno-istovarnih kolica pretovarnih mosnih dizalica. Koeficijent opadanja krutosti se neznatno smanjuju sa porastom koeficijenta ψ.

d) Sa porastom koeficijenta φ dolazi do porasta krutosti ramova utovarno-istovarnih kolica, pri čemu je nagib linija koje predstavljaju zakone promena krutosti skoro konstantan.

ZAHVALNOST

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LITERATURA

[1] Trifković S., Zdravković N., Gašić M., Savković;., Marković G.: Analysis of the Influence Parameters on the Support Structure Stiffness of Large Radial-Axial Bearings, Strojniški vestnik - Journal of Mechanical Engineering 65(2019)6, 366-374, DOI:10.5545/sv-jme.2019.6006.

[2] Karnovsky I.A.: Theory of arched structures strength, vibration, springer, New York, 2012

[3] Karnovsky I.A., Lebed O.: Advanced methods of structural analysis, springer, New York, London, 2010

[4] M. Gašić, M. Savković, G. Marković, N. Zdravković, G. Bošković, M. Nikolić: “Prilog geometrijskoj identifikaciji nosača aksijalnih ležajeva velikih prečnika”, IMK-14-Istraživanje i razvoj u teškoj mašinogradnji 23 (2017) 4, SR 117-121, UDK 621 ISSN 0354-6829

[5] Zupan, S., Prebil, I. (2001). Carrying Angle and Carrying Capacity of a Large Single Row Ball Bearomg as a Function of Geometry Parameters of Rolling Contact and Supporting Structure Stiffness, Mechanism and Machine Theory, 38, p. 479-496.

[6] Smolnicki, T., Stanco, M., Pietrusiak, D. (2013). Distribution of loads in the large size bearing-problems of identification, Technical Gazette; Vol. 20, no. 5, p. 831-836.

[7] Jerman, B. Hladnik, J., Resman, F., Landschutzer, C. (2018). Optimization of the support structure of large axial-radial bearing of overhead type manipulator, FME

Transactions, vol. 46, no. 2, pp. 386-391, DOI: 10.5937/fmet1803386J

[8] Guanci, C., Cunzhu, W., Zhengming, X. (2016). Effects of supporting Structure and bolt connection on the fatigue life and carrying capacity of a slewing bearing, Jornal of Engineering Tribology, Vol. 2331(6), 1-17, DOI: 10.1177/1350650116677606

[9] Gašić, M., Savković, M., Bulatović, R. (2011). Optimization of trapezoidal cross section of the truck crane boom by Lagrange's multipliers and by differential evolution algorithm (de), Strojniški vestnik - Journal of Mechanical Engineering, Vol. 57, No. 4, p. 304-312, DOI: 10.5545/sv-jme.2008.029.

[10] Karnovsky, I.A., LEbed, O. (2010). Advanced methods of structural analysis, Springer, New York, London.

[11] Бабин Ј.Н., Владић М.Ј., Бркљач Б.Н., Шостаков С.Р.: Металне конструкције у машинству, ФТН издаваштво, Нови Сад, 2012.

[12] Vasiljević R., Gašić M., Savković M.: Parameters Influencing the Dynamic Behaviour of the Carrying Structures of a Type H Portal Crane, Strojniški vestnik - Journal of Mechanical Engineering, vol. 66, no. 10, p. 591-602, 2016. DOI: 10.5545/sv-.jme.2016.3553

[13] Trifković, S., Gašić, M., Radić, N., Milutinović, M.: The Equations of Motion of the Crane with Loading-unloading Trolley on the Slewing Platform, VII International Conference “Heavy Machinery-HM 2014”, Zlatibor, 25-28 June 2014, A.183-186

[14] S. Trifković, M. Gašić, N. Radić, M. Milutinović: “Dynamic Analysis of the Crane with Loading-unloading Trolley on Slewing Platform”, IMK-14 - Research & Development in Heavy Machinery 21(2015)1, EN1-6, UDC 621 ISSN 0354-6829

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Contribution to the Analysis of the Frame Stiffness of the Cranes with Loading-unloading Trolley

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The paper analyzes the dependence of the stiffness of the support frames of the cranes with loading-unloading trolley on the change of geometrical parameters of the frames.

The calculation model of the frames with transverse stiffness was formed because it additionally stiffens the supporting surface of the frames for connection with the axial-radial bearing.

The relations of characteristic geometrical sizes of longitudinal and transverse elements were established, at which the necessary stiffness of the frames of loading-unloading trolley was achieved.

Also, the variables and constants of geometric size were introduced, as well as the constant values of appropriate coefficients, the variation of which provides the possibility to form the optimal structure of the frames of loading-unloading trolley.

Keywords: Geometry of the support, Ratios of moments of inertia, Transverse stiffness, Parameter of sheet thickness, Diagram dependence

1. INTRODUCTION

With the advent of machines with loading-unloading trolley and conveyor belt, there are new conceptual solutions for transport machines in the form of bridge cranes which significantly increase the capacity of the work performed. The application of construction solutions with the belt belongs to the group with less complex machines, but also with a significantly reduced service space, because the crane is also moved in the process of transport. This shortcoming can be eliminated by using transport machines in which loading-unloading trolley with a rotating platform is installed on the upper side of the crane. Such solutions are up-to-date and can achieve much higher transport capacity. The loading-unloading trolley 2 has the platform with the boom 4 (Fig. 1). The axial-radial bearing 1 is installed between the platform and the trolley.

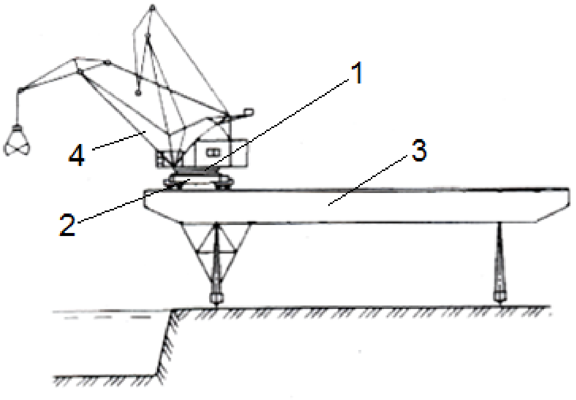

Figure 1. Crane with loading-unloading trolley

Trolley 2 moves along the upper belt of the main supports 3.

Loads in the vertical direction are transferred from the rotating platform, through the axial-radial bearing 1 which lies on the frame of the loading-unloading trolley, to the main longitudinal supports 3 and further through the legs to the crane tracks. It can be seen that the functional capacity of the cranes depends on the reliability of the connection between the rotating platform and the frame of the loading-unloading trolley. Therefore, in order to achieve the reliability of the connection between the rotating platform and the frame of the loading-unloading trolley by means of the radial-axial bearing, it is necessary to establish the dependence of the moment of inertia of the cross sections of box-like beam supports so that the condition of stiffness of the frame of the loading-unloading trolley is met, which provides the required horizontality of the surface on which the axial-radial bearing lies.

Hence, it is necessary to form the calculation model of the frame which will provide the possibility to vary the variable coefficients which depend on the following:

- geometric identification of the frame of loading-unloading trolley,
- the ratio of moments of inertia of the cross section of the segments of the supporting frame of the loading-unloading trolley,
- the length of the segment of longitudinal supports of the frame of the loading-unloading trolley (coefficient w)
- the length of the first segment of the transverse support according to the length of the middle segment of the transverse support (coefficient ψ),
- the ratio of the moment of inertia of the initial part of the transverse support to the moment of inertia of the longitudinal support (φ),
- the ratio of the moment of inertia of the short transverse support to the moment of inertia of the section of the transverse support (coefficient β).

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In addition to the variable coefficients, constant coefficients are introduced and they refer to the thickness of vertical sheets (t_1) and horizontal sheets (t_2), distance of transverse supports (l_2), distance of the middle longitudinal supports (L_2) and clearance between the crane supports and crane track (Δ). This clearance causes another clearance in one of the frame supports of the loading-unloading trolley (e.g. the support C). The constant coefficients representing the thickness ratios of horizontal and vertical sheets are denoted by (λ).

2. FORMATION OF THE MODEL OF THE FRAME OF LOADING-UNLOADING TROLLEY

The entire system of the crane can be divided into two parts:

- the first part consists of the supporting frame of the loading-unloading trolley on which the rotating platform with the boom lies;
- the second part consists of the crane supporting structure which receives all loads from the first part and transfers them to the crane track.

The schematic representation of the support structure of the loading-unloading trolley with supports at the points ABCD is given in Figure 2.

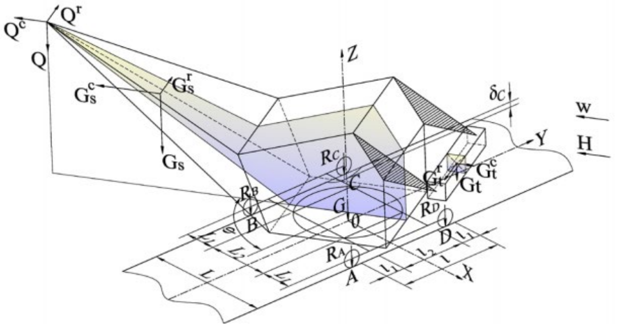

Figure 2: The schematic representation of the support structure of the loading-unloading trolley

The model of the frame of the loading-unloading trolley is shown in Figure 3. If the support structure of the frame of the loading-unloading trolley is considered rigid, then the supports ABCD lie in the plane π. Due to the elastic deformations of the frame elements, the supports ABCD turn from the plane π into the A'B'C'D which are now in the plane π_1.

If it is assumed that the support C has decreased by the size δ_c, then in the supports on the diagonal BD there will be an increase in the reaction between R_B and R_D [1], i.e.:

$$R = R_K \pm c\delta \tag{1}$$

where:

- R_K - the pressures in supports when all supports lie in one plane (π),
- c - the stiffness of the frame of loading-unloading trolley.

The sign (-) in the expression (1) refers to diagonal supports with clearance, while the sign (+) refers to the other two supports.

The determination of the additional force in the support can be done using canonical equations:

$$A\vec{X} = \vec{\delta}_c \tag{2}$$

where:

- A - the quadratic matrix composed of influential coefficients $a_{ik}\left(i,k = 1,2,3,\ldots,7\right)$
- $\vec{X}$ - the vector column of unknown forces and moments $X_i\left(i = 1,2,3,\ldots,7\right)$
- $\vec{\delta}_c$ - the vector column of the displacement in the vertical direction $\delta_i\left(i = 1,2,3,\ldots,7\right)$.

To solve the system of equations (2), it is necessary to determine the members of the quadratic matrix, i.e. the influential coefficients.

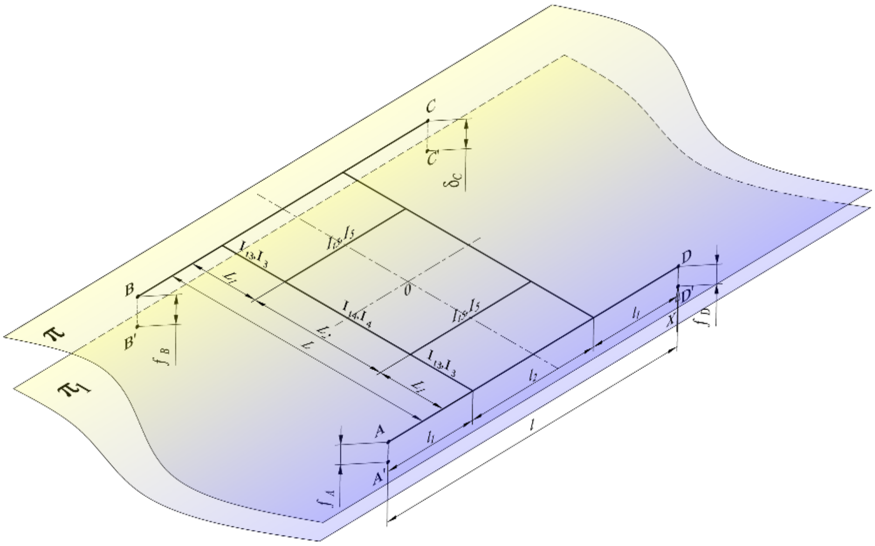

Figure 3: Model of the frame of the loading-unloading trolley

They can be determined by the Maxwell-Mohr integral [2,3]. For the considered case, the general expression for the Maxwell-Mohr integral has the following form:

$$a_{ik} = \int_l \frac{M_{zk} \cdot M_{zi}}{GI_t} dx + \int_l \frac{M_{xk} \cdot M_{xi}}{EI_x} dx \tag{3}$$

By forming the moment diagrams of the equations of forces and moments and by integrating over the entire contour, the influential coefficients a_{ik} are obtained.

For the case under consideration, these coefficients are:

$$\begin{gathered}
a_{11} = \frac{4 \cdot l_1^3}{3 \cdot E \cdot I_1} + \frac{2 \cdot l_1 \cdot l_2^2 \cdot \left(1 + \frac{l_1}{l_2} + \frac{l_2}{3 \cdot l_1}\right)}{E \cdot I_2} + \frac{2 \cdot L_1 \cdot l_2}{G \cdot I_{t3}} + \frac{L_2 \cdot l_2}{G \cdot I_{t4}} \\
a_{22} = \frac{l_2^3}{3 \cdot E \cdot I_2} + \frac{L_1^3}{6 \cdot E \cdot I_3} + \frac{L_1^2 \cdot l_2}{4 \cdot G \cdot I_{t5}} + \frac{l_2^3}{3 \cdot E \cdot I_5} \\
a_{33} = \frac{l_2}{E \cdot I_2} + \frac{l_2}{E \cdot I_5} + \frac{2 \cdot L_1}{G \cdot I_{t3}} \\
a_{44} = \frac{2 \cdot l_2^3}{3 \cdot E \cdot I_5} + \frac{L_2^3}{6 \cdot E \cdot I_4} + \frac{L_2^2 \cdot l_2}{2 \cdot G \cdot I_{t5}}; a_{55} = \frac{2 \cdot l_2}{E \cdot I_5} + \frac{2 \cdot L_2}{G \cdot I_{t5}} \\
a_{66} = \frac{l_2^3}{3 \cdot E \cdot I_2} + \frac{l_2^3}{3 \cdot E \cdot I_5} + \frac{L_1^3}{6 \cdot E \cdot I_3} + \frac{L_1^2 \cdot l_2}{4 \cdot G \cdot I_{t2}} + \frac{L_1^2 \cdot l_2}{4 \cdot G \cdot I_{t5}} \\
a_{77} = \frac{l_2}{E \cdot I_2} + \frac{l_2}{E \cdot I_5} + \frac{2 \cdot L_1}{G \cdot I_{t3}}; a_{12} = \frac{l_2^2 \cdot (3 \cdot l_1 + 2 \cdot l_2)}{6 \cdot E \cdot I_2} \\
a_{13} = -\frac{L_1 \cdot l_2}{G \cdot I_{t3}} \\
a_{14} = 0; a_{15} = -\frac{L_2 \cdot l_2}{G \cdot I_{t4}}; a_{16} = \frac{l_2^2 \cdot (3 \cdot l_1 + 2 \cdot l_2)}{6 \cdot E \cdot I_2} \\
a_{17} = -\frac{l_2 \cdot (2 \cdot l_1 + l_2)}{2 \cdot E \cdot I_2} - \frac{L_1 \cdot l_2}{G \cdot I_{t3}}; a_{23} = -\frac{l_2^2}{2 \cdot E \cdot I_2} - \frac{l_2^2}{2 \cdot E \cdot I_5} \\
a_{24} = -\frac{l_2^3}{3 \cdot E \cdot I_5} + \frac{L_1 \cdot l_2 \cdot L_2}{4 \cdot G \cdot I_{t5}}; a_{25} = \frac{l_2^2}{2 \cdot E \cdot I_5}; a_{26} = a_{27} = 0 \\
a_{34} = \frac{l_2^2}{2 \cdot E \cdot I_5}; a_{35} = \frac{l_2}{E \cdot I_5}; \ a_{36} = a_{37} = 0 \\
a_{45} = -\frac{l_2^2}{E \cdot I_5}; a_{46} = -\frac{l_2^3}{3 \cdot E \cdot I_5} + \frac{L_1 \cdot l_2 \cdot L_2}{4 \cdot G \cdot I_{t5}}; a_{47} = \frac{l_2^2}{2 \cdot E \cdot I_5} \\
a_{56} = \frac{l_2^2}{2 \cdot E \cdot I_5}; a_{57} = -\frac{l_2}{E \cdot I_5}; a_{67} = -\frac{l_2^2}{2 \cdot E \cdot I_5} - \frac{l_2^2}{1 \cdot E \cdot I_2}
\end{gathered} \tag{4}$$

As the cross sections of the support structure of the frame of the loading-loading trolley are box-like, the relation applies to them [1]:

$$\frac{E \cdot I_x}{G \cdot I_t} = k\sqrt{3} \tag{5}$$

where *k* is the ratio of the height *h* and the width *b* of the cross section of the box-like support. Its value ranges from 1 to 3.

In addition to the coefficient *k*, the ratio of bending and torsional stiffness is also affected by the coefficient λ which represents the ratio of the thicknesses of the belt and vertical sheets. Thus, the expression (5) is transformed and has the following form:

$$\frac{E \cdot I_x}{G \cdot I_t} = 1,6 \cdot k \cdot \lambda^{0,35} \tag{6}$$

It should be noted that the values of the coefficient λ range from 1.0 to 1.5. The optimal value of the coefficient λ is $\lambda = 1,3$.

By entering the dependence (6) in the canonical equations (2), the values of the additional forces can be calculated with the occurrence of the clearance δ_c below the support C.

3. STIFFNESS OF THE SUPPORTING STRUCTURE OF THE FRAME OF LOADING-UNLOADING TROLLEY

The ratio of the lengths of the segments l_1 and l_2 (Fig. 3) of the longitudinal supports of the supporting structure of the frames of the loading-unloading trolley is defined by the coefficient:

$$w = \frac{l_1}{l_2} \tag{7}$$

The ratio of the lengths of the first segment L_1 to the length of the middle segment L_2 is defined by the coefficient:

$$\psi = \frac{L_1}{L_2} \tag{8}$$

The coefficient φ defines the ratio of the moments of inertia I_3 of the initial part of the support to the moment of inertia I_2 of the longitudinal support, i.e.:

$$\varphi = \frac{I_3}{I_2} \tag{9}$$

Finally, the coefficient β is introduced because it defines the ratio of the moments of inertia I_4 of the short support to the moment of inertia I_2 of the longitudinal support:

$$\beta = \frac{I_4}{I_2} \tag{10}$$

The study of the change in the stiffness of the support structure of the frames of loading-unloading trolley is conducted for the following values of variable coefficients:

$$w = 0,4; 0,6; 0,8; 1,0$$
$$\psi = 0,4; 0,6; 0,8; 1,0$$
$$\varphi = 1,0; 1,5; 2,0$$
$$\beta = 1,0; 1,5; 2,0$$

By varying the coefficients β, w, φ and ψ, for $\lambda = 1,3$, the values of additional forces in the frame of the unloading-loading trolley can be determined from the canonical equations, when the clearance δ_c appears below the support C (Fig. 3).

As the values of additional forces X_1 change in the support D_1, the stiffness of the frame of loading-unloading trolley also changes in the same ratio with different coefficients (11) of the calculated values for additional forces in the support D (Tables 1, 2, 3 and 4).

Table 1: $T_1 - X_1 = f(\beta, w)$

β	w	X_1(kN)
1,0	0,4	93,74
	0,6	84,02
	0,8	74,29
	1,0	64,57
1,5	0,4	120,00
	0,6	106,68
	0,8	93,37
	1,0	80,05
2,0	0,4	142,92
	0,6	126,30
	0,8	109,69
	1,0	93,07

Table 2: $T_2 - X_1 = f(\psi, \varphi)$

ψ	φ	X_1(kN)
0,4	1,0	72,58
	1,5	77,79
	2,0	83,00
0,6	1,0	66,72
	1,5	72,82
	2,0	78,91
0,8	1,0	60,90
	1,5	67,91
	2,0	74,92
1,0	1,0	56,26
	1,5	63,64
	2,0	71,02

Table 3: $T_3 - X_1 = f(w, \psi)$

w	ψ	X_1(kN)
0,4	0,4	72,76
	0,6	66,22
	0,8	60,18
	1,0	54,10
0,6	0,4	64,11
	0,6	60,12
	0,8	56,13
	1,0	52,84
0,8	0,4	59,76
	0,6	57,38
	0,8	55,00
	1,0	52,62
1,0	0,4	56,55
	0,6	54,12
	0,8	52,01
	1,0	50,40

Table 4: $T_4 - X_1 = f(\varphi, \beta)$

β	w	X_1(kN)
1,0	1,0	81,11
	1,5	99,94
	2,0	118,76
1,5	1,0	84,91
	1,5	104,67
	2,0	124,44
2,0	1,0	87,60
	1,5	109,20
	2,0	130,80

The diagram of the change in the stiffness of the frames of loading-unloading trolley as the function of the change of the coefficients β, w, φ and ψ is given in Figure 4.

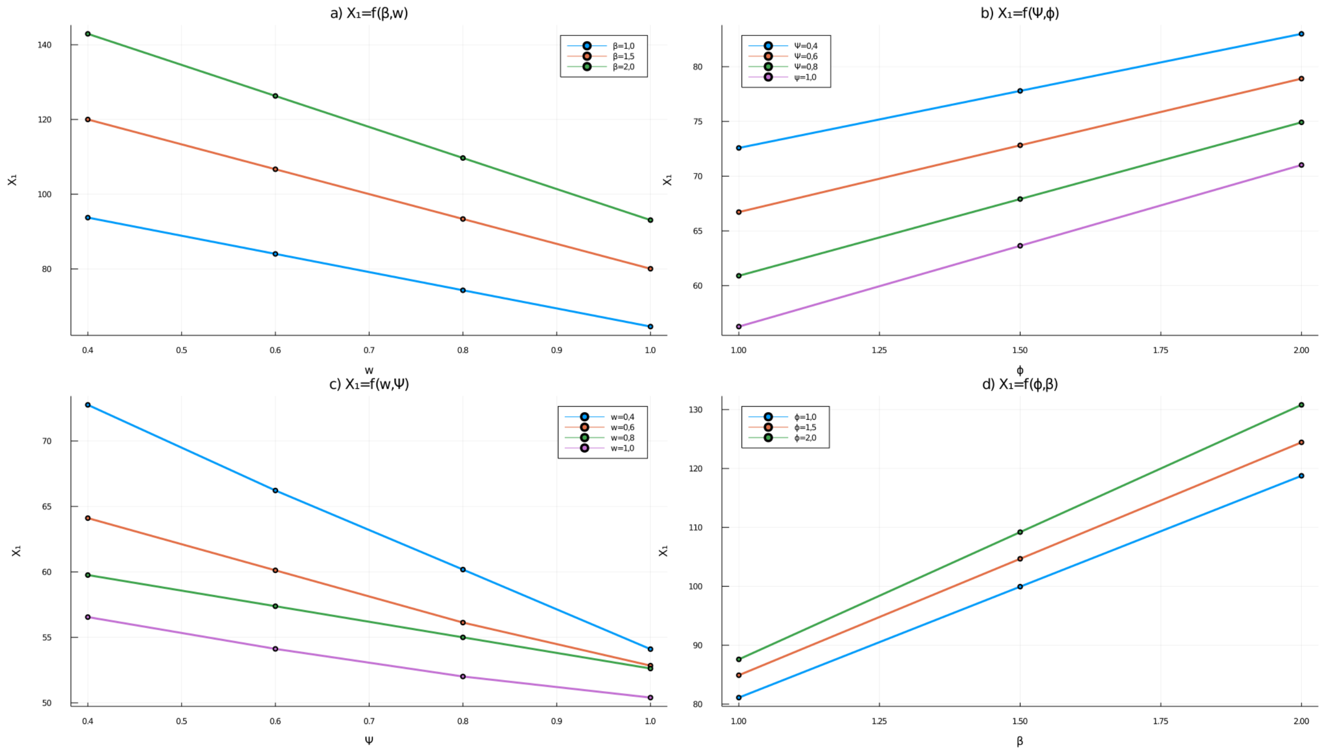

Figure 4. Dependence of the frame stiffness of loading-unloading trolley as the function of the change of coefficients β, w, φ and ψ, with $\lambda = 1,3$

4.CONCLUSION

By analyzing the changes in the frame stiffness of the cranes with loading-unloading trolley for the optimal values of the thickness ratio of the belt and vertical sheets of box-like supports $\lambda = 1,3$, the following conclusions can be formed:

a) As the coefficient β increases, the frame stiffness of the loading-unloading trolley also increases. It should be noted too that the slope of the lines representing the law of stiffness change increases along with the increase of the coefficient β.

b) As the coefficient w increases, the frame stiffness of the loading-unloading trolley decreases, and the coefficient of the slope of the lines representing the law of stiffness change also decreases.

c) As the coefficient ψ increases, the stiffness of the frames of the loading-unloading trolley decreases. The coefficient of stiffness decreases slightly with the increase of the coefficient ψ.

d) As the coefficient φ increases, the frame stiffness of the loading-unloading trolley increases, where the slope of the lines representing the laws of stiffness change is almost constant.

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REFERENCES

[1] Trifković S., Zdravković N., Gašić M., Savković;., Marković G.: Analysis of the Influence Parameters on the Support Structure Stiffness of Large Radial-Axial Bearings, Strojniški vestnik - Journal of Mechanical Engineering 65(2019)6, 366-374, DOI:10.5545/sv-jme.2019.6006.

[2] Karnovsky I.A.: Theory of arched structures strength, vibration, springer, New York, 2012

[3] Karnovsky I.A., Lebed O.: Advanced methods of structural analysis, springer, New York, London, 2010

[4] M. Gašić, M. Savković, G. Marković, N. Zdravković, G. Bošković, M. Nikolić: "Prilog geometrijskoj identifikaciji nosača aksijalnih ležajeva velikih prečnika", IMK-14-Istraživanje i razvoj u teškoj mašinogradnji 23 (2017) 4, SR 117-121, UDK 621 ISSN 0354-6829

[5] Zupan, S., Prebil, I. (2001). Carrying Angle and Carrying Capacity of a Large Single Row Ball Bearomg as a Function of Geometry Parameters of Rolling Contact and Supporting Structure Stiffness, Mechanism and Machine Theory, 38, p. 479-496.

[6] Smolnicki, T., Stanco, M., Pietrusiak, D. (2013). Distribution of loads in the large size bearing-problems of identification, Technical Gazette; Vol. 20, no. 5, p. 831-836.

[7] Jerman, B. Hladnik, J., Resman, F., Landschutzer, C. (2018). Optimization of the support structure of large axial-radial bearing of overhead type manipulator, FME Transactions, vol. 46, no. 2, pp. 386-391, DOI: 10.5937/fmet1803386J

[8] Guanci, C., Cunzhu, W., Zhengming, X. (2016). Effects of supporting Structure and bolt connection on the fatigue life and carrying capacity of a slewing bearing, Jornal of Engineering Tribology, Vol. 2331(6), 1-17, DOI: 10.1177/1350650116677606

[9] Gašić, M., Savković, M., Bulatović, R. (2011). Optimization of trapezoidal cross section of the truck crane boom by Lagrange's multipliers and by differential evolution algorithm (de), Strojniški vestnik - Journal of Mechanical Engineering, Vol. 57, No. 4, p. 304-312, DOI: 10.5545/sv-jme.2008.029.

[10] Karnovsky, I.A., LEbed, O. (2010). Advanced methods of structural analysis, Springer, New York, London.

[11] Бабин Ј.Н., Владић М.Ј., Бркљач Б.Н., Шостаков С.Р.: Металне конструкције у машинству, ФТН издаваштво, Нови Сад, 2012.

[12] Vasiljević R., Gašić M., Savković M.: Parameters Influencing the Dynamic Behaviour of the Carrying Structures of a Type H Portal Crane, Strojniški vestnik - Journal of Mechanical Engineering, vol. 66, no. 10, p. 591-602, 2016. DOI: 10.5545/sv-.jme.2016.3553

[13] Trifković, S., Gašić, M., Radić, N., Milutinović, M.: The Equations of Motion of the Crane with Loading-unloading Trolley on the Slewing Platform, VII International Conference "Heavy Machinery-HM 2014", Zlatibor, 25-28 June 2014, A.183-186

[14] S. Trifković, M. Gašić, N. Radić, M. Milutinović: "Dynamic Analysis of the Crane with Loading-unloading Trolley on Slewing Platform", IMK-14 - Research & Development in Heavy Machinery 21(2015)1, EN1-6, UDC 621 ISSN 0354-6829