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Observer-Based Fault Estimation in Steer-by-Wire Vehicle

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ABSTRACT

In this paper, the mechanism for the fault estimation (FE) problem for a steer-by-wire (SBW) vehicle with sensor and actuator faults is investigated. To deal with the design issues, we transformed the nonlinear model of SBW vehicle into a new coordinate system to jointly estimate the sensor and actuator faults. In the new coordinate system, the Lipschitz conditions and system uncertainties are also considered. The proposed schemes essentially transform the original system into two subsystems, where subsystem-1 includes the effects of actuator faults but is free from sensor faults and subsystem-2 only has sensor faults. Then two sliding mode observers (SMOs) are designed to estimate actuator and sensor faults, respectively. The sufficient conditions for the existence of the proposed observers with H_∞ performance are derived and expressed as an LMI optimization problem such that the upper bounds of the state and fault estimation errors can be minimized. Finally, the numerical example with simulation results is provided to validate the practicability and efficacy of the developed estimation strategy.

KEYWORDS

Joint fault estimation, Sensor faults, Actuator faults, Steer-by-wire vehicle, Sliding-mode observer

1. INTRODUCTION

Modern control systems are prone to faults, which can damage the systems themselves or the environments in which they operate. For this reason, fault detection and isolation (FDI) algorithms become essential, since they enable fault-tolerant actions that minimize the effect of faults and improve the overall system's reliability and safety. The research on FDI has received considerable attention during the last two decades due to the increasing demand for safety and reliability of automatic control systems [1-2]. Naturally, the nonlinear control problem for different dynamics systems is always a hot topic and has attracted compelling attention from scholars, see [3-5].

The approaches of FDI developed in the past can be grouped into three fundamental categories: knowledge-based FDI methods, signal-based FDI and model-based FDI [6]. Among the possible approaches, the model-based FDI ones have been recognized as a handy tool by the scientific community and have been applied successfully to many physical systems, for example, lateral dynamics of a vehicle [7], aircraft [8], satellites [9], unmanned aerial vehicles (UAVs)[10], and wind turbines [11], among others. The basic idea of the observer-based FDI approaches, which fall into the category of model-based FDI, is to generally compare the actual system's behavior with the predicted or estimated behavior based on its mathematical model. Hence, the success of this type of technique directly relates to the mathematical model, which in reality is not a perfect representation. As a consequence, any discrepancies between the system and the model, which appear in the form of system uncertainties in the model, can cause a misleading alarm and make the FDI ineffective. Therefore, there is a need for a robust FDI which is sensitive to faults but insensitive to model uncertainties [12].

On the other side, fault estimation (FE) is different from the majority of fault detection and isolation (FDI) in the sense that it not only detects and isolates the fault, but also provides details of the fault, such as the location, size, and duration. Thus, it is especially useful for incipient faults and slow drifts, which are very difficult to detect. Also FE is vital in fault-tolerant control (FTC) systems which improve the system's performance. During the last two decades, considerable research results have been reported on FE. In [13], an online estimation approach based on adaptive observer technique was adopted to reconstruct the faults with an incipient time profile. A descriptor system approach was introduced to investigate sensor fault diagnosis for nonlinear systems in [14], which is applicable for sensor faults of any forms. Most physical systems are nonlinear, and therefore FDI should employ nonlinear algorithms, which are difficult to generalize and apply in many cases. It is well known that sliding mode techniques offer good potential for increasing the robustness of FDI by including a nonlinear discontinuous term that depends on the output estimation error into the observer [15-16]. During the last two decades, the research on sliding-mode observers (SMO)-based FE has received considerable attention. Several results have been reported on this topic [17-21].

Among various types of FE methods that have been developed, SMO-based FE has been proven to be an effective way to estimate/reconstruct faults for many systems [22-25]. These researches deal separately with actuator or sensor faults. However, in many practical systems, actuators and sensors are jointly prone to faults. Misinterpretation of actuator and sensor faults may cause a high rate of false alarm and unnecessary maintenance. Therefore, it would be desirable to consider actuator and sensor faults under a unified framework.

In this paper, an estimation fault scheme is considered for uncertain Lipschitz nonlinear systems that are subject to actuator and sensor faults simultaneously. The proposed schemes essentially transform the original system into two subsystems, where subsystem-1 includes the effects of actuator faults but is free from sensor faults and subsystem-2 only has sensor faults. Using an integral observer-based approach [26], sensor faults in subsystem-2 are transformed such that they appear as actuator faults. The augmented subsystem-2 is further transformed by a linear coordinate transformation such that a specific structure can be imposed to the sensor fault distribution matrix. In the first scheme, it is assumed that the fault distribution matrix satisfies the matching condition. Then two SMOs are designed to estimate actuator and sensor faults, respectively. However, this assumption is restrictive and sometimes it is difficult to find such matrices to satisfy both the Lyapunov equation and matching condition. To overcome the restriction imposed by the matching condition, many attempts have been made using high-order SMOs [27-28]. The sufficient conditions for the existence of the proposed observers with H∞ performance are derived and expressed as an LMI optimization problem such that the upper bounds of the state and fault estimation errors can be minimized.

The remainder of the paper is organized as follows: Section 2 briefly describe model of the steer-by-wire (SBW) vehicle under faults, disturbances and noise. Section 3 describes the mathematical preliminaries required for developing the FDI method and design procedure of the implemented SMO. The results of the simulation are shown in Section 4 and conclusions are given in Section 5.

2. DESCRIPTION OF THE SBW VEHICLE

Let's consider the following model of a steer-by-wire (SBW) vehicle subject to actuator and sensor faults, disturbances and measurement noise:

$$\begin{aligned}
\dot{\beta}_s &= -\frac{c_0}{mu_x}\beta_s + \left(\frac{c_1}{mu_x^2} - 1\right)\gamma_s + \frac{c_{\alpha f}}{mu_x}\left(\delta_s + f_a\right) + d \\
\dot{\gamma}_s &= -\frac{c_1}{I_z}\beta_s - \left(\frac{c_1}{I_z u_x}\right)\gamma_s + \frac{c_{\alpha f}a}{I_z}\left(\delta_s + f_a\right) + d \\
y_{01} &= \beta_s + w \\
y_{02} &= \gamma_s + f_s + w
\end{aligned} \tag{1}$$

with $c_0 = c_{\alpha f} + c_{\alpha r}$, $c_1 = c_{\alpha r}b - c_{\alpha f}a$, $c_{\alpha f}a^2 + c_{\alpha r}b^2$. The different parameters m, I_z, $c_{\alpha f}$, $c_{\alpha r}$, a and b are given with corresponding numerical values in Table 1. The slide-slip angle at the center of gravity is represented by β_s, γ_s is the yaw rate, δ_s is the front steering angle of the vehicle, and u_x represents the longitudinal velocity which is assumed constant and equal to 8 m/s. Also, $x_1 = \beta_s$ and $x_2 = \gamma_s$ are the state variables, $u = \delta_s$ is the control input of the system, f_a is the actuator fault, f_s is the sensor fault, d represents the disturbance corrupting the dynamics and w is the measurement noise.

Table 1. Parameters of the steer-by-wire vehicle model.

Notations	Denotes
$m = 350\ kg$	Mass of the vehicle
$I_z = 260\ kgm^2$	Moment of inertia of the front wheel system
$c_{\alpha f} = 2700\ N/rad$	Front cornering stiffness
$c_{\alpha r} = 2700\ N/rad$	Rear cornering stiffness
$a = 0.65\ m$	Distance from the center of gravity to the front axle
$b = 0.83\ m$	Distance from the center of gravity to the rear axle

3. JOINT ESTIMATION OF ACTUATOR AND SENSOR FAULTS USING SMO

Consider a non-linear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x,t) + B\big(u(t) + f_a(t)\big) + E\Delta\psi(t) \\ y &= Cx(t) + Df_s(t) \end{aligned} \tag{2}$$

where the state vector is $x \in R^n$, the input vector is $u \in R^m$, the output vector is $y \in R^p$, the actuator fault is denoted as $f_a \in R^m$, the sensor fault is denoted as $f_s \in R^q$. The known constant matrices are $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{p\times q}$, $E \in R^{n\times r}$ (p ≥ q) and B and D both being of full rank.

Remark 1: It should be noted that the system uncertainty under consideration is unstructured, which is more general than the type of structured uncertainty that has been considered for fault diagnosis of Lipschitz nonlinear systems in the literature [30]. In the case of structured system uncertainty, certain rank conditions of the uncertainty distribution matrix are assumed to be satisfied such that the fault can be completely decoupled from the uncertainty.

For the objective of achieving fault diagnosis, the following assumptions and Lemmas are introduced. The nonlinear term $f(x,t) \in R^n$ is assumed to be known and Lipschitz about x uniformly $\forall x, \hat{x} \in R^n$.

$$\left\| f(x,t) - f(\hat{x},t) \right\| \le \mathrm{L}_{ff} \left\| x - \hat{x} \right\| \tag{3}$$

where L_{ff} is the known Lipschitz constant. The unknown nonlinear term $\Delta\psi(t)$ is structured modelling uncertainty, but bounded, and satisfies $\Delta\psi(t) \le \xi$. Also, the unknown sensor fault and its derivative are norm bounded, i.e. $f_s(t) \le \rho$ and $\dot{f}_s(t) \le \rho_s$.

Lemma 1. [30] Let's assume $\text{rank}(CB) = \text{rank}(B)$. There exist state and output transformations of coordinates

$$h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = Tx \text{ and } w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = Sy \tag{4}$$

such that in the new coordinates, the system matrices become

$$TAT^{-1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix},\ TB = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},\ TE = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix},\ SCT^{-1} = \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix},\ SD = \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \tag{5}$$

where $T \in R^{n\times n}$, $S \in R^{p\times p}$, $T_1 \in R^{m\times n}$, $S_1 \in R^{m\times p}$, $h_1 \in R^m$, $w_1 \in R^m$, $A_1 \in R^{m\times m}$, $A_4 \in R^{(n-m)\times(n-m)}$, $B_1 \in R^{m\times m}$, $E_1 \in R^{m\times r}$, $C_1 \in R^{m\times m}$, $C_4 \in R^{(p-m)\times(n-m)}$ and $D_2 \in R^{(p-m)\times q}$. B_1 and C_1 is invertible.

After introducing the state and output transformations (4), the original system is converted into two subsystems:

$$\text{Subsystem 1:} \begin{cases} \dot{h}_1 = A_1 h_1 + A_2 h_2 + T_1 f(T^{-1}h,t) + B_1(u + f_a) + E_1 \Delta\psi(t) \\ w_1 = C_1 h_1 \end{cases} \tag{6}$$

$$\text{Subsystem 2:}\begin{cases}\dot{h}_2 = A_3h_1 + A_4h_2 + T_2f(T^{-1}h,t) + E_2\Delta\psi \\ w_2 = C_4h_2 + D_2f_s\end{cases} \tag{7}$$

For Subsystem 2, we define a new state $h_3 = \int_0^t w_2(\tau)d\tau$ so that $\dot{h}_3 = C_4h_2 + D_2f_s$. An augmented system with the new state h_3 is therefore given as:

$$\begin{aligned}&\dot{h}_0 = A_0h_0 + \bar{A}_3h_1 + \bar{T}_2f(T^{-1}h,t) + D_0f_s + E_0\Delta\psi \\ &w_3 = C_0h_0\end{aligned} \tag{8}$$

in which $h_0 = \begin{bmatrix} h_2 & h_3\end{bmatrix}^T \in R^{n+p-2m}$, $w_3 \in R^{p-m}$, $A_0 = \begin{bmatrix} A_4 & 0 \\ C_4 & 0\end{bmatrix} \in R^{(n+p-2m)\times(n+p-2m)}$, $\bar{A}_3 = \begin{bmatrix} A_3 \\ 0\end{bmatrix} \in R^{(n+p-2m)\times m}$, $E_0 = \begin{bmatrix} E_2 \\ 0\end{bmatrix} \in R^{(n+p-2m)\times r}$, $D_0 = \begin{bmatrix} 0 \\ D_2\end{bmatrix} \in R^{(n+p-2r)\times q}$, $C_0 = \begin{bmatrix} 0 & I_{p-m}\end{bmatrix} \in R^{(p-m)\times(n+p-2m)}$, $\bar{T}_2 = \begin{bmatrix} T_2 \\ 0\end{bmatrix} \in R^{(n+p-2m)\times n}$.

Lemma 2.[31-32] The pair (A_0, C_0) is observable if and only if the minimum phase condition holds

$$\text{rank}\begin{bmatrix} sI_n - A & B \\ C & 0\end{bmatrix} = n + \text{rank}(B) \tag{9}$$

for every complex number s with nonnegative real part.

Now, a scheme which consists of two SMOs to estimate actuator and sensor faults is developed. One SMO is designed to estimate actuator faults while the other one is designed to estimate sensor faults.

By taking the state transformation of $z \triangleq T_0h_0 = [z_1^T z_2^T]^T$ with

$$T_0 = \begin{bmatrix} I_{n-p} & P_{01}^{-1}P_{02} \\ 0 & I_{p-m}\end{bmatrix} \tag{10}$$

where $z_1 \in R^{n-m}$ and $z_2 \in R^{p-m}$.

Subsystem (8) can be rewritten as in a more compact form as

$$\begin{aligned}&\dot{z} = A_zz + T_0A_3h_1 + T_0\bar{T}_2f(T^{-1},t) + D_zf_s + E_z\Delta\psi \\ &w_3 = C_zz\end{aligned} \tag{11}$$

where

$$A_z = \begin{bmatrix} A_4 + P_{01}^{-1}P_{02}C_4 & -A_4P_{01}^{-1}P_{02} - P_{01}^{-1}P_{02}C_4P_{01}^{-1}P_{02} \\ C_4 & -C_4P_{01}^{-1}P_{02}\end{bmatrix},\ D_z = \begin{bmatrix} P_{01}^{-1}P_{02}D_2 \\ D_2\end{bmatrix},\ E_z = \begin{bmatrix} E_2 \\ 0\end{bmatrix},\ C_z = [0 \quad I_{p-m}] \tag{12}$$

Lemma 3.[31][32] If there exists an arbitrary matrix $F_0 \in R^{q(p-m)}$ such that

$$D_0^TP_0 = F_0C_0 \tag{13}$$

then we have

$$P_{02}D_2 = 0, \tag{14}$$

$$F_0 = D_2^T\bar{P}_{03}. \tag{15}$$

Therefore, Systems (6) and (11) can be rewritten respectively as

$$\text{Subsystem 1:}\begin{cases}\dot{h}_1 = A_1h_1 + A_2z_1 + A_2P_{01}^{-1}P_{02}w_3 + T_2f(T^{-1}h,t) + B_1(u+f_a) + E_1\Delta\psi(t) \\ w_1 = C_1h_1\end{cases} \tag{16}$$

$$\text{Subsystem 2}:\begin{cases}\dot{z}_1=\bar{A}_1z_1+\bar{A}_2z_2+A_3h_1+T_2f(T^{-1}h,t)+E_2\Delta\psi\\ \dot{z}_2=\bar{A}_3z_1+\bar{A}_4z_2+D_2f_s\\ w_3=z_2\end{cases} \tag{17}$$

For the above systems it is constructed the following two SMOs:

$$\begin{cases}\dot{\hat{h}}_1=A_1\hat{h}_1+A_2\hat{z}_1-A_2P_{01}^{-1}P_{02}w_3+T_2f(T^{-1}\hat{h},t)+B_1(u+v_1)+(A_1-A_{1s})C_1^{-1}(w_1-\hat{w}_1)\\ \hat{w}_1=C_1\hat{h}_1\end{cases} \tag{18}$$

$$\begin{cases}\dot{\hat{z}}_1=\bar{A}_1\hat{z}_1+\bar{A}_2w_3+A_3C_1^{-1}w_1+T_2f(T^{-1}\hat{h},t)\\ \dot{\hat{z}}_2=\bar{A}_3\hat{z}_1+\bar{A}_4\hat{z}_2+(\bar{A}_4-L)(\bar{w}_3-w_3)+D_2v_2\\ \hat{w}_3=\hat{z}_2\end{cases} \tag{19}$$

where $\hat{h}_1,\hat{z}_1,\hat{z}_2,\hat{w}_1$ and $\hat{w}_3$ denote, respectively, the estimated h_1,z_1,z_2,w_1 and w_3. $A_1^s\in R^{m\times m}$ is a stable matrix, and $L\in R^{(p-m)\times(p-m)}$ is the observer gain. It is worth noting that $\hat{h}\triangleq\mathrm{col}(C_1^{-1}S_1y,\ \hat{z}_1-P_{01}^{-1}P_{02}w_3)$ and does not represent the state estimate vector $\mathrm{col}(\hat{h}_1,\hat{h}_2)$. The discontinuous output error injection terms v1 and v2 are defined by

$$v_1=\begin{cases}(\rho_a+\eta_1)\dfrac{B_1^TP_1\left(C_1^{-1}S_1y-\hat{h}_1\right)}{\left\|B_1^TP_1\left(C_1^{-1}S_1y-\hat{h}_1\right)\right\|}, & C_1^{-1}S_1y-\hat{h}_1\neq 0\\ 0 & \text{otherwise}\end{cases} \tag{20}$$

$$v_2=\begin{cases}(\rho_s+\eta_2)\dfrac{D_2^T\bar{P}_{03}\left(w_3-\hat{w}_3\right)}{\left\|D_2^T\bar{P}_{03}\left(w_3-\hat{w}_3\right)\right\|}, & w_3-\hat{w}_3\neq 0\\ 0 & \text{otherwise}\end{cases} \tag{21}$$

where $P_1\in R^{m\times m}$ is the symmetric definite Lyapunov matrix for A_1^s, and η_1 and η_2 are two positive scalars which are to be determined.

If the state estimation errors are defined as $e_1=h_1-\hat{h}_1$, $e_2=z_1-\hat{z}_1$, and $e_3=z_2-\hat{z}_2$ then the controlled estimation error can be obtained as

$$r=He=H\begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix} \tag{22}$$

We now present Theorem 1 which establishes the sufficient conditions for the existence of the proposed SMOs in the form of (18) and (19) with a prescribed performance

$$\|H\|_\infty\triangleq\sup_{\|\Delta\psi\|_{L_2}\neq 0}\frac{\|r\|_{L_2}^2}{\|\Delta\psi\|_{L_2}^2}\leq\mu, \tag{23}$$

where μ is a small positive constant.

Theorem 1. Let's consider system (2) with introduced lemmas. If there exist matrices A_1^s, L, $P_1=P_1^T>0$, $P_{01}=P_{01}^T>0$, $P_{03}=P_{03}^T>0$, and P_{02}, and positive scalars α_1, α_2 and μ such that

$$P_{02}D_2=0 \tag{24}$$

$$\Lambda\triangleq\begin{bmatrix}\Pi_1+H_1^TH_1 & P_1A_2 & 0 & P_1E_1\\ A_2^TP_1 & \Pi_2+H_2^TH_2 & C_4^T\bar{P}_{03} & P_{01}E_2\\ 0 & \bar{P}_{03}C_4 & \Pi_3+H_3^TH_3 & 0\\ T_1^TP_1 & T_2^TP_{01} & 0 & -\mu I_r\end{bmatrix}<0 \tag{25}$$

where $\Pi_1 = A_1^{sT} P_1 + P_1 A_1^s + \frac{1}{\alpha_1} P_1 T_1 T_1^T P_1$, $\Pi_2 = A_4^T P_{01} + P_{01} A_4 + C_4^T P_{02}^T + P_{02} C_4 + \frac{1}{\alpha_2} P_{01} T_2 T_2^T P_{01} + (\alpha_1+\alpha_2) \mathrm{L}_f^2 \left\| T^{-1} \right\|^2 I_{n-m}$ and $\Pi_3 = \bar{P}_{03} L + L^T \bar{P}_{03}$, then the estimation error dynamics are asymptotically stable with the prescribed H_∞ tracking performance $\|r\|_{\mathrm{L}_2} \le \sqrt{\mu} \|\Delta\psi\|_{\mathrm{L}_2}$. The Proof of Theorem 1 can be seen in [33].

Theorem 2. Let's consider system (2) with introduced lemmas. If there exist matrices A_1^s, X, Y, Z, $P_1 = P_1^T > 0$, $P_{01} = P_{01}^T > 0$, $P_{03} = P_{03}^T > 0$, and positive scalars α_1, α_2 such that the following LMI feasibility problem has a solution:

$$\begin{bmatrix} X + X^T + H_1^T H_1 & P_1 A_2 & 0 & P_1 E_1 & P_1 T_1 & 0 \\ A_2^T P_1 & \Pi_4 + H_2^T H_2 & C_4^T \bar{P}_{03} & P_{01} E_2 & 0 & P_{01} T_2 \\ 0 & \bar{P}_{03} C_4 & Y + Y^T + H_3^T H_3 & 0 & 0 & 0 \\ E_1^T P_1 & E_2^T P_{01} & 0 & -\mu I & 0 & 0 \\ T_1^T P_1 & 0 & 0 & 0 & -\alpha_1 I & 0 \\ 0 & T_2^T P_{01} & 0 & 0 & 0 & -\alpha_2 I \end{bmatrix} < 0 \tag{26}$$

where

$$\Pi_4 = A_4^T P_{01} + P_{01} A_4 + (\alpha_1 + \alpha_2) \mathrm{L}_f^2 \left\| T^{-1} \right\|^2 I_{n-m} + Z C_4 - Z D_2 D_2^+ C_4 + (Z C_4)^T - (Z D_2 D_2^+ C_4)^T$$

then the estimation error dynamics is asymptotically stable with the prescribed H_∞ tracking performance $\|r\|_{\mathrm{L}_2} \le \sqrt{\mu} \|\Delta\psi\|_{\mathrm{L}_2}$. The Proof of Theorem 2 can be seen in [33].

The effect of system uncertainties on the estimation errors is decided by the value of μ. The accuracy of the fault estimation increases with smaller value of μ. The minimization of μ can be found by solving the following LMI optimization problem:

$$\begin{gathered} \text{minimize } \mu \text{ subject to} \\ X < 0,\ P_1 > 0,\ P_{01} > 0,\ \bar{P}_{03} > 0 \text{ and (26).} \end{gathered} \tag{27}$$

We have proved that the proposed observers are asymptotically stable with the prescribed H_∞ performance in Theorem 2. The objective now is to determine the constant gain η_1 in (20) and η_2 in (21) such that the error systems can be directed to the sliding surface S which is defined as

$$\mathrm{S} = \left\{ (e_1, e_2, e_3) \middle| e_1 = 0,\ e_3 = 0 \right\} \tag{28}$$

in finite time and maintain on it thereafter.

Theorem 3. Let's consider system (2) with introduced assumptions, lemmas and the proposed observers (18) and (19). Then the error dynamics

$$\dot{e}_1 = A_1^s e_1 + A_2 e_2 + T_1 \left(f(T^{-1}h,t) - f(T^{-1}\hat{h},t) \right) + B_1 (f_a - v_1) + E_1 \Delta\psi \tag{29}$$

$$\dot{e}_2 = \bar{A}_1 e_2 + T_2 \left(f(T^{-1}h,t) - f(T^{-1}\hat{h},t) \right) + E_2 \Delta\psi \tag{30}$$

$$\dot{e}_3 = \bar{A}_3 e_2 + L e_3 + D_2 (f_s - v_2) \tag{31}$$

can be driven to the sliding surface (28) in finite time and remain on it if LMI optimization problem formulated in (27) is solvable and the gains η_1 and η_2 satisfy

$$\eta_1 \ge \left\| B_1^{-T} \right\| \left(\|A_2\| \|e_2\| + \mathrm{L}_f \|T_1\| \left\| T^{-1} \right\| \|e_2\| + \|E_1\| \xi \right) + \eta_3, \tag{32}$$

$$\eta_2 \ge \frac{\|C_4 e_2\| + \eta_4}{\left\| D_2^T \bar{P}_{03} e_3 \right\|}, \tag{33}$$

where η_3 and η_4 are positive scalars. The Proof of Theorem 3 can be seen in [33].

Given the observers in the form of (18) and (19), the objective now is to simultaneously estimate actuator and sensor faults. From Theorem 3, we know that an ideal sliding motion (28) will take place after some finite time if the conditions (32) and (33) are satisfied. During the sliding motion, (29) becomes

$$0 = A_2 e_2 + T_1\left(f(T^{-1}h,t) - f(T^{-1}\hat{h},t)\right) + B_1\left(f_a - v_{1eq}\right) + E_1 \Delta\psi \tag{34}$$

where v_{1eq} is the equivalent output error injection signal to maintain the sliding motion [33], and can be approximated to any degree of accuracy by replacing (20) with

$$v_1 \approx (\rho_a + \eta_1)\frac{B_1^T P_1\left(C_1^{-1}S_1 y - \hat{h}_1\right)}{\left\|B_1^T P_1\left(C_1^{-1}S_1 y - \hat{h}_1\right)\right\| + \delta_1} \tag{35}$$

where δ_1 is a small positive scalar to reduce the chattering effect.

Since B_1 is invertible, (34) can be further rewritten as

$$f_a - v_{1eq} = -B_1^{-1}\left(A_2 e_2 + T_1\left(f(T^{-1}h,t) - f(T^{-1}\hat{h},t)\right)\right) + E_1 \Delta\psi \tag{36}$$

Computing the L_2 norm of (36) yields

$$\begin{aligned}\left\|f_a - v_{1eq}\right\|_{\mathrm{L}_2} &= \left\|B_1^{-1}\left(A_2 e_2 + T_1\left(f(T^{-1}h,t) - f(T^{-1}\hat{h},t)\right)\right) + E_1 \Delta\psi\right\|_{\mathrm{L}_2} \le \left(\sigma_{max}\left(B_1^{-1}A_2\right) + \sigma_{max}\left(B_1^{-1}T_1\right)\mathrm{L}_f\left\|T^{-1}\right\|\right)\|e\|_{\mathrm{L}_2} + \\ &+ \sigma_{max}\left(B_1^{-1}E_1\right)\|\Delta\psi\|_{\mathrm{L}_2}\end{aligned} \tag{37}$$

Since $\|e\|_{\mathrm{L}_2} \le \sigma_{max}\left(H^{-1}\right)\sqrt{\mu}\|\Delta\psi\|_{\mathrm{L}_2}$, it can be obtained that

$$\left\|f_a - v_{1eq}\right\|_{\mathrm{L}_2} = \left(\sqrt{\mu}\left(\sigma_{max}\left(B_1^{-1}A_2\right) + \sigma_{max}\left(B_1^{-1}T_1\right)\mathrm{L}_f\left\|T^{-1}\right\|\right)\right)\cdot\sigma_{max}\left(H^{-1}\right) + \sigma_{max}\left(B_1^{-1}E_1\right)\|\Delta\psi\|_{\mathrm{L}_2} \tag{38}$$

It follows that

$$\sup_{\|\Delta\psi\|_{\mathrm{L}_2}\neq 0}\frac{\left\|f_a - v_{1eq}\right\|_{\mathrm{L}_2}}{\|\Delta\psi\|_{\mathrm{L}_2}} \le \mu\beta_1 + \beta_2, \tag{39}$$

where $\beta_1 = \left(\sigma_{max}\left(B_1^{-1}A_2\right) + \sigma_{max}\left(B_1^{-1}T_1\right)\mathrm{L}_f\left\|T^{-1}\right\|\right)\sigma_{max}\left(H^{-1}\right)$ and $\beta_2 = \sigma_{max}\left(B_1^{-1}E_1\right)$.

Thus for a small $\left(\mu\beta_1 + \beta_2\right)\|\Delta\psi\|_{\mathrm{L}_2}$, the actuator faults f_a can be approximated as

$$\hat{f}_a \approx (\rho + \eta_1)\frac{B_1^T P_1\left(C_1^{-1}S_1 y - \hat{h}_1\right)}{\left\|B_1^T P_1\left(C_1^{-1}S_1 y - \hat{h}_1\right)\right\| + \delta_1} \tag{40}$$

Similarly, we can get that

$$\sup_{\|\Delta\psi\|_{\mathrm{L}_2}\neq 0}\frac{\left\|f_a - v_{2eq}\right\|_{\mathrm{L}_2}}{\|\Delta\psi\|_{\mathrm{L}_2}} \le \sqrt{\mu}\sigma_{\max}\left(H^{-1}\right)\sigma_{\max}\left(D_2^{+}\bar{A}_3\right) \tag{41}$$

where v_{2eq} is the equivalent output error injection signal which can be approximated as

$$v_2 \approx (\rho_s + \eta_2)\frac{D_2^T \bar{P}_{03}\left(w_3 - \hat{w}_3\right)}{\left\|D_2^T \bar{P}_{03}\left(w_3 - \hat{w}_3\right)\right\| + \delta_2} \tag{42}$$

where δ_2 is a small positive scalar. Thus for a small $\sqrt{\mu}\sigma_{\max}\left(H^{-1}\right)\sigma_{\max}\left(D_2^{+}\bar{A}_3\right)\|\Delta\psi\|_{\mathrm{L}_2}$, the sensor faults f_s can be approximated as

$$\hat{f}_s \approx (\rho_s + \eta_2)\frac{D_2^T \bar{P}_{03}\left(w_3 - \hat{w}_3\right)}{\left\|D_2^T \bar{P}_{03}\left(w_3 - \hat{w}_3\right)\right\| + \delta_2}. \tag{43}$$

4. SIMULATION RESULTS

It is assumed that there is a complex sensor fault composed of a combination of abrupt, ramp, and sine-type faults, which is represented as:

$$f_s = \begin{cases} 0 & t < 50 \\ 0.05 \cdot \sin(0.15\pi t) & 50 \le t < 90 \\ 0 & 90 \le t < 100 \end{cases}, \tag{44}$$

$$f_a = \begin{cases} 0 & t < 20 \\ 0.2 \cdot (t - 20) & 20 \le t < 50 \\ 0.5 + 0.2 \cdot \sin(0.2\pi t) & 50 \le t < 100 \end{cases}. \tag{45}$$

For simulation purposes, we set the control input $u(t)$ as sinus function $\sin(\pi t)$ and the system uncertainty $\Delta\psi(t)$ as

$$\Delta\psi(t) = \begin{bmatrix} 0.1\sin(0.2t)^2 \\ 0.1\sin(0.1t)^2 \\ 0.2\sin(t)^2 \\ 0.3\sin(t)^2 \end{bmatrix}. \tag{46}$$

The results of sensor and actuator faults estimation are illustrated in Fig.1 and Fig.2. These figures clearly demonstrate that the proposed SMO based estimation method is able to estimate the faults successfully, irrespective of uncertainties in the system. In addition to gradual and slow faults, such as a ramp and sinusoidal faults, it should be noted that f_a changes abruptly at time instant 50s, which indicates that the proposed approach has the ability to track abrupt faults.

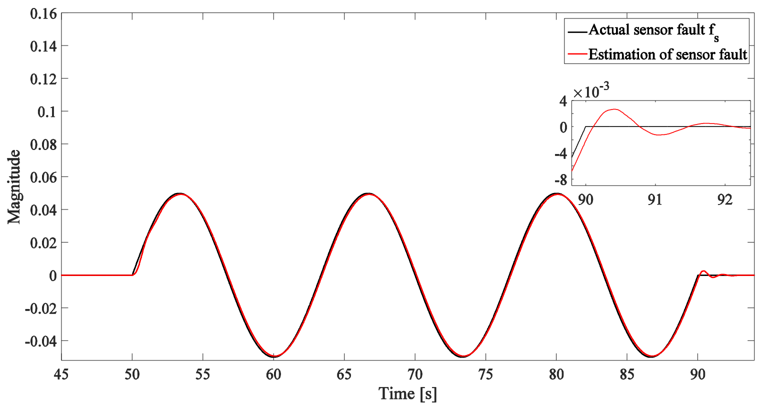

Figure 1: Estimation of the sensor fault

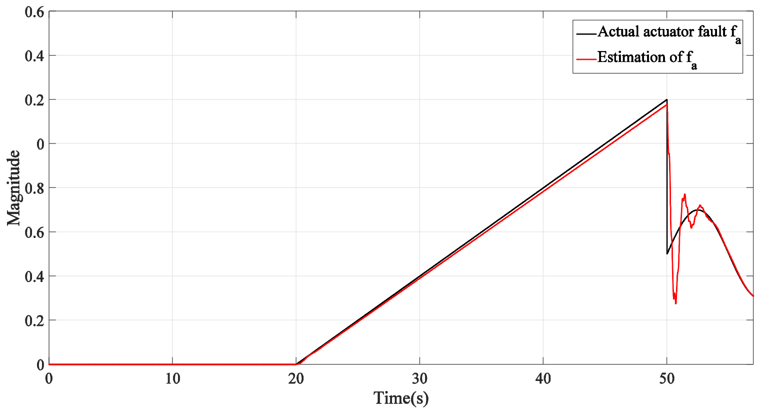

Figure 2: Estimation of the actuator fault

Fig. 3 shows the trajectories of the actual states and their estimates. It can be seen from the figures that the proposed SMO approach can estimate the states accurately, before and after the occurrence of any fault.

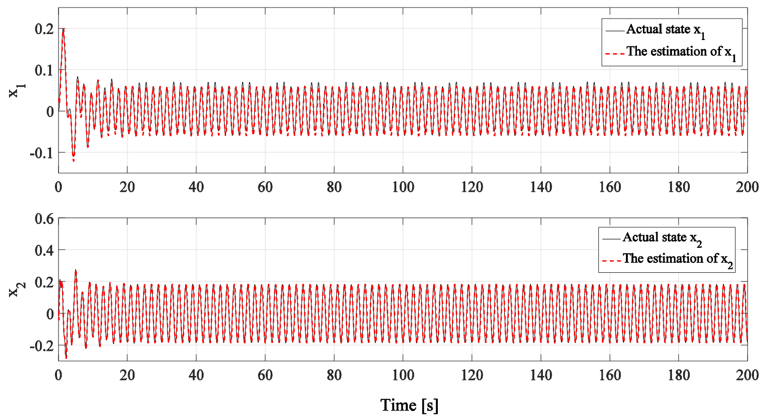

Figure 3: System states and their estimates

It can be seen that despite the presence of system uncertainties and measurement noises, the tracking performances of states x and actuator and sensor fault have achieved an ideal performance.

5. CONCLUSIONS

In this paper, an algorithm for jointly estimating actuator and sensor faults for uncertain Lipschitz nonlinear systems is considered. The proposed method uses two SMOs and can effectively estimate both the actuator and sensor faults provided certain matching condition is satisfied. Moreover, H∞ filtering is integrated into the algorithm to attenuate the effects of the system uncertainties on state and fault estimates. The simulation results show that proposed approach can successfully estimate actuator and sensor faults simultaneously despite the existence of system uncertainties. When the outputs are contaminated by noises, sensor fault estimates can still be obtained. However, real-world applications are not focused exclusively on successful FE. When a system is subject to a set of possible faults, fault-tolerant control (FTC) is necessary to maintain stability and reliability. We will study this issue in future.

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REFERENCES

[1] Zhang, J., A. K. Swain, and S. K. Nguang, "Detection and isolation of incipient sensor faults for a class of uncertain nonlinear systems", IET Contr. Theory Appl., Vol. 6, pp. 1870–1880, https://doi.org/10.1049/iet-cta.2011.0440, (2012).

[2] Zhang, P. and J. Zou, "Observer-based fault diagnosis and self-restore control for systems with measurement delays", Asian J. Control, Vol. 14, pp. 1717–1723, https://doi.org/10.1002/asjc.458, (2012)

[3] Jayamaha DK, Lidula NW, Rajapakse AD "Wavelet-multi resolution analysis based ANN architecture for fault detection and localization in DC microgrids", IEEE Access, Vol.3(7), pp. 145371–84, https://doi.org/10.1109/ACCESS.2019.2945397, (2019)

[4] Z. Wang, Y. Shen, X. Zhang, "Actuator fault estimation for a class of nonlinear descriptor systems", Int J Syst Sci, Vol.45, pp. 487–496, https://doi.org/10.1080/00207721.2012.724100, (2014).

[5] J. Yang, Y. Guo, W. Zhao, "Long short-term memory neural network based fault detection and isolation for electro-mechanical actuators", Neurocomputing, Vol. 360, pp. 85–96, https://doi.org/10.1016/j.neucom.2019.06.029, (2019).

[6] F. Ferracuti, V. Artists, A. Monteriù, "Algorithms for Fault Detection and Diagnosis", MDPI AG, Basel, Switzerland, https://doi.org/10.3390/books978-3-0365-0463-6, (2021).

[7] S. Gómez-Peñate, G. Valencia-Palomo, FR. López-Estrada, CM. Astorga-Zaragoza, R.A. Osornio-Rios, I. Santos-Ruiz, "Sensor fault diagnosis based on a sliding mode and unknown input observer for Takagi-Sugeno systems with uncertain premise variables", Asian J Control, Vol. 21(1), pp. 339-353, https://doi.org/10.1002/asjc.1913, (2019).

[8] D. Henry, J. Cieslak, A. Zolghadri, D. Efimov, "H∞/ H-LPV solutions for fault detection of aircraft actuator faults: bridging the gap between theory and practice", Int J Robust Nonlinear Control, Vol.25(5), pp. 649-672, https://doi.org/10.1002/rnc.3157, (2015).

[9] Q. Jia, W. Chen, Y. Zhang, X. Chen, "Robust fault reconstruction via learning observers in linear parameter-varying systems subject to loss of actuator effectiveness", IET Control Theory Appl, Vol. 8(1), pp. 42-50, https://doi.org/10.1049/iet-cta.2013.0417, (2014).

[10] D. Rotondo, A. Cristofaro, T.A. Johansen, F. Nejjari, V. Puig, "Robust fault and icing diagnosis in unmanned aerial vehicles using LPV interval observers", Int J Robust Nonlinear Control, Vol. 29(16), pp. 5456-5480, https://doi.org/10.1002/rnc.4381, (2019).

[11] S. Li, H. Wang, A. Aitouche, N. Christov, "Unknown input observer design for faults estimation using linear parameter varying model. application to wind turbine systems", Paper presented at: Proceedings of the 2018 7th International Conference on Systems and Control, Valencia, Spain, pp. 45-50, https://doi.org/10.1109/ICoSC.2018.8587778, (2018).

[12] J. Zhang, A.K. Swain, S.K. Nguang, "Robust sliding mode observer based fault estimation for certain class of uncertain nonlinear systems", Asian Journal of Control, https://doi.org/10.1002/asjc.987, (2015).

[13] C. Bonivento, A. Isidoria, L. Marconia, A. Paolia, "Implicitfault-tolerant control: application to induction motors", Automatica, Vol.40(3), pp. 355–371, https://doi.org/10.1016/j.automatica.2003.10.003, (2004).

[14] Z.W. Gao, S.X. Ding, "Sensor fault reconstruction and sensor compensation for a class of nonlinear state-space systems via a descriptor system approach", IET Control Theory Appl, Vol.1, pp. 578–585, https://doi.org/10.1049/iet-cta:20050509, (2007).

[15] C.P. Tan, C. Edwards, "An LMI approach for designing sliding mode observers", Int J Control, Vol. 74(16), pp. 1559-1568, https://doi.org/10.1080/00207170110081723, (2001).

[16] Vl. Utkin, "Sliding Modes in Control and Optimization", Berlin, Germany, Springer Science & Business Media, https://doi.org/10.1007/978-3-642-84379-2, (2013).

[17] C.P. Tan, C. Edwards, "Sliding mode observers for detection and reconstruction of sensor faults", Automatica, Vol. 38, pp. 1815–1821, https://doi.org/10.1016/S0005-1098(02)00098-5, (2002).

[18] B. Jiang, M. Staroswiecki, V. Cocquempot, "Fault estimation in nonlinear uncertain systems using robust/sliding-mode observers", IEE Proc Control Theory Appl, Vol. 151(1), pp. 29–37, https://doi.org/10.1049/ip-cta:20040074, (2004).

[19] F.E. Thomas, I. Castillo, R.F. Benito, "Robust model-based fault detection and isolation for nonlinear processes using sliding modes", Int J Robust Nonlinear Control, Vol. 22, pp. 89–104, https://doi.org/10.1002/rnc.1807 (2012).

[20] CP. Tan, C. Edwards, "Sliding mode observers for robust detection and reconstruction of actuator and sensor faults", Int J Robust Nonlinear Control, Vol. 13, pp. 443–463, https://doi.org/10.1002/rnc.723, (2003).

[21] C.P. Tan, C. Edwards, "A robust sensor fault tolerant control scheme implemented on a crane", Asian J Control, Vol. 9(3), pp. 340–344, https://doi.org/10.1111/j.1934-6093.2007.tb00420.x, (2007).

[22] C. Edwards, S.K. Spurgeon, R.J. Patton, "Sliding mode observers for fault detection and isolation", Automatica, Vol. 36, pp. 541–553, https://doi.org/10.1016/S0005-1098(99)00177-6, (2000).

[23] X.G. Yan, C. Edwards, "Sensor fault detection and isolation for nonlinear systems based on a sliding mode observer", Int J Adapt Control Signal Process, Vol. 21, pp. 657–673, https://doi.org/10.1002/acs.967, (2007).

[24] H. Alwi, C. Edwards, C.P. Tan, "Sliding mode estimation schemes for incipient sensor faults", Automatica, Vol. 45, pp.1679–1685, https://doi.org/10.1016/j.automatica.2009.02.031, (2009).

[25] X.G. Yan, C. Edwards, "Robust sliding mode observer-based actuator fault detection and isolation for a class of nonlinear systems", Int J Syst Sci, Vol. 39(4), pp. 349–359, https://doi.org/10.1080/00207720701778395, (2008).

[26] J. Zhang, A.K. Swain, S.K. Nguang, "Detection and isolation of incipient sensor faults for a class of uncertain nonlinear systems", IET Control Theory Appl, Vol. 6, pp. 1870–1880, https://doi.org/10.1049/iet-cta.2011.0440, (2012).

[27] T. Floquet, C. Edwards, S.K. Spurgeon, "On sliding mode observers for systems with unknown inputs", Int J Adapt Control Signal Process, Vol. 21, pp. 638–656, https://doi.org/10.1002/acs.958, (2007).

[28] F. Zhu, "State estimation and unknown input reconstruction via both reduced-order and high-order sliding mode observers", J Process Control, Vol. 22, 296–302, https://doi.org/10.1016/j.jprocont.2011.07.007, (2012)

[29] M. Jelali, A. Kroll, "Hydraulic Servo-systems: Modelling, Identification and Control", Berlin: Springer; https://doi.org/10.1007/978-1-4471-0099-7, (2003).

[30] R. Raoufi, H.J. Marquez, A.S.I. Zinober, "H∞ sliding mode observer for uncertain nonlinear Lipschitz systems with fault estimation synthesis", Int J Robust Nonlinear Control, Vol. 20, pp.1785–1801, https://doi.org/10.1002/rnc.1545, (2010).

[31] M. Corless, J. Tu, "State and input estimation for a class of uncertain systems", Automatica, Vol. 34, pp. 757–764, https://doi.org/10.1016/S0005-1098(98)00013-2, (1998).

[32] S. Hui, S.H. Zak, "Observer design for system with unknown inputs", Int. J. Appl. Math. Comput. Sci, Vol.15(4), pp. 431–446, (2005).

[33] J. Zhang, A.K. Swain, S.K. Nguang, "Robust Observer-Based Fault Diagnosis for Nonlinear Systems Using MATLAB®", Springer International Publishing, Switzerland, https://doi.org/10.1007/978-3-319-32324-4, (2016).

[34] Z.W. Gao, S.X. Ding, "Sensor fault reconstruction and sensor compensation for a class of nonlinear state-space systems via a descriptor system approach", IET Control Theory Application, Vol. 1, pp. 578–585, https://doi.org/10.1049/iet-cta:20050509, (2007).

[35] C. Gao, Q. Zhao, G. Duan, "Robust actuator fault diagnosis scheme for satellite attitude control systems", J Frankl Inst, Vol. 350(9), pp, 2560–2580, https://doi.org/10.1016/j.jfranklin.2013.02.021, (2013).

[36] F. Zhu, Z. Han, "A note on observers for lipschitz nonlinear systems", IEEE Trans Autom Control, Vol. 47, pp.1751–1754, https://doi.org/10.1109/TAC.2002.803552, (2002)