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Vehicle Suspension System with Integrated Inerter - Extended Analysis

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Abstract. Inerter devices are used in Formula 1 vehicles for a decade to reduce vertical and rolling displacement at the corner of the sharp racing curves and provide higher tire grip on racing challenges. To improve dynamics Formula SAE Car, several topological designs for suspension system are already proposed and published [1]. Based on our passive suspension system for quarter-car model, we carried out analysis for different parameters and under the impact from the road disturbance and demonstrated when integrated inerter device can increase ride comfort and decrease vertical displacement. Proposed solution system with integrated inerter mechanism can improve vehicle suspension systems development and have effects on dynamics and stability of a vehicle.

1. Introduction

Inerter devices are used in Formula 1 vehicles (figure 1) for a decade to reduce vertical and rolling displacement at the corner of the sharp racing curves and provide higher tire grip on racing challenges. Its application to Formula SAE race car (SAE – Society of Automotive Engineers) has not been studied enough. Some pioneering work in this field has been done by Tran et al. [1-3]. In general engineering practice, suspension systems with passive, semi-active and active elements have been used to improve vehicle ride comfort [4,5]. Active suspension systems are the most investigated in the literature [6], but in order to increase robustness of the system, passive suspension systems are the most desirable [7]. Smith [8] introduced inerter elements including practical realization and Chen et al. [9] studied different mechanical network topologies with springs, dampers and inerter elements. In our study, we investigated passive suspension systems which can improve rolling stability by reducing vertical displacement and increase drive comfort. Our passive suspension system containing springs, dampers and inerter elements and discusses its influence on behavior of both the sprung and un-sprung mass of the vehicle subjected to road disturbance. The vertical displacement of vehicle body can be reduced by optimizing the springs, dampers and inerter elements which represent parameters of tire and suspension.

2. Physical model

The force produced by inerter is:

$$F = b(\ddot{z}_2 - \ddot{z}_1), \tag{1}$$

where b is the inertance, and $\ddot{z}_1$, $\ddot{z}_2$ are accelerations on two terminals (figure 2). For inerter model presented in figure 2, the constant b can be obtained as:

$$b = m\alpha_1\alpha_2 \tag{2}$$

with

$$\alpha_1 = \frac{\gamma}{r_3},\ \alpha_2 = \frac{r_2}{r_1}. \tag{3}$$

Another parameters are: r_1 - the radius of the rack pinion, r_2 - the radius of the gear wheel, r_3 - the radius of the flywheel pinion, γ - the radius of gyration of the flywheel, m - the mass of the flywheel.

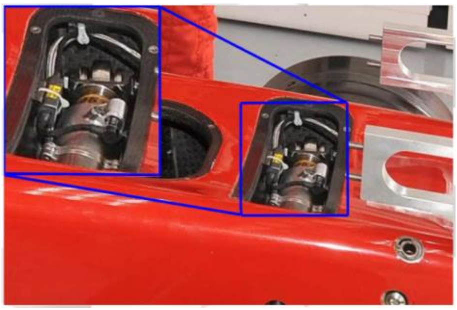

Figure 1. Ferrari inerter fitted on F10 racer, 2010.

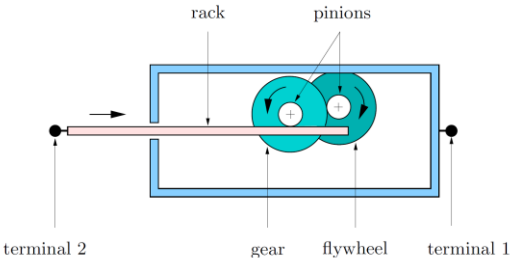

Figure 2. Inerter model proposed by Smith [8]

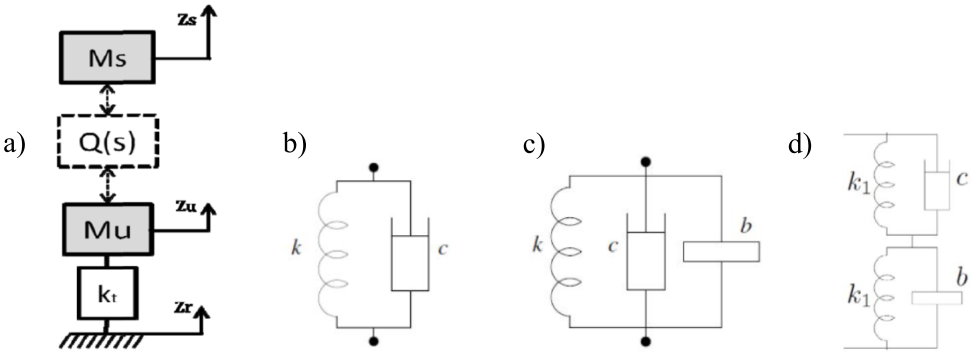

Figure 3. The quarter-car model with suspension function block Q(s) in Laplace domain (a), and different arhitectures for Q block: b) basic, c) parallel, d) series

In the figure 3a is presented quarter car model composed of two masses. k_t is tire stiffness. M_u is mass of the wheel (unsprung mass) and M_s is mass of the chassis (sprung mass). Function block $Q(s)$ in figure 3a is suspension function in Laplace domain and can have one of the forms presented in figure 3b, 3c or 3d. Constants k, k_1 are spring stiffnesses, c is damping and b is inertance.

3. Mathematical model

Equations of motion for the basic quarter-car model in time-domain are:

$$M_s\ddot{z}_s(t) = F_k(t) + F_c(t) \tag{4}$$
$$M_u\ddot{z}_u(t) = F_{kt}(t) - (F_k(t) + F_c(t)) \tag{5}$$

where forces due to suspension spring stiffness (F_k), suspension damper (F_c) and tire stiffness (F_{kt}) are defined as:

$$F_k(t) = k(z_u(t) - z_s(t)) \tag{6}$$

$$F_c(t) = c(\dot{z}_u(t) - \dot{z}_s(t)) \tag{7}$$

$$F_{kt}(t) = k_t(z_r(t) - z_u(t)) \tag{8}$$

z_r, z_u, z_s are displacement due to road roughness, displacement of unsprung mass (wheal) and displacement of the sprung mass (chassis), respectively, in time domain. Over dots are time derivatives. Variables Z_r, Z_u, Z_s represent displacements z_r, z_u, z_s, but in Laplace transformed domain.

The equations of motion (4,5) can be transformed in the Laplace domain leading to:

$$M_s s^2 Z_s = -sQ(s)(Z_s - Z_u) \tag{9}$$

$$M_u s^2 Z_u = sQ(s)(Z_s - Z_u) + k_t(Z_r - Z_u) \tag{10}$$

where $Q(s)$ is suspension function represented in Laplace transformed domain. From equations (9,10) are computed relevant transfer functions in the following text and equations, because they are suitable for studying our dynamic problem. Influence of the road disturbance z_r to the displacement of the sprung mass z_s is described as

$$T_{z_r \to z_s} = T_{Z_r \to Z_s} = \frac{k_t Q(s)}{M_s s(M_u s^2 + k_t) + \left((M_u + M_s)s^2 + k_t\right)Q(s)} \tag{11}$$

Influence of the road disturbance z_r to the acceleration of the sprung mass $\ddot{z}_s$ is described with the following transfer function:

$$T_{z_r \to \ddot{z}_s} = s^2 T_{Z_r \to Z_s} = \frac{s^2 k_t Q(s)}{M_s s(M_u s^2 + k_t) + \left((M_u + M_s)s^2 + k_t\right)Q(s)} \tag{12}$$

Influence of the road disturbance z_r to the displacement of the unsprung mass z_u is described as

$$T_{z_r \to z_u} = T_{Z_r \to Z_u} = \frac{k_t(M_s s + Q(s))}{(M_u s^2 + k_t)\left(M_s s + Q(s)\right) + M_s s^2 Q(s)} \tag{13}$$

Influence of the road disturbance z_r to the acceleration of the unsprung mass $\ddot{z}_u$ is described with the following transfer function:

$$T_{z_r \to \ddot{z}_u} = s^2 T_{Z_r \to Z_u} = \frac{s^2 k_t(M_s s + Q(s))}{(M_u s^2 + k_t)\left(M_s s + Q(s)\right) + M_s s^2 Q(s)} \tag{14}$$

The conventional suspension function represented in Laplace transformed domain:

$$Q(s) = Y_k + Y_c = \frac{k}{s} + c \tag{15}$$

The parallel suspension function represented in Laplace transformed domain:

$$Q(s) = Y_k + Y_c + Y_b = \frac{k}{s} + c + bs \tag{16}$$

The series suspension function represented in Laplace transformed domain:

$$Q(s) = \left(\frac{1}{Y_c+Y_{k_1}} + \frac{1}{Y_b+Y_{k_2}}\right)^{-1} = \left(\frac{1}{c+\frac{k_1}{s}} + \frac{1}{bs+\frac{k_2}{s}}\right)^{-1} \tag{17}$$

where Y_k, Y_c, Y_b are stiffness, damping and inertance in Laplace domain. In our special case two springs have equal stiffness leading to $Y_{k1} = Y_{k2}$. Ride comfort is defined as:

$$J_1 = E\left[\ddot{Z}_s^2(t)\right] = r.m.s \text{ body vertical acceleration} \tag{18}$$

4. Numerical results

With data from the table 1 are obtained results presented on figures 4-7. In the figure 4 are presented displacement and acceleration response of sprung body mass in 3 different configurations presented on the figure 3. Since the parallel configuration gave the best results, it is used in the further analysis. figures 6 – 7. present displacement and acceleration response of sprung mass for 4 different values of inertance parameter (0,20,40,60 kg). Figure 8. presents maximal displacement and maximal acceleration of sprung mass and Figure 9. ride comfort as a function of inertance. This diagram can serve for engineering design purposes.

Table 1. Used parameters for Formula SAE quarter-car.

Symbols	Parameters	Values
M_s	Mass of body	63 kg
M_u	Mass of tire	12 kg
k	Stiffness coefficient	24000 N/m
k_1	Stiffness coefficient	48000 N/m
c	Damping coefficient	1200 Ns/m
c_1	Damping coefficient	2400 Ns/m
b	Mass of inertance	20 kg
k_t	Stiffness coefficient of tyre	70000 N/m
H_0	Road disturbance hump	0.05 m

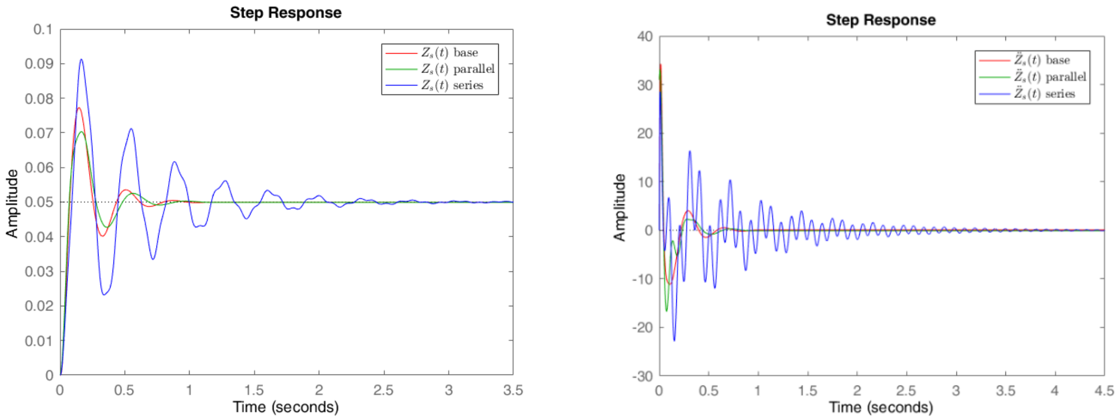

Figure 4. Time response diagrams for a) displacement and b) acceleration of sprung mass for 3 different quarter-car models subjected to road disturbance hump

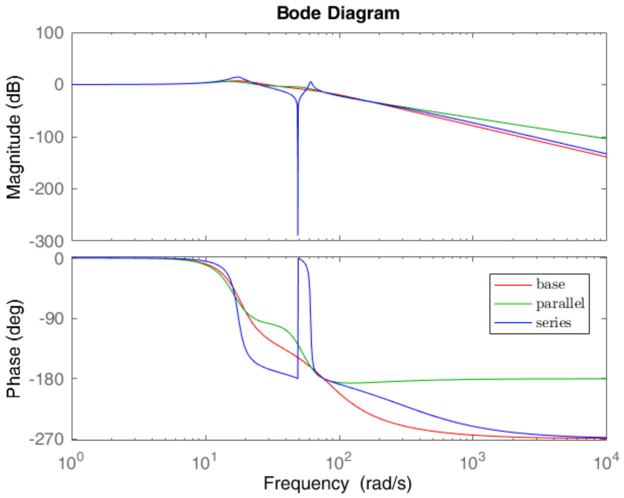

Figure 5. Bode diagrams for 3 different quarter car models

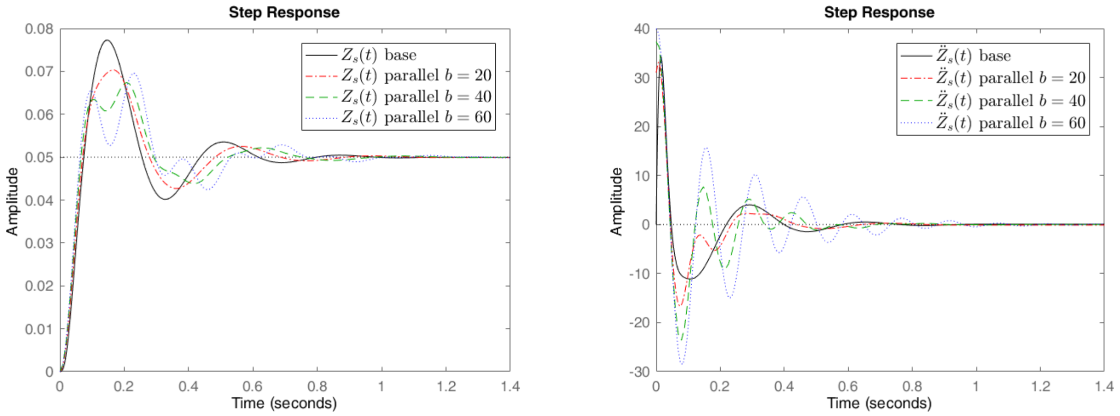

Figure 6. Time response of sprung mass of quarter-car model for different values of inertance paramer in parallel configuration: a) displacement and b) acceleration

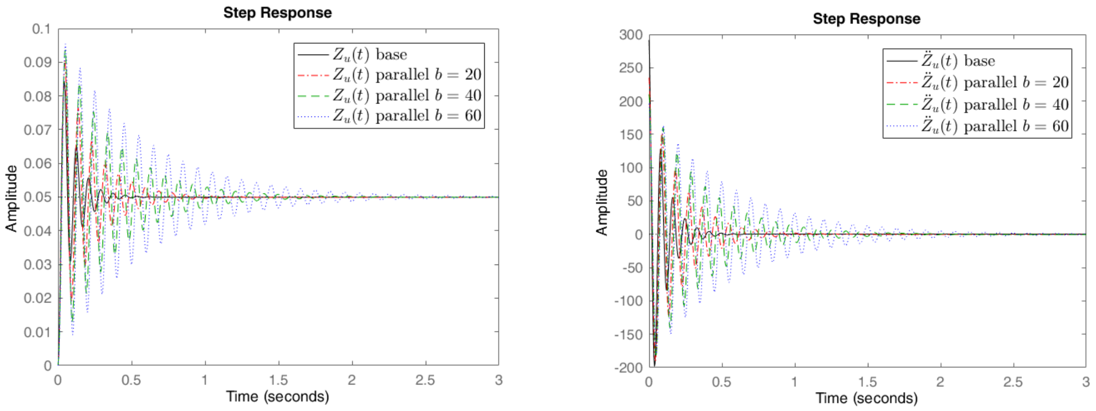

Figure 7. Time response of unsprung mass of quarter-car model for different values of inertance paramer in parallel configuration: a) displacement and b) acceleration

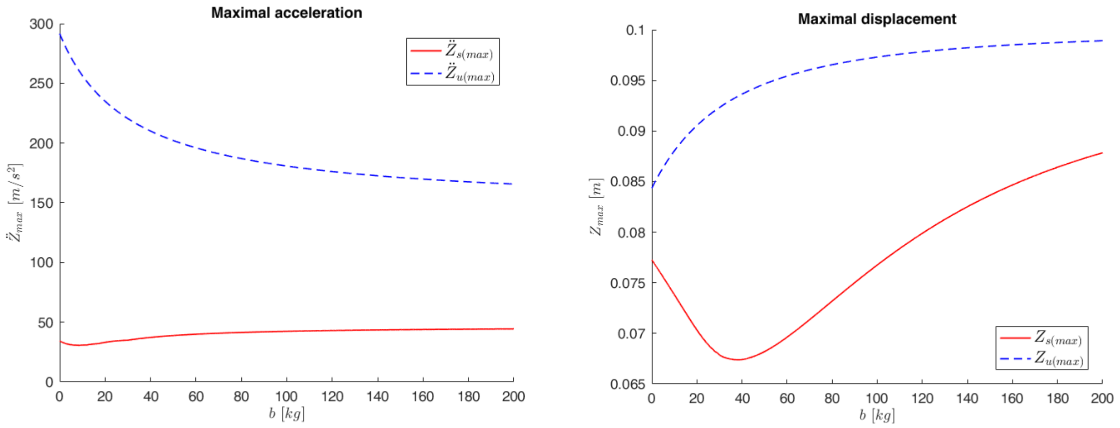

Figure 8. Parallel quarter car model: a) Maximal acceleration, b) maximal displacement

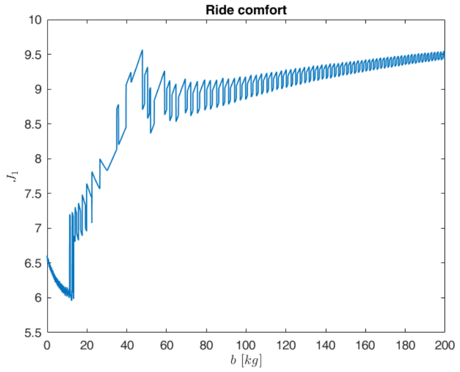

Figure 9. Parallel quarter car model: ride comfort as a function of inertance b

5. Conclusion

This paper demonstrated that quarter-car Formula SAE passive suspension model with inerter can reduce the oscillation, comparing to conventional spring and damper model. Influence of inertance on dynamic behavior is also presented which can serve for an optimal suspension design. Based on our passive suspension system for quarter-car model, we carried out analysis for different parameters and under the impact from the road disturbance and demonstrated when integrated inerter device can increase ride comfort and decrease vertical displacement. Therefore, proposed solution system with integrated inerter mechanism can improve vehicle suspension systems development and have effects on dynamics and stability of a vehicle. For more profound investigation, this study can be extended with half-car and full car model.

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