

DYNAMIC MODEL OF CYCLOIDAL SPEED REDUCER

Mirko Blagojević¹, Zorica Đorđević², Miloš Matejić³, Nenad Kostić⁴, Nenad Petrović⁵

UDC:621.833.058:[519.6]

INTRODUCTION

Thanks to the application of internal spur gears, planetary gearboxes are characterized by extreme compactness of the structure, high transmission ratios, as well as a good balance of the dynamic forces. Consequences of the smaller gear dimensions used in planetary speed reducer are mainly smaller rotational speeds and the sliding and rolling speed on the teeth. As a result, the dynamic load and vibrations in the elements of the planetary gear trains are less. The sliding and rolling speeds on the teeth in the planetary gear are lower by approximately 30% to 40% compared to conventional speed reducer with parallel axes. Also, it should be noted significantly less rotating mass (up 75%), lower moment of inertia, the less impact force when starting and stopping gear, [4, 17].

Cycloidal speed reducer belongs to a group of modern planetary gear. Thanks to its good working characteristics (large gear ratio, long and reliable service life, compact design, high efficiency, low vibration, low noise, ...), cycloidal speed reducers have very widely application in modern industry.

In order to describe the dynamic behaviour of the cycloidal speed reducer, the first were analysed well known dynamic models of planetary gears classical concepts. The dynamic models of planetary speed reducers are presented in papers [9, 10]. The same author (*Kahraman*) with a group of co-authors analysed the influence of the ring gear rim thickness on planetary gear set behaviour, [11, 12]. Determination of gear mesh stiffness and internal dynamic forces at planetary gear drives is presented in papers [1, 2, 3, 15]. The elasto-dynamic model of internal gear planetary transmissions is described in paper [19]. Determination of planetary gear natural frequencies and vibration modes is presented in papers [7, 14].

In the papers [5, 13, 17] the various procedures for calculating of the cycloidal speed reducer efficiency have presented. Determination of stress and strain state of cycloidal speed reducer elements using the FEM in static and dynamic conditions is presented in papers [6, 8, 18]. The procedure for torsion stiffness calculation of cycloidal speed reducer is defined in paper [16].

A dynamic model of a single - stage cycloidal speed reducer is developed in this paper.

¹*Mirko Blagojević, Associate Professor, Faculty of Engineering, University of Kragujevac, Sestre Janjić 6, Kragujevac, mirkob@kg.ac.rs*

²*Zorica Đorđević, Associate Professor, Faculty of Engineering, University of Kragujevac, Sestre Janjić 6, Kragujevac, zoricadj@kg.ac.rs*

³*Miloš Matejić, Ph.D student, Faculty of Engineering, University of Kragujevac, Sestre Janjić 6, Kragujevac, mmatejic@kg.ac.rs*

⁴*Nenad Kostić, Ph.D student, Faculty of Engineering, University of Kragujevac, Sestre Janjić 6, Kragujevac, nkostic@kg.ac.rs*

⁵*Nenad Petrović, Ph.D student, Faculty of Engineering, University of Kragujevac, Sestre Janjić 6, Kragujevac, npetrovic@kg.ac.rs*

DYNAMIC MODEL OF ONE - STAGE CYCLOIDAL SPEED REDUCER

The main generators of internal dynamic forces in gear drives are: changes of deformations in meshing process, collisions of teeth in the meshing process, profile shape deviations, tooth wear, etc. Cycloidal speed reducer is a very complex mechanical system and for studying its dynamic behavior it is necessary to take into accounts all of its peculiarities. The greatest impact on the internal dynamic forces at cycloidal speed reducer have: errors that occur during the production of cycloid disc's teeth, as well as other elements of cycloidal speed reducer, unequal distribution of load at cycloid disc teeth as well as at stationary central gear rollers and output rollers, elastic deformations of case and other elements,...

On the basis of a very careful analysis of dynamic models of external involute tooth gearing, as well as dynamic model of planetary gear trains, the dynamic model of single - stage cycloidal speed reducer was developed in this paper. This model is presented in Figure 1.

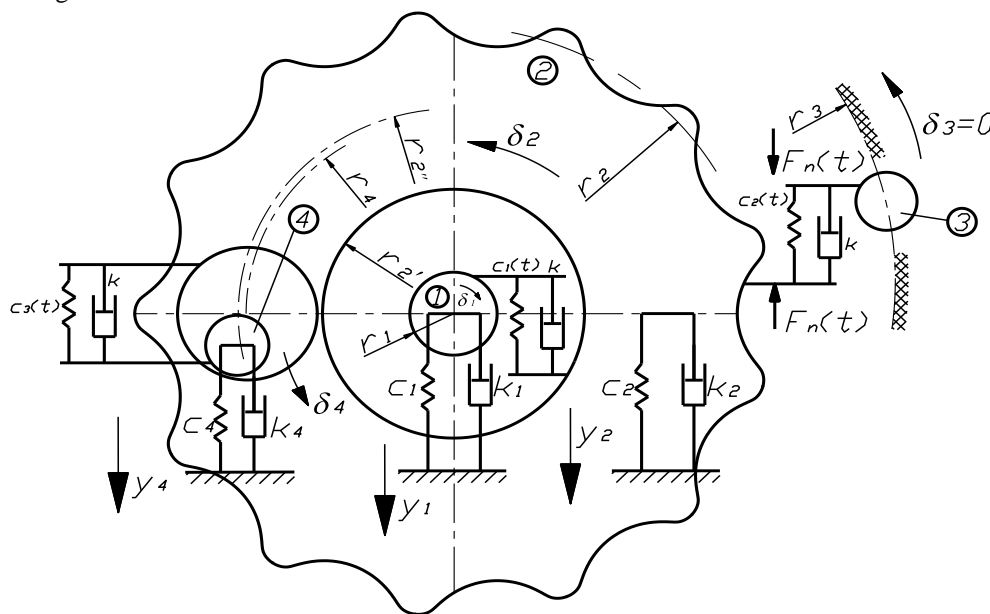


Figure 1 Dynamic model of a single-stage cycloidal speed reducer (1-input shaft with the eccentric cam, 2-cycloid disc, 3-stationary central gear roller, output roller)

The geometric dimensions from figure 1 are:

- r_1 – external radius of the eccentric cam,
- r_2 – radius of the pitch circle of the cycloid disc,
- r_2' – radius of the central hole of the cycloid disc,
- r_2'' – radius of the cycloid disc circle with the holes for the output rollers,
- r_3 – radius of the pitch circle of the stationary central gear,
- r_4 – radius of the output shaft flange with the holes for corresponding rollers.

The cycloidal speed reducer elements are connected in their supports in the following way:

- Input shaft with eccentric cam: elastic connect of the stiffness c_1 and a damper with the coefficient of damping k_1 ,
- Cycloid disc: elastic connection of the stiffness c_2 and a damper with coefficient of damping k_2 ,
- Stationary central gear roller: elastic connection of the stiffness c_3 and a damper with the coefficient of damping k_3 ,
- Output roller: elastic connection of the stiffness c_4 and a damper with the coefficient of damping k_4 .

The contacts between the corresponding elements of the cycloidal speed reducer are described in the following way:

- Input shaft with eccentric cam – cycloid disc: elastic connection of the variable stiffness $c_1(t)$ and damper with the coefficient of damping k ,
- Cycloid disc – stationary central gear roller: elastic connection of the variable stiffness $c_2(t)$ and a damper with the coefficient of damping k ,
- Cycloid disc – output roller: elastic connection with the variable coefficient of stiffness $c_3(t)$ and a damper with the coefficient of damping k .

The excitation force $F_n(t)$ occurs in the contact between the cycloid disc tooth and the stationary central gear roller. The excitation force is calculated using the following expression, [1, 2, 3]:

$$F_n(t) = c \cdot w(t) \cdot b, \quad (1)$$

where:

c - connected teeth stiffness,

$w(t)$ - deformation of the cycloid disc tooth,

b - cycloid disc width.

For as much as deformation's magnitude periodical time's function, the same case is for excitation force. The total displacement x in the contact between the cycloid disc tooth and the stationary central gear roller can be expressed as follows:

$$x = r_2 \cdot \delta_2 - y_2, \quad (2)$$

The result of the excitation force action is a dynamic force which loads the cycloid disc teeth and the stationary central gear rollers:

$$F_e = c \cdot x + b \cdot \dot{x}, \quad (3)$$

The kinetic system energy is:

$$E_k = \frac{1}{2} m_1 \cdot \dot{y}_1^2 + \frac{1}{2} J_1 \cdot \dot{\delta}_1^2 + \frac{1}{2} m_2 \cdot \dot{y}_2^2 + \frac{1}{2} J_2 \cdot \dot{\delta}_2^2 + \frac{1}{2} m_4 \cdot \dot{y}_4^2 + \frac{1}{2} J_4 \cdot \dot{\delta}_4^2, \quad (4)$$

$$E_p = \frac{1}{2} c_1 \cdot \dot{y}_1^2 + \frac{1}{2} c_2 \cdot \dot{y}_2^2 + \frac{1}{2} c_4 \cdot \dot{y}_4^2 + \frac{1}{2} c_1(t) \cdot [(y_2 - r_2 \cdot \dot{\delta}_2) - (y_1 + r_1 \cdot \dot{\delta}_1)]^2 + \frac{1}{2} c_2(t) \cdot [(y_2 - r_2 \cdot \dot{\delta}_2)]^2 + \frac{1}{2} c_3(t) \cdot [(y_4 + r_4 \cdot \dot{\delta}_4) - (y_2 + r_2 \cdot \dot{\delta}_2)]^2 \quad (5)$$

The dissipation system function is:

$$\Phi = \frac{1}{2} k_1 \cdot \dot{y}_1^2 + \frac{1}{2} k_2 \cdot \dot{y}_2^2 + \frac{1}{2} k_4 \cdot \dot{y}_4^2 + \frac{1}{2} k \cdot [(\dot{y}_2 - r_2 \cdot \dot{\delta}_2) - (\dot{y}_1 + r_1 \cdot \dot{\delta}_1)]^2 + \frac{1}{2} k \cdot [(\dot{y}_2 - r_2 \cdot \dot{\delta}_2)]^2 + \frac{1}{2} k \cdot [(\dot{y}_4 + r_4 \cdot \dot{\delta}_4) - (\dot{y}_2 + r_2 \cdot \dot{\delta}_2)]^2 \quad (6)$$

The virtual work of the conservative forces is:

$$\delta A = F_n(t) \cdot r_2 \cdot \delta \delta_2, \quad (7)$$

The conservative force is:

$$Q_{\delta_2} = F_n(t) \cdot r_2, \quad (8)$$

The signs m_1 , m_2 and m_3 stand for masses of correspondent elements, and J_1 , J_2 and J_3 stand for a rectangular moment of inertia of the same elements.

Stiffnesses in the corresponding contacts and the dynamic force are variable time functions. However, in order to enable the analysis of influence of deviation of geometric dimensions, it is adopted that the mentioned stiffnesses are constant and equal to some average values, and the excitation force stay a variable time function.

$$\begin{aligned} c_1(t) &= c_{01} = \text{const.} \\ c_2(t) &= c_{02} = \text{const.} \\ c_3(t) &= c_{03} = \text{const.} \end{aligned} \quad (9)$$

The differential equation of the system motion in matrix form is:

$$\begin{bmatrix} J_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_4 \end{bmatrix} \begin{bmatrix} \ddot{\delta}_1 \\ \ddot{y}_1 \\ \ddot{\delta}_2 \\ \ddot{y}_2 \\ \ddot{\delta}_4 \\ \ddot{y}_4 \end{bmatrix} + \begin{bmatrix} k \cdot r_1^2 & k \cdot r_1 & k \cdot r_1 \cdot \dot{r}_2 & -k \cdot r_1 & 0 & 0 \\ 0 & k \cdot r_1 & k_1 + k & -k & 0 & 0 \\ k \cdot r_1 \cdot \dot{r}_2 & k \cdot r_2 & k \cdot \left[(r_2^2 + (\dot{r}_2^2) + r_2^2) \right] & k \cdot (-r_2 + r_2 \cdot \dot{r}_2) & -k \cdot r_4 \cdot \dot{r}_2 & -k \cdot r_2 \\ k \cdot r_1 & k & k \cdot (r_2 + r_2 \cdot \dot{r}_2) & k_2 - k & k \cdot r_4 & -k \\ 0 & 0 & -k \cdot r_4 \cdot \dot{r}_2 & -k \cdot r_4 & k \cdot r_4^2 & k \cdot r_4 \\ 0 & 0 & -k \cdot \dot{r}_2 & -k & k \cdot r_4 & k + k_4 \end{bmatrix} \begin{bmatrix} \dot{\delta}_1 \\ \dot{y}_1 \\ \dot{\delta}_2 \\ \dot{y}_2 \\ \dot{\delta}_4 \\ \dot{y}_4 \end{bmatrix} + \begin{bmatrix} c_{01} \cdot r_1^2 & c_{01} \cdot r_1 & c_{01} \cdot r_1 \cdot \dot{r}_2 & -c_{01} \cdot r_1 & 0 & 0 \\ c_{01} \cdot r_1 & c_1 + c_{01} & c_{01} \cdot \dot{r}_2 & -c_{01} & 0 & 0 \\ c_{01} \cdot r_1 \cdot \dot{r}_2 & c_{01} \cdot \dot{r}_2 & c_{01} \cdot (r_2^2 + c_{02} \cdot \dot{r}_2^2 + c_{03} \cdot (\dot{r}_2^2) & -c_{01} \cdot \dot{r}_2 + c_{03} \cdot \dot{r}_2 - c_{02} \cdot r_2 & -c_{03} \cdot r_4 \cdot \dot{r}_2 & -c_{03} \cdot r_2 \\ c_{01} \cdot r_1 & c_{01} & c_{01} \cdot \dot{r}_2 + c_{03} \cdot \dot{r}_2 + c_{02} \cdot \dot{r}_2 & -c_{01} + c_2 - c_{02} + c_{03} & -c_{03} \cdot r_4 & -c_{03} \\ 0 & 0 & -c_{03} \cdot r_4 \cdot \dot{r}_2 & -c_{03} \cdot r_4 & c_{03} \cdot r_4^2 & c_{03} \cdot r_4 \\ 0 & 0 & -c_{03} \cdot \dot{r}_2 & -c_{03} & c_{03} \cdot r_4 & c_{03} \end{bmatrix} \begin{bmatrix} \delta_1 \\ y_1 \\ \delta_2 \\ y_2 \\ \delta_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = F_n(t) \cdot r_2 \quad (10)$$

DYNAMIC BEHAVIOUR OF ONE CONCRETE ONE - STAGE CYCLOIDAL SPEED REDUCER

The differential equation of the system motion was solved using MATLAB - SIMULINK for one concrete one - stage cycloidal speed reducer with following working parameters:

- Input power: $P_{in} = 0,25$ kW;
- Input rotations per minute: $n_{in} = 1390$ min⁻¹;
- Reduction ratio: $u = 11$.

The input system parameter is the excitation force $F_n(t)$, and the output parameters are: total displacement (x) in the point of contact between a cycloid disc tooth and a stationary central gear roller, the adequate velocity (\dot{x}) and the dynamic force F_e .

The values of the coefficients of damping as well as adequate stiffnesses are adopted from literature [1, 2, 3, 15]:

$k = 5500$ Ns/m, $k_1 = 2700$ Ns/m, $k_2 = 200$ Ns/m, $k_3 = 1800$ Ns/m, $k_4 = 1000$ Ns/m, $c_{01} = 1,4 \cdot 10^8$ N/mm, $c_{02} = 1,5 \cdot 10^7$ N/mm, $c_{03} = 1,6 \cdot 10^8$ N/mm, $c_1 = 1,9 \cdot 10^9$ N/mm, $c_2 = 1,7 \cdot 10^8$ N/mm, $c_3 = 2,1 \cdot 10^9$ N/mm, $c_4 = 1,6 \cdot 10^8$ N/mm.

The masses of elements, the rectangular moments of inertia and the adequate radii have the following values: $m_1 = 0,17$ kg, $m_2 = 0,74$ kg, $m_4 = 0,70$ kg, $J_1 = 26$ kgmm², $J_2 = 1422,3$ kgmm², $J_4 = 762,5$ kgmm², $r_1 = 17,5$ mm, $r_2 = 62$ mm, $r_2' = 20$ mm, $r_2'' = 39$ mm, $r_4 = 39$ mm.

Diagram of excitation force is presented in Figure 2, and the results of calculation for total displacement, velocity and dynamic force are presented in Figures 3,4 and 5.

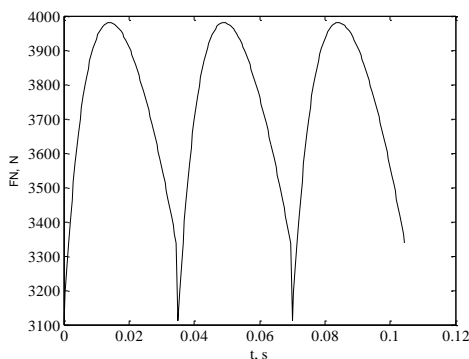


Figure 2 Diagram of excitation force $F_n(t)$

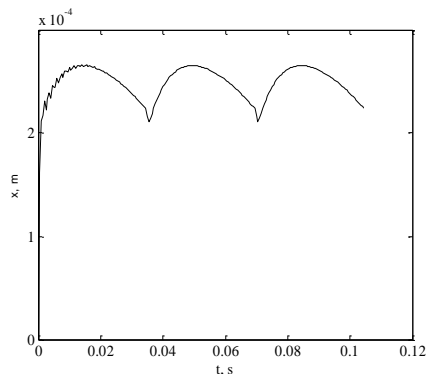


Figure 3 Diagram of the total displacement x

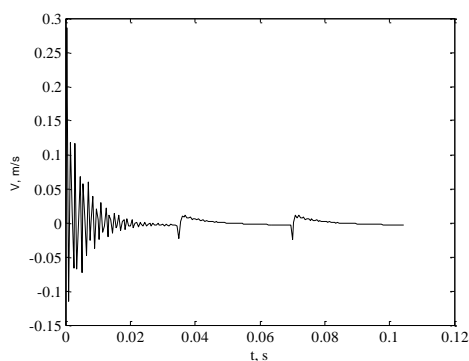


Figure 4 Diagram of the velocity \dot{x}

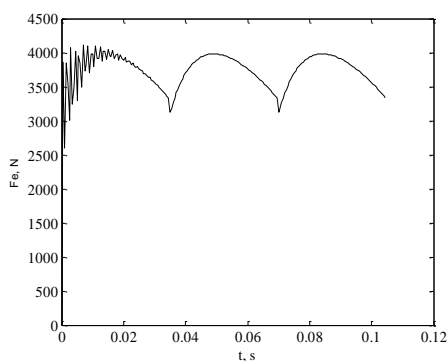


Figure 5 Diagram of the dynamic force F_e

The presented results refer to the most critical case - the case of single meshing (one cycloid disc tooth and one stationary central gear roller are in the contact).

CONCLUSIONS

When the values for the coefficients of damping and adequate stiffnesses are selected from a recommended range, vibrations with damping in time disappear very quickly and only forced vibrations remain.

Forasmuch as the excitation force is a periodical time function, the same goes for the other parameters (total displacement, velocity and the dynamic force).

The biggest influence on dynamic behaviour of one - stage cycloid speed reducer has the coefficient of the damping during the contact between the cycloid disc tooth and the stationary central gear roller.

REFERENCES

- [1] Batinic, V.: "Determination of Gear Mesh Stiffness in Planetary Gearing", Military Technical Courier, Vol.56, No.2, 2008, pp. 227-236.
- [2] Batinic, V.: "Dynamic Stability of Planetary Gear Drives", International Conference Research and Development of Mechanical Elements and Systems - IRMES, 2008, Beograd, Serbia, pp. 539-544.
- [3] Batinic, V.: "Internal Dynamic Forces at Planetary Gear Drives", Scientific Technical Review, Vol.LI, No.3, 2001, pp. 43-48.
- [4] Blagojevic, M., Marjanovic, N., Djordjevic, Z., Stojanovic, B., Disic, A.: "A New Design of a Two-Stage Cycloidal Speed Reducer", Journal of Mechanical Design, Vol.133, No.8, 2011, pp. 085001-1-085001-7.
- [5] Blagojevic, M., Kocic, M., Marjanovic, N., Stojanovic, B., Djordjevic, Z., Ivanovic, L., Marjanovic, V.: "Influence of the Friction on the Cycloidal Speed Reducer Efficiency", Journal of the Balkan Tribological Association, Vol.18, No.2, 2012, pp. 217-227.
- [6] Blagojevic, M., Marjanovic, N., Djordjevic, Z., Stojanovic, B., Marjanovic, V., Vujanac, R., Disic, A.: "Numerical and Experimental Analysis of the Cycloid Sisc Stress State", Technical Gazette, Vol.21, No.2, 2014, pp. 377-382.

- [7] Guo, Y., Parker, R.: "Sensitivity of General Compound Planetary Gear Natural Frequencies and Vibration Modes to Modal Parameters", *Journal of Vibration and Acoustics*, Vol.132, No.1, 2010, pp. 011006-1-011006-13.
- [8] Hsieh, C.F.: "The Effect on Dynamic of Using a New Transmission Design for Eccentric Speed Reducer", *Mechanism and Machine Theory*, Vol. 80, 2014, pp. 1-16.
- [9] Kahraman, A.: "Load Sharing Characteristics of Planetary Transmissions", *Mechanism and Machine Theory*, Vol.29, No.8, 1994, pp. 1151-1165.
- [10] Kahraman, A.: "Free Torsional Vibration Characteristics of Compound Planetary Gear Sets", *Mechanism and Machine Theory*, Vol.36, No.8, 2001, pp. 953-971.
- [11] Kahraman, A., Kharazi, A.A., Umrani, M.: "A Deformable Body Dynamic Analysis of Planetary Gears With Thin Rims", *Journal of Sound and Vibration*, Vol.262, No.3, 2003, pp. 752-768.
- [12] Kahraman, A., Ligata, H., Singh, A.: "Influence of Ring Gear Rim Thickness on Planetary Gear Set Behaviour", *Journal of Mechanical Design*, Vol.132, No.2, 2010, pp. 021002-1-021002-8.
- [13] Kosse, V.: "Using Hysteresis Loop and Torsional Shock Loading to Assess Damping and Efficiency of Cyclo-drive", 14th International Congress on Sound and Vibration, 2007., Cairns, Australia, pp. 1-8.
- [14] Lin, J., Parker, R.G.: "Sensitivity of Planetary Gear Natural Frequencies and Vibration Modes to Model Parameters", *Journal of Sound and Vibration*, Vol.228, No.1, 1999, pp. 109-128.
- [15] Lin, J., Parker, R.G.: "Planetary Gear Parametric Instability Caused by Mesh Stiffness Variation", *Journal of Sound and Vibration*, Vol.249, No.1, 2002, pp. 129-145.
- [16] Liu, J., Matsumura, S., Chen, B., Houjoh, H.: "Torsional Stiffness Calculation of Double-Enveloping Cycloid Drive", *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, Vol.6, No.1, 2012, pp. 2-14.
- [17] Malhotra, S.K., Parameswaran, M.A.: "Analysis of a Cycloid Speed Reducer", *Mechanism and Machine Theory*, Vol.18, No.6, 1983, pp. 491-499.
- [18] Thube, S.V., Bobak, T.R.: "Dynamic Analysis of a Cycloidal Gearbox Using Finite Element Method", *AGMA Technical Paper*, 2012, pp. 1-13.
- [19] Yang, J., Zhang, C.: "Elasto-dynamics of Internal Gear Planetary Transmissions", *Mechanism and Machine Theory*, Vol.40, No.10, 2005, pp. 1107-1125