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# ERRORS IN PROCESSING OF VEHICLE VIBRATION DATA

**ABSTRACT:** Vehicle vibration data are acquired during experimental measurements. Unfortunately, every measurement is associated with certain amount of measurement error. Since the validity of experimental results and their analysis directly depends on the amount of errors contained in the measurement process, error identification is a very important part of experimental research.

After a brief discussion of possible measurement errors, the main attention in the paper has been given to statistical errors in analysis of the measurement results. Theoretical expressions for statistical errors of data processing are taken from the related literature and presented in a form that corresponds to real conducted measurement. Experimental data of vehicle vibrations – vertical accelerations at the centres of all four wheels and at the connection point between the front left damper and the car body were acquired during the road investigations of the vehicle. Statistical errors made during data processing are calculated, graphically presented and discussed. Finally, conclusions on reducing the amount of errors during vehicle investigations are drawn.

**KEYWORDS:** vehicle, vibration, measurement, statistical errors

## INTRODUCTION

Vibrations are often found as direct or indirect objects of measurements conducted on the motor vehicle. Measurands are the vibration frequencies of individual vehicle elements due to excitations coming from the vehicle travelling over a rough road, the vehicle's engine operation, the influence of the random side wind, the unknown shape of the vehicle's trajectory, etc. Eigenfrequencies of the vehicle elements and different resonant effects are specially analysed.

Vehicle vibration data, necessary for further analyses and research, are acquired during measurements, with the use of corresponding measurement instrumentation. Each measurement is the process with more or less pronounced effects that cause measurement error - one of the major factors that can affect a research results. An error is, by its nature, an indeterminate quantity and its value can therefore only be estimated. Thus, it is necessary to examine its nature of origin carefully and its causes and to classify them and determine its influence on measurement result reliability. Therefore, error identification presents a very important step in experimental research. In fact, measurements that do not report the range of possible measurement errors contain only limited information.

The theory of measurement errors is explained, not only in general literature that deals with experimental methods [2, 3, 5, 7], instrumentation [9] or analysis of the results [1, 4], but also in literature related to motor vehicle investigation, e.g. [8]. This paper deals with statistical errors in processing and analysis of vehicle vibration measurement data. Experimental data on vehicle vibrations were acquired during complex investigations of interaction between the steering and the suspension system of the motor vehicle [6]. The analysis of the statistical errors is based on the theory presented in [1].

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# A BRIEF REVIEW OF MEASUREMENT ERRORS

There are two classification systems of measurement errors in use today [8]: the *ISO classification* and the *engineering classification*.

The ISO classification system recognises *Type A errors* (if there are data to calculate a sample standard deviation) and *Type B errors* (if there are not data to calculate a sample standard deviation, so the sample standard deviation is obtained, for example, from experience or manufacturer's specifications). The ISO classification does not provide information how to improve measurements. Thus, the engineering classification is the preferred approach.

The engineering classification recognizes that there are two kinds of errors that arise in measurements: 1) systematic errors (bias) and 2) random errors. *Systematic errors* are constant in sign and intensity, or they change following a certain law in each repeated measurement of the same quantity. They arise due to the improper selection of the measurement method, the use of inaccurate instruments or the neglecting of influence of environmental factors. Systematic measurement errors may be completely or partially eliminated with appropriate corrections. *Random errors* are variable in sign and intensity. They arise due to simultaneous effects of many causes that contribute to small variation of the results. Thus, practically, they arise from unknown reasons. They are more difficult to detect and they can not be completely or partially removed or corrected.

Measurement errors arise from many sources. Since the measurement necessitates the interaction between humans (experimenter or participant), experimental installation and (more or less) controlled environment, the main sources of measurement errors are humans, instrumentation and environment.

*Human errors* are individual and they depend on training, competence, liability and routine of the experimenter (or the participant). They mostly arise from incompetence or carelessness of the experimenter. Some examples of human errors in measurements are: selection of improper measurement procedure and installation, incorrect layout, misuse and miscalibration of the instruments and incorrect reading or recording of the results. These errors may be difficult to recognize. Even the most skilful experimenters who carefully plan experiments, collect, manage, and analyze experimental data may still make mistakes in interpretation of data.

*Instrumentation errors* often amount to a few percent, although most statistical analyses, and many of the researchers, assume that instruments are error free. In fact, instrumentation has its limitations or deficiencies. Some examples of instrumentation errors are: instrument overload, failure of the segment of the installation, instrument inaccuracy (due to spontaneous miscalibration), drift, hysteresis, saturation, nonlinearity and other variations of characteristics due to changes in environmental and experimental conditions.

Variable environmental conditions may cause *measurement errors due to environmental influences*. Variations of temperature, atmospheric pressure, air humidity, precipitation, natural wind and the influence of magnetic fields of the Earth and of other external sources, may indirectly produce measurement errors, for they may influence operation of the instrumentation and measurement results and some of environmental conditions may even affect the humans.

Considering the magnitude, measurement errors may be *large*, *medium, small*, or *insignificant*. Large errors (20 % or more) are usually easy to detect, but finding the source can be difficult. Medium errors (*5* to l0 %) are much more difficult to detect and can only be eliminated by understanding the limitations of the used method or equipment. Small errors (1% and less) are contained in many measurement methods. Fortunately, in many situations these small errors are not significant. High accuracy of measurements (with errors less than 0.l %), are not attainable and also not necessary in many aspects of engineering, including the investigation of motor vehicles.

# NORMALIZED STATISTICAL ERRORS IN DATA PROCESSING

Most experimental investigations of motor vehicles vibration imply measurement of several quantities called "inputs" and several quantities called "outputs" of the vehicle as a system or of some vehicle's subsystem. Thus, in time domain, the subject of investigation may be considered (modelled) as multiple input/output system with $x_i(t)$, $i=1,2,\ldots,m$ inputs and $y_j(t)$, $j=1,2,\ldots,n$ outputs. If spectral domain is considered, such multiple input/output systems may be presented as $n$ multiple input / single output systems (for each output), depicted in Figure 1. Quantities shown in Figure 1a are: $X_i$ – Fourier transform of input $x_i$, $i=1,2,\ldots,m$, $H_{iy}$ – ordinary frequency response function of a transfer channel $x_i$-$y$, $Y$ – Fourier transform of output $y(t)$ and $N$ – Fourier transform of measurement noise, $n(t)$, at the output $Y$, which includes all possible deviations from the ideal model (actually, quantity $N$ denotes the influence of all possible errors that may occur during data acquisition on output $Y$). Conditioned spectral analysis produces quantities presented in Figure 1b: $X_{i\cdot(i-1)}$, $i=2,3,\ldots,m$ - uncorrelated inputs and $L_{iy}$ – optimum frequency response function of a transfer channel $X_i$-$Y$, $i=1,2,\ldots m$, while $Y$ and $N$ mean the same as in Figure 1a.

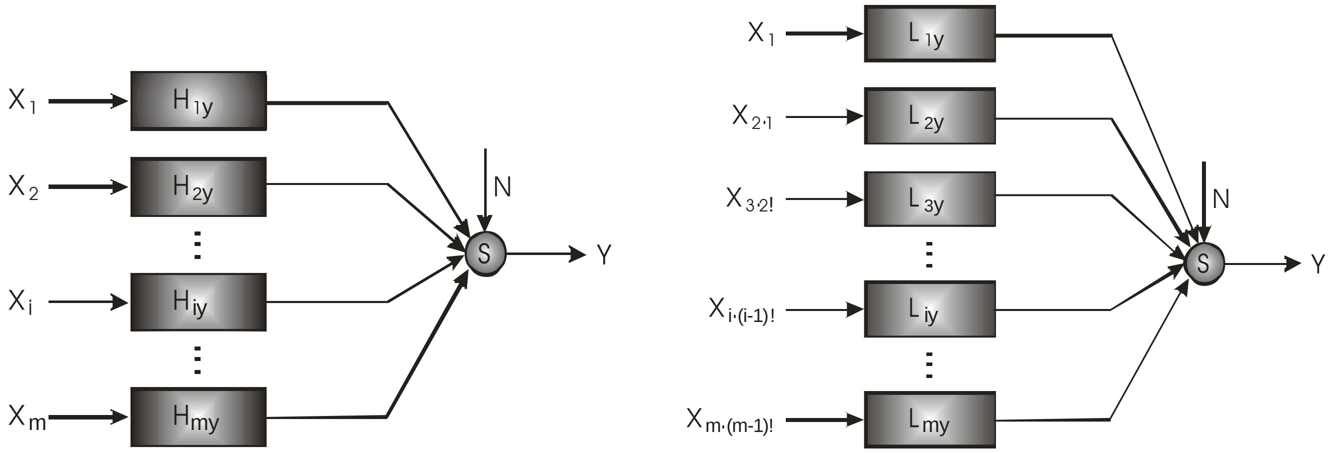


***Figure 1*** *Multiple input/single output model of experiment: a) correlated inputs, b) uncorrelated inputs*

During the analysis of random data, acquired during investigation of multiple input/output systems (like the motor vehicle or its subsystems), two types of errors occur: random errors and systematic (bias) errors.

***Random error*** is random dissipation of the results of the analysis of different samples of the same random quantities. It arises because averaging operations must be performed on a definite number of samples, N, or on one data record of definite length, T. This means that every analysis comprises random errors. The main sources of random errors in data analysis are [1]:

- "measurement noise" in sensors and other instrumentation ("input noise") and "computational noise" in digital calculations,
- other unmeasured inputs that contribute to the output and that are not correlated to measured input and
- non-linearities in the system between the inputs and the outputs.

Analytical expressions for normalised random errors are [1]:

- random error of auto-spectral density estimate, $S_{xx}$ , of a measured input, $x$:

$$\varepsilon_r\left[S_{xx}(f)\right]=\frac{1}{\sqrt{n_d}} \qquad (1)$$

where: $n_d$ - is a number of averages used in the processing,

- random error of cross-spectral density estimate, $S_{xy}$, of input, $x$, and output, $y$:

$$\varepsilon_r\left[\left|S_{xy}(f)\right|\right]=\frac{1}{\left|\gamma_{xy}(f)\right|\sqrt{n_d}} \qquad (2)$$

where: $\left|\gamma_{xy}(f)\right|$ - is a positive square root of ordinary coherence function, $\gamma^2_{xy}$ ,

- random error of ordinary coherence function estimate, $\gamma^2_{xy}$:

$$\varepsilon_r\left[\gamma^2_{xy}(f)\right]=\frac{\sqrt{2}\left[1-\gamma^2_{xy}(f)\right]}{\left|\gamma_{xy}(f)\right|\sqrt{n_d}} \qquad (3)$$

- random error of ordinary frequency response function intensity estimate, $H_{xy}(f)$:

$$\varepsilon_r\left[\left|H_{xy}(f)\right|\right]=\frac{\sqrt{1-\gamma^2_{xy}(f)}}{\left|\gamma_{xy}(f)\right|\sqrt{2n_d}} \qquad (4)$$

- random error of multiple coherence function estimate, $\gamma^2_{y:x}(f)$:

$$\varepsilon_r\left[\gamma^2_{y:x}(f)\right]=\frac{\sqrt{2}\left[1-\gamma^2_{y:x}(f)\right]}{\left|\gamma_{y:x}(f)\right|\sqrt{n_d-m}} \qquad (5)$$

where m – is the total number of measured inputs,

- random error of partial coherence function estimate, $\gamma^2_{iy\cdot(i-1)!}(f)$:

$$\varepsilon_r\left[\gamma^2_{iy\cdot(i-1)!}(f)\right] = \frac{\sqrt{2}\left[1-\gamma^2_{iy\cdot(i-1)!}(f)\right]}{\left|\gamma_{iy\cdot(i-1)!}(f)\right|\sqrt{n_d+1-i}} \tag{6}$$

where *i=1, 2,…, m* – is the ordinal number of input,

- random error of optimal frequency response function intensity estimate, $L_{iy}$ :

$$\varepsilon_r\left[\left|L_{iy}(f)\right|\right] = \frac{\sqrt{1-\gamma^2_{iy\cdot(i-1)!}(f)}}{\left|\gamma_{iy\cdot(i-1)!}(f)\right|\sqrt{2(n_d+1-i)}} \tag{7}$$

***Systematic (bias) error*** is an error that occurs with the same intensity and in the same direction from analysis to analysis. It is defined as the difference between the expected value of the observed quantity (average of the repeated estimates) and the real value of the estimated quantity. Generally, bias errors arise due to [1]:

- unknown “noise” at the input, not passing through the system,
- resolution of spectral density estimate,
- nonlinear system parameters and
- other unmeasured inputs which contribute to the output and which are not correlated to the measured input.

Normalised bias error of the auto-spectral density estimate of random input, *x*, is [1]:

$$\varepsilon_b\left[S_{xx}(f)\right] \approx \frac{B_e^2}{24}\frac{S''_{xx}(f)}{S_{xx}(f)} \tag{8}$$

where: $B_e$ – is frequency resolution of the auto-spectral density estimate and $S''_{xx}(f)$ - is the second derivative of the auto-spectral density estimate, $S_{xx}(f)$.

## STATISTICAL ERRORS IN THE ANALYSIS OF VEHICLE VIBRATION DATA

In order to analyse statistical errors that occur during processing of vehicle vibration data, measurement data from a large scale vehicle investigation of interaction between the steering and the suspension system of a passenger car [5] were used. Results from a test of a straight line drive with constant speed of 70 [kmh$^{-1}$] along the highway were observed.

Attention will be given to the following measured quantities:

- vertical acceleration at the centre of a front left wheel, $\ddot{z}_1(t)$, at the input, Figure 2,
- vertical acceleration at the connection point between the front left damper and a car body, $\ddot{z}_A(t)$ at the output, Figure 3.

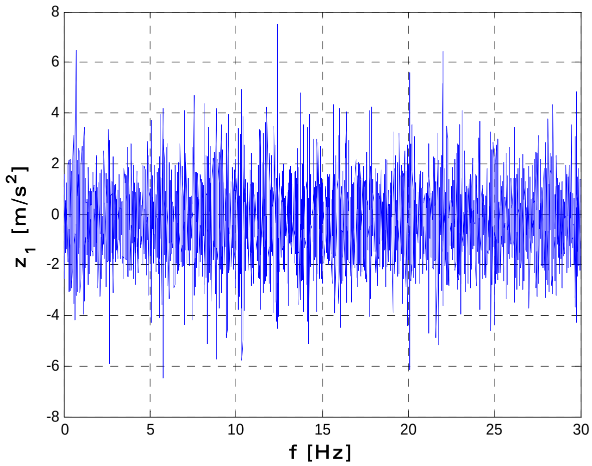

***Figure 2*** *Time series of vertical acceleration* $\ddot{z}_1(t)$

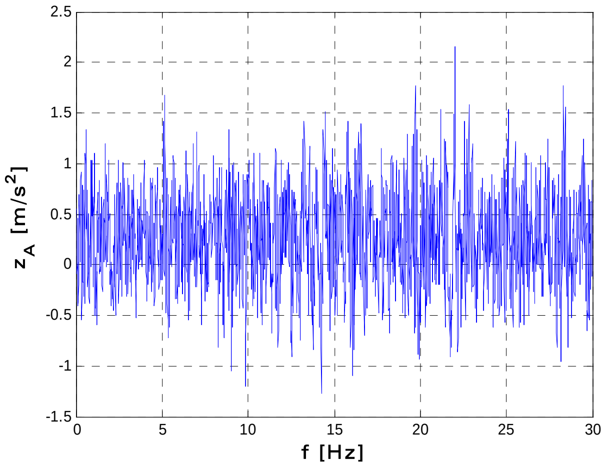

***Figure 3*** *Time series of vertical acceleration* $\ddot{z}_A(t)$

The results of the processing of these two quantities may be used in investigations of the front left wheel suspension system's behaviour under the influence of vertical excitation coming from the road roughness.

Measurement data were subjected to spectral analysis and estimates of all important spectral quantities were obtained. The main consideration here is to calculate the amount of errors made during data processing or to determine how reliable calculated estimates really are.

Data were sampled for a period of $T_{total} = 30\ [s]$ with the sample frequency $f_s = 100\ [Hz]]$ , giving the sum of 3000 samples in a record. Time resolution interval between the samples was $\Delta t = 0.01\ [s]$. Analog anti-aliasing filters with a cut-off frequency of 50 [Hz] were used. To execute the analysis in practice, a subdivision of data from the record into $n_d$ sub-records (for the same number of averages) was obtained. It should be noted here, that overlap averaging reduces the accuracy of acquired data and must be used with caution. The number of samples for Fast Fourier Transform was $n_{fft} = 2048\ [-]$. Thus, the frequency resolution for all observed estimates was $B_e = \dfrac{1}{2048 \cdot 0.01}\ [Hz] \approx 0.05\ [Hz]$.

By analysis of expression (1), it may be concluded that, in order to obtain small values of normalized random error of auto-spectral density estimate (less than 5%), the whole record must be divided into a minimum of $n_d = 400\ [-]$ sub-records. Three different numbers of averages were used: $n_d = 10\ [-]$ ($\varepsilon_r(S_{11}) \approx 0.32\ [-]$), $n_d = 100\ [-]$ ($\varepsilon_r(S_{11}) = 0.1\ [-]$) and $n_d = 400\ [-]$ ($\varepsilon_r(S_{11}) = 0.05\ [-]$), in order to investigate the influence of number if averages to the final results of analysis.

Figure 4 shows the results for estimates of: cross-spectral density, auto-spectral density, frequency response function and ordinary coherence function for the three different numbers of averages. In the names of the estimates, index 1 denotes the vertical acceleration at the centre of the front left wheel and index 2 denotes the vertical acceleration at the connection point between the front left damper and a car body.

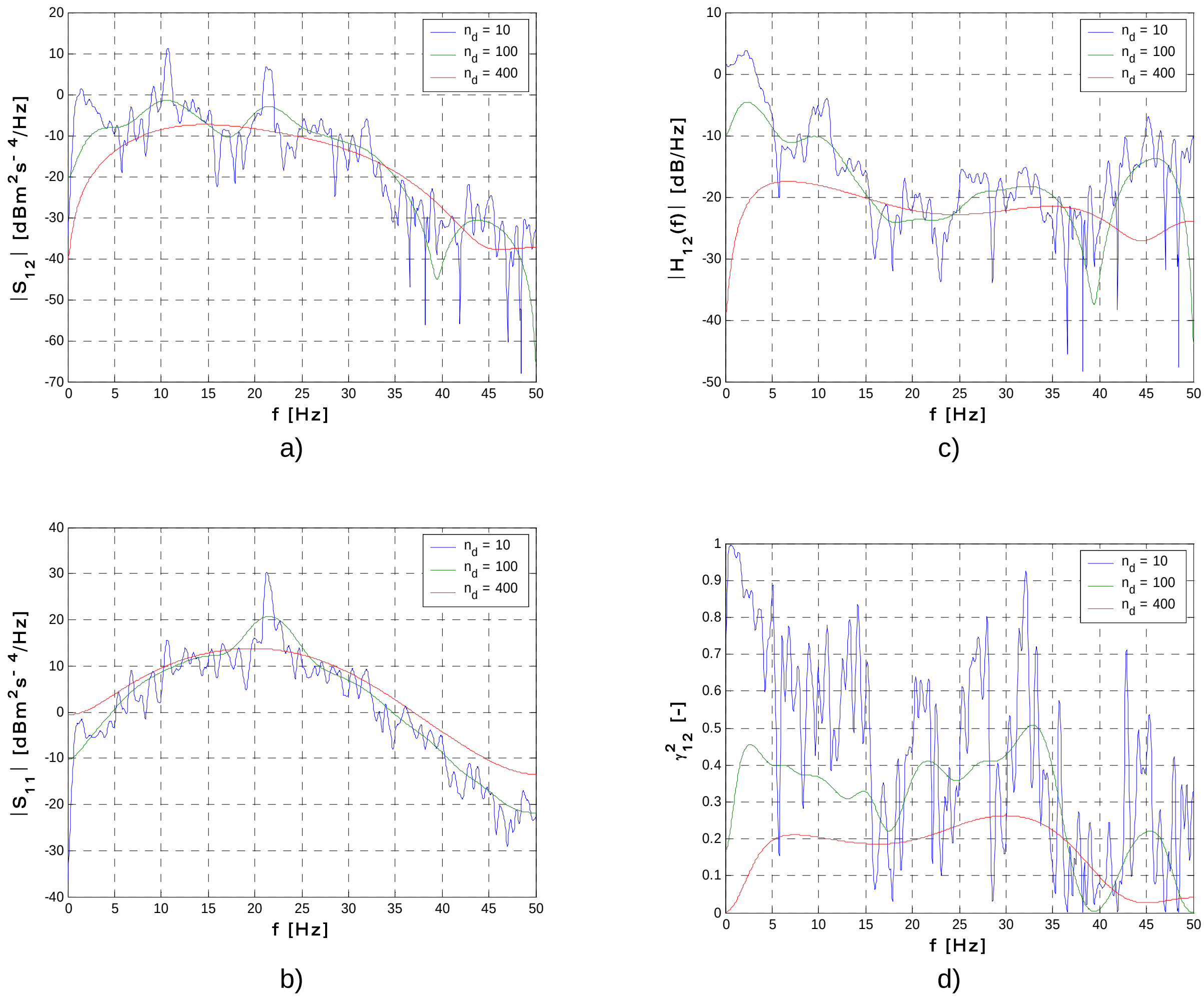


***Figure 4*** *Estimates of: a) cross-spectral density, b) auto-spectral density, c) frequency response function, d) ordinary coherence function*

The resulting random error of the frequency response function estimate $|H_{12}(f)|$ (gain factor – magnitude) directly depends on the values of the coherence function estimate, $\gamma_{12}^2(f)$, and the number of averages, $n_d$, used in calculations. In the frequency range $f = 0 \div 5$ $[Hz]$, where the frequency response function is not near its minimum value, while the auto-spectral density estimate is relatively small, input noise should be suspected. It may be the consequence of a smaller sensitivity of the used sensors in a low frequency range. Beside enhanced random error, this will also lead to bias error. The decrease of the frequency response function in the frequency range $f = 15 \div 20$ $[Hz]$ where the magnitude of frequency response function is relatively small usually implies output noise due to “measurement noise” and/or due to the contribution of other uncorrelated inputs.

The coherence function estimate usually has sharp peaks at frequencies at which the frequency function estimate also has peaks. This phenomenon is related to the fact that signal-to-noise ratio is the largest at these frequencies. From Figure 4c, it is obvious that frequency response function has peaks at approximately 2 [Hz] and 11 [Hz] which are clearly the system resonances. At other frequencies, coherence function does not have peaks or even has “notches” which points to bias errors due to resolution problems or to some non-linearity in the system (more rarely).

Figure 4 also clearly shows the influence of the averaging procedure on the results of analysis. Small number of averages ($n_d = 10$ $[-]$) gives results with considerable amount of variance, but there are clear peaks at resonance frequencies. Larger number of averages produces smoother estimates, but eventually, clear peaks are lost (e.g. for $n_d = 400$ $[-]$). This could lead to difficulties in data analysis and making conclusions about the measurements. Increase in number of averages has obviously lead to smaller overall values of the coherence function. The coherence function is extremely sensitive to relatively minor resolution errors, so the improved frequency resolution should be considered.

Figures 5 and 6 present the calculated values of normalized random error of the cross-spectral density estimate and ordinary coherence function, respectively, while Figure 7 shows the values of normalized bias error for auto-spectral density estimate.

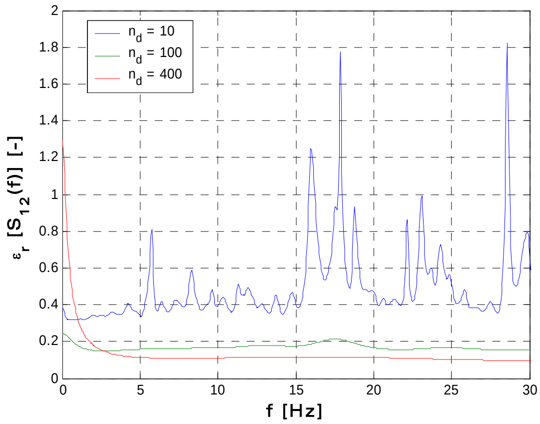


***Figure 5*** *Normalized random error of cross-spectral density estimate*

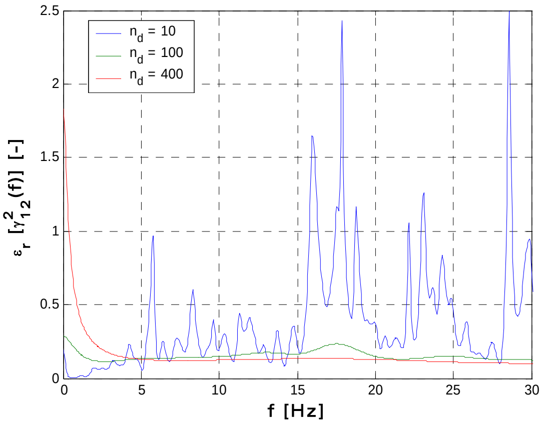


***Figure 6*** *Normalized random error of ordinary coherence function estimate*

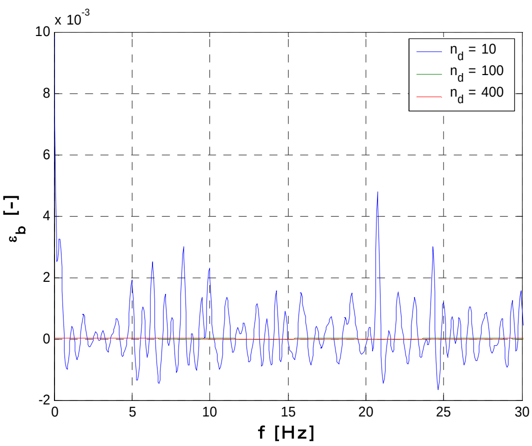


***Figure 7*** *Normalized bias error of auto-spectral density estimate*

Larger number of averages brings smaller random errors into data analysis. Values of random errors are considered “not too large” [1] if they are smaller than 0.2. They are obtained with higher number of averages. Extremely large values of normalized error are obtained for extremely small values of the coherence function estimates (e.g. in the frequency range $f = 15 \div 20\ [Hz]$), Figures 5 and 6.

For this measurement, bias errors are very small (portions of one percent) and for larger number of averages even negligible, Figure 7. A conclusion may be reached that this measurement had no significant systematic errors.

## CONCLUSIONS

Errors are always part of any measurement. They arise from many sources and may be random or systematic. Researchers should determine as early as possible what are likely to be the dominant sources of error in the measurement task and to devote sufficient time to find ways of reducing these errors. Random errors are more difficult to detect and they can not be completely or partially removed or corrected. Systematic (bias) measurement errors may be completely or partially eliminated with appropriate corrections.

Some of the ways to reduce measurement errors are:

- pilot testing of the instruments, to make an appropriate choice from the equipment available (or to design a more appropriate instrument),
- careful study of the acquired data, because all data entry for computer analysis should be verified,
- the use of statistical procedures (from rather simple formulas to very complex modelling procedures for modelling the error and its effects),
- the use multiple measures of the same quantities.

With the use of digital signal processing on the computers, it is often possible to improve the accuracy of a poor quality measurement through the use of *estimation techniques*. These methods range from simple averaging or low-pass filtering to cancel out random errors, to more sophisticated techniques such as Wiener or Kalman filtering and model-based estimation techniques. The increasing capability and lowering cost of computation makes it increasingly attractive to use lower performance sensors with more sophisticated estimation techniques in many applications.

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