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COMPARATIVE OVERVIEW OF CALCULATION OF NORMAL FORCE ON CYCLOIDAL GEAR TOOTH

ABSTRACT: The Fourth Industrial Revolution is bringing about major and important changes to the world we know today. One of the most significant results of this revolution is the development of "smart factories" in which the manufacturing process will be fully automated and will take place without the presence of people. Industrial robots will certainly form the backbone of these production systems. An industrial robot is a very complex mechatronic system that, in addition to some mechanical, electrical, electronic and other types of components, has a number of very precise high gear ratio speed reducers. In addition to the planetary and harmonic drive reducer, cycloid gears are most commonly used as drive systems for industrial robots.

Speed reducers in industrial robots are expected to be very precise, reliable, efficient, and dynamically stable. In order to do a comprehensive analysis of a speed reducer, it is primarily necessary to know the loads its vital elements are exposed to. For this reason, the subject of this paper is the procedure of calculating the normal force that occurs in the contact of the cycloid gear tooth and the stationary ring gear roller. In the load calculation procedure, apart from getting accurate results, it is also essential that the procedure be quick and simple. So far, several algorithms have been developed to calculate the normal force on cycloid gear, and this paper will present the results of two the most relevant procedures for calculating this force (the *Kudrijavcev*'s and the *Lehmann*'s method).

KEYWORDS: cycloidal speed reducer, cycloid gear, normal force, ring gear

INTRODUCTION

The Fourth Industrial Revolution brought about tectonic changes in the world's major industrial systems. The usage of robots in the world is growing rapidly every year. One of the most important mechanical components in robots is the high-precision reducer. Although industrial robots consist of a large number of mechanical and electrical components, high-precision reducers participate with 25% in the total price of the robot, [17]. In addition to the classic speed reducers, which still dominate the world industry, completely new solutions of mechanical power transmissions have been developed in recent decades. When it comes to high-precision reducers, planetary reducers, wave reducers and cycloidal speed reducers are most often used, [17]. All three types of gearboxes have their advantages and disadvantages. Due to their extremely good working characteristics (compact construction, high transmission ratio, dynamic stability, high efficiency, reliability in operation, steady and quiet operation...) cycloidal speed reducers are increasingly used in industrial robots. The disassembled single-stage cycloidal speed reducer is shown in Figure 1. It can be noted that in recent years the use of cycloidal speed reducers in electric cars has become increasingly important.

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In recent decades, there are very intensive researches conducted in the field of these power transmissions. Researchers are engaged in defining new concepts of cycloidal speed reducers [3,6,16], analysis of stress-strain state of its vital elements [4,7,10,19], dynamic analysis [20,21], determining the efficiency [2,13,15], by modifying the gear profile and the gear side clearance [8,12,14], etc. Numerous researches are dedicated to the load distribution in the gearbox, as in the case of ideal tooth meshing (when all tooth transfer the load) [9,11], as well as in the case of taking into account the gap on the load distribution [1,5,18]. In the framework of this paper, a comparative analysis of the calculation results of the normal force on cycloid gear in the case of ideal tooth meshing was conducted. The values of normal forces were calculated using two basic analytical methods most commonly used in practice (the *Kudrijavcev*'s method and the *Lehmann*'s method).

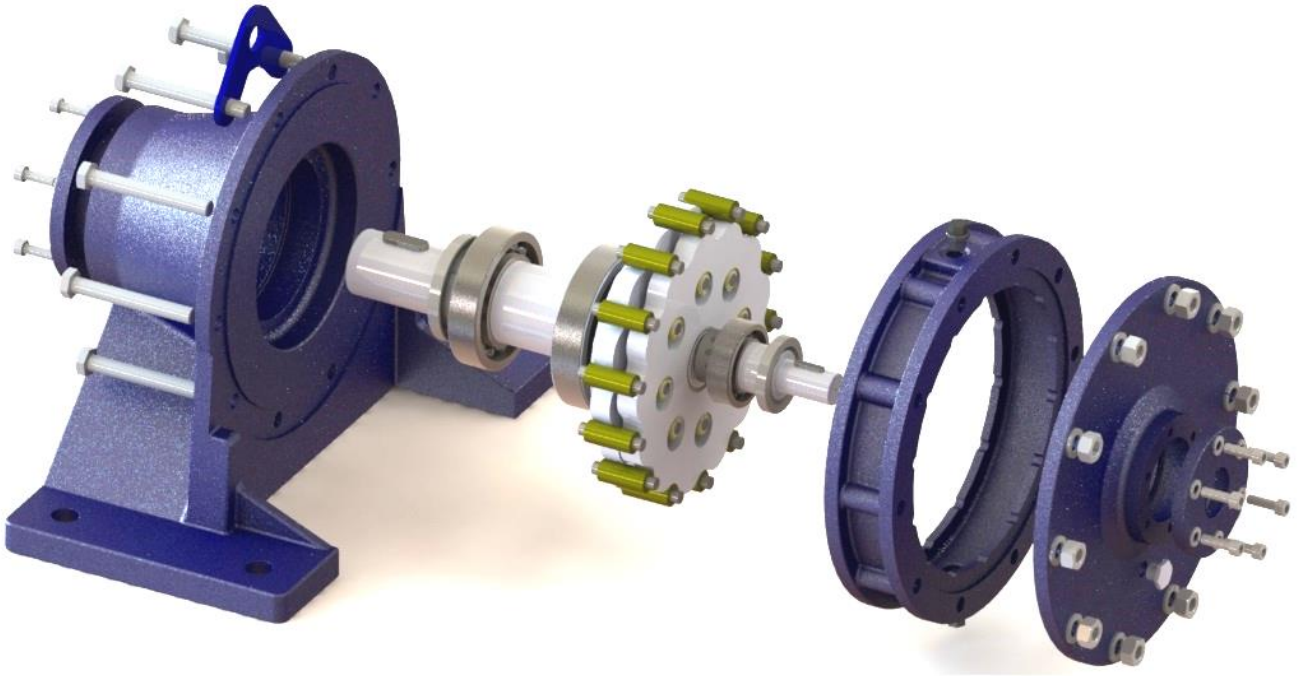

Figure 1 Disassembled single-stage cyclodial speed reducer

ANLYTICAL CALCULATION OF THE NORMAL FORCE

In order to perform the analysis of the dynamic behavior of the cycloid speed reducer, determine the efficiency, noise and vibration level as well as the distribution of stresses and strains, it is necessary to know the loads on its vital elements. The cycloidal speed reducer is a very complex system, both from the aspect of geometric and from the aspect of its kinematic and dynamic structure. The central element of the cycloid speed reducer is the cycloid gear. Figure 2 shows the forces acting on the cycloid gear, [9, 11]:

- F_E – *eccentricity force* (force whose vertical component F_{EV} make the input torque T_1 due to eccentric rotation),
- F_N – normal force in the meshing point between cycloid gear and ring gear (*normal force*),
- F_K – normal force in the meshing point between cycloid gear hole and output roller (*output force*).

Torques are:

- T_1 – cycloid gear input torque,
- T_2 – ring gear torque,
- T_3 – cycloid gear output torque.

It is obvious that this is a statically indeterminate system. The only known force is the vertical component of the eccentric force, F_{EV}. *Kudrijavcev* defined the first procedure for the forces calculation in cyclodial speed reducers in his book *Planetary Gear Train* in 1966, [9]. *Lehmann* later elaborated this procedure in his doctoral dissertation, [11]. Both *Kudrijavcev* and *Lehmann* analyzed the ideal meshing case (there is no gap between the teeth of the cyclodal gear and the rollers of the stationary ring gear and half of them transfer the load). In reality, of course, this is not the case because the existence of a gap is necessary for several reasons (possibility of assembling, provision of adequate lubrication...). In addition, *Kudrijavcev* and *Lehmann* introduced another important assumption, and that the cycloid gear is a rigid and non-deformable body, which does not correspond to real conditions.

Numerous researchers have in recent years been engaged in the analysis of load distribution in cyclodal speed reducers, taking into account the gaps that occur between the cyclodal gear and the corresponding elements, [2,8,12,18]. They also took into account the influence of the stiffness of the contact elements, [1]. In this way, a system of very complex equations is obtained, the solution of which implies the use of complex algorithms. In addition, it takes a lot of time. Given the fact that time is a very important factor in the product development process, it is justified to calculate the cycloid gear forces on the basis of the procedures defined by *Kudrijavcev* and *Lehmann*, [9,11,1]. In this paper, a comparative analysis of the results of normal force calculations using the *Kudrijavcev*'s method and the *Lehmann*'s method is made.

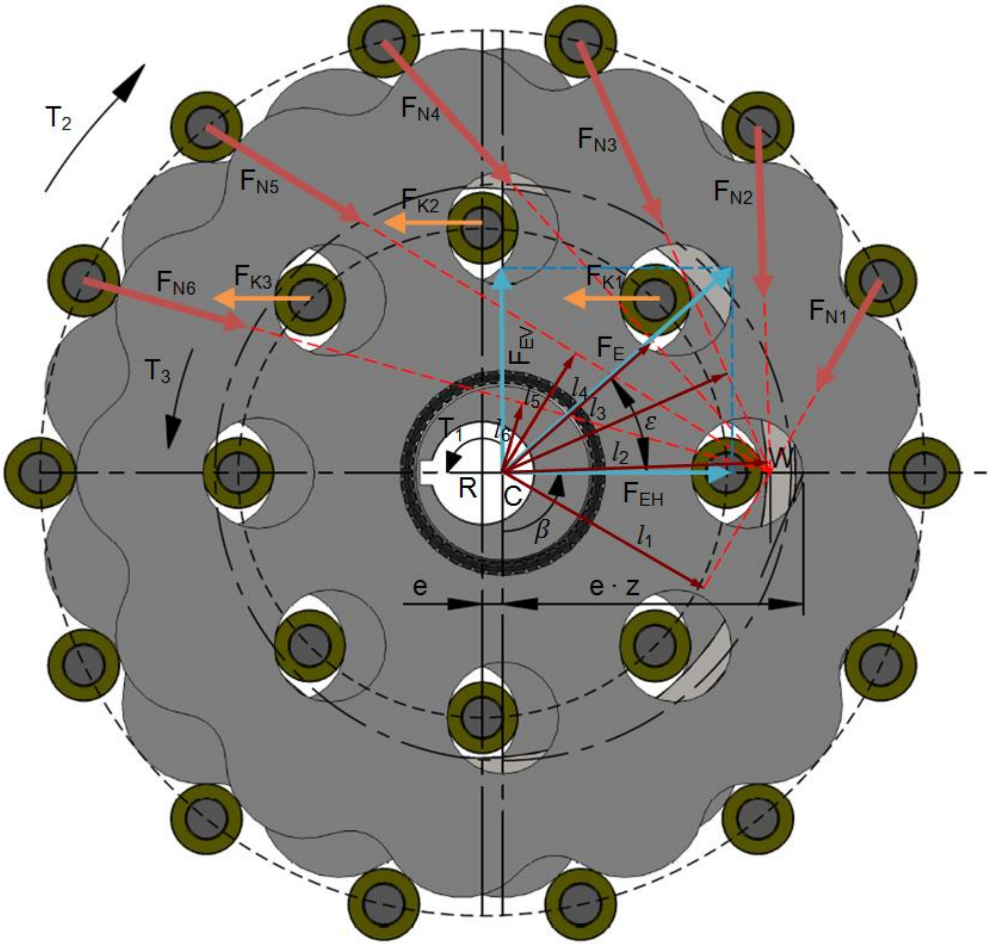

Figure 2 Cyloidal gear acting forces distribution

The *Kudrijavtcev*'s procedure for the normal force calculation that occurs in the contact of the cycloid gear tooth and the fixed ring gear roller is described in detail in the literature, [9]. Only basic expressions are given in this paper. The normal force on the *i*-th roller is calculated based on the expression:

$$F_{Ni} = F_{N\max} \cdot \frac{l_i}{r_1} \tag{1}$$

where are: $F_{N\max}$ – maximum value of the normal force, l_i – normal distance from cycloid gear center to pair normal force (Figure 2), r_1 – fixed circle radius.

Since the output torque is on the cycloid gear T_3 :

$$T_3 = F_{N\max} \cdot r_1 \cdot z_2 \cdot \frac{\sum_i l_i^2}{r_1^2 \cdot z_2} \tag{2}$$

the maximum value of the normal force after solving the equation (2) is calculated as:

$$F_{N\max} = \frac{4 \cdot T_3}{r_1 \cdot z_2} \tag{3}$$

Where z_2 – number of ring gear rollers.

Manfred Lehmann elaborated *Kudrijavcev*'s procedure in details in his doctoral dissertation, [11]. He defined a complete algorithm for normal force calculation that takes into account all geometric and kinematic parameters. Based on the *Lehmann*'s procedure, the values of the normal force as a function of the drive angle *β* are obtained. In order to determine the values of normal forces with this method, it is necessary to write an appropriate computer program based on the developed algorithm.

Normal force (*Lehmann*) can be calculated by equation:

$$F_{Ni} = (c \cdot \Delta\beta) \cdot r_i \cdot \sin\psi_i \tag{4}$$

where are: c – ring gear roller stiffness, $\Delta\beta$ – elemental angle movement of the cycloid gear, r_i – distance between contact point of *i* – th ring gear roller and cycloid gear center B_i [11], ψ_i – angle between direction of normal force and direction of line B_iC (Figure 3), [11].

Product $(c \cdot \Delta\beta)$ is calculated by following expression:

$$(c \cdot \Delta\beta) = \frac{F_{EV}}{\sum_i r_i \cdot \sin\psi_i \cdot \cos x_i} \tag{5}$$

x_i – angle between normal force *i* – th ring gear roller and vertical direction (Figure 3), F_{EV} – vertical component of the eccentricity force.

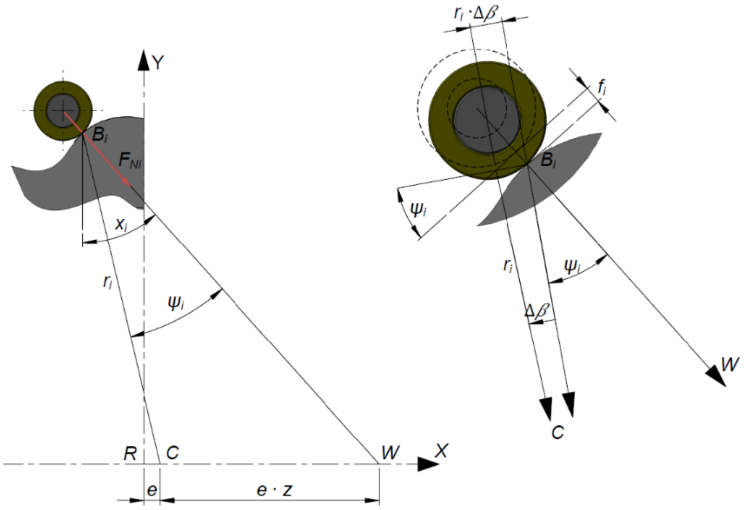

Figure 3 Determining of ring gear roller deflection

In Figure 4 is shown application segment for normal force calculation according to *Lehmann*. Application is written in *Fortran* programming language.

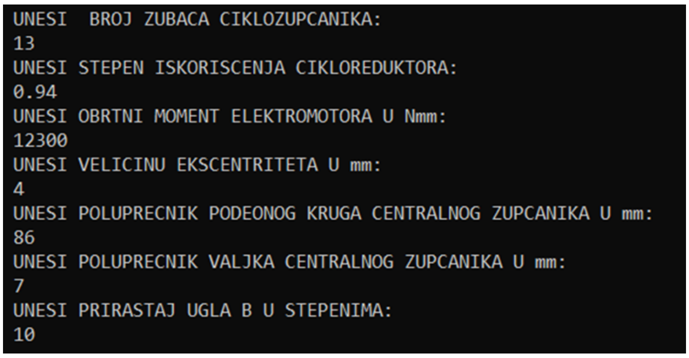

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Figure 4 Application segment for normal force calculation (*Lehmann*’s method)

NORMAL FORCE CALCULATION RESULTS

In order to compare the values of normal forces obtained using the methods of *Kudrijavcev* and *Lehmann*, a single-stage cycloidal reducer with the following operating parameters was chosen: input power (P = 4 kW), transmission ratio (u_{CR} = 13), number of fixed ring gear rollers (z_2 = 14), eccentricity size (e = 4 mm), radius of the ring gear pitch circle (r = 86 mm). The calculation was performed for the values of the drive angle β in the interval from 0° to 180°. The number of rollers in contact varies from 6 to 7. By rotation of the cycloid gear, the rollers of the ring gear which are in contact with the teeth of the cycloid gear and which transfer the load are also changing. The following tables show the most relevant calculation results, as well as the deviation percentage between these two methods.

Table 1 Normal force values (β = 0°)

Roller no., *i*	Normal force F_{Ni}, N		Deviation, %
	Kudrijavcev	*Lehmann*	
1	861.22	835.70	2.96
2	993.29	963.70	2.98
3	909.77	882.75	2.97
4	740.24	718.14	2.98
5	519.67	504.19	2.98
6	267.58	259.61	2.98

Table 2 Normal force values (β = 60°)

Roller no., *i*	Normal force F_{Ni}, N		Deviation, %
	Kudrijavcev	*Lehmann*	
3	691.31	662.18	4.21
4	986.79	945.10	4.22
5	948.95	909.99	4.11
6	803.50	769.62	4.22
7	595.93	572.28	3.97
8	354.16	339.14	4.24
9	90.02	86.16	4.29

Table 3 Normal force values (β = 130°)

Roller no., *i*	Normal force F_{Ni}, N		Deviation, %
	Kudrijavcev	*Lehmann*	
6	839.62	810.91	3.42
7	993.67	959.66	3.42
8	917.22	885.69	3.44
9	751.32	725.31	3.46
10	532.48	514.57	3.36
11	282.10	272.43	3.43
12	15.29	14.49	5.23

If it is carefully looked at the values of normal forces obtained using the methods of *Kudrijavcev* and *Lehmann* in tables 1, 2 and 3, it can be easily noticed that the deviations are minimal and its amounts are approximately from 3% to 5%. This further means that when it is necessary to perform a calculation of normal forces in the contact of the cycloid gear tooth and the fixed ring gear roller, it is possible to use either of these two described methods. *Kudrijavcev*'s method is definitely simpler and faster. *Lehmann*'s method requires writing an appropriate computer program with making numerous simulations, which is much more time taking. However, once a program is written, it can be used to calculate normal forces for different values of input parameters and in that case the *Lehmann*'s method is much simpler and faster.

CONCLUSIONS

Cycloidal speed reducers as gearboxes with high gear ratio and high precision have been increasingly used in modern mechanical engineering in recent decades. Their usage in the robot and electric car industry is especially significant. According to given reason, the number of researches in the field of cyclodial speed reducers has increased.

In order to make the correct choice of cycloidal speed reducer for a specific industrial purpose, it is necessary to know the loads on its vital elements. A large number of researches are dedicated to determining the loads in cycloidal speed reducers. Although they do not take into account the influence of stiffness and the gap between the elements in contact, the methods of *Kudrijavcev*'s - and *Lehmann*'s - as the oldest methods still have the greatest application today.

This paper presents the methods *Kudrijavcev*'s and *Lehmannn*'s for the calculation of the normal force in the contact of the cycloid gear tooth and the fixed ring gear roller. After that, for one specific single-stage cycloidal speed reducer, the values of normal forces for different values of the angle β as well as percentage deviations were calculated using these two methods.

Since the deviations of the results range from 3% to 5%, both methods can be used with great reliability. Compared to the *Kudrijavcev*'s method, which is mathematically much simpler, *Lehmann*'s method involves writing a mathematical program for calculating normal force, which means that the *Kudrijavcev*'s method is significantly faster. However, when a program exists, it can be used to calculate normal forces for different values of input parameters and in that case the *Lehmann*'s method has an advantage.

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