



## TEACHING-LEARNING-BASED OPTIMIZATION ALGORITHM FOR SOLVING MACHINE DESIGN CONSTRAINED OPTIMIZATION

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*Abstract: The contemporary Teaching-learning based optimization method (TLBO), is used for solving machine design optimization problems. The method consists of two phases, teaching phase and learning phase. This method requires a small number of known facts which describe the problem in order to be able to find a solution. This research is oriented on software implementation of TLBO methods and problems in real world application for solving optimization problems. An original software has been developed which uses TLBO method. Testing the method is conducted on machine design examples from literature, and the results are compared to optimal solutions from literature. TLBO is a modern and very significant method, and this paper shows its convergence characteristics and practical implementation for engineering problems.*

*Key words: Constrained optimization, Machine design optimization, Optimization software, Teaching-learning-based optimization*

### 1 INTRODUCTION

Machine design presents a creative process with clearly defined goals with the simultaneous fulfilment of certain constraints and needs for adequate decision making. In order for this process to be successful, an optimal design solution, preferable for real application, must be made. Optimization is implemented with a clear definition of the objective function, optimization variables, existing constraints, feasible solutions and optimization method. Optimization is finding an adequate, possible solution from a group of alternative possible solutions. Heuristic methods are preferred for use in engineering problems due to their favorable characteristics, such as their ability to work with a large number of variables, overcoming local extremes, speed and efficiency of work, field of use, prerequisite knowledge of the problem being solved, etc.

There is a large number of heuristic methods such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), Ant Colony

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Optimization (ACO), Teaching-Learning-Based Optimization (TLBO), etc. In common for all these methods is the principle of mimicking natural occurrences and processes. TLBO is a modern heuristic method initially developed for solving engineering problems. The algorithm was developed in 2011 [1] and consists of two basic phases, Teacher Phase and Learner Phase. The algorithm has been tested on numerous problems [2,3], which are very complex for optimizing, thereby verifying this method.

Rao and Patel [4] presented the TLBO algorithm which is used on complex optimization problems. The same authors improved the base TLBO algorithm [3], and used it to solve unconstrained problems. Certain authors [5,6] directed their research to determining performances of some developments of the TLBO algorithm. Huang et al. [7] analyzed the effects of modifying TLBO in engineering practice. Regardless of the fact that the algorithm is still young, it is a frequent theme of research and is increasing in implementation in practice. An overview of research with details of papers and analyses of what has been achieved, was presented by Rao [8], where he considered over 200 distinguished research papers from this field, concluding with the year 2015.

The motivation behind this research is in understanding, implementation, and adaptation of an efficient heuristic algorithm for solving machine design optimization problems. The contemporaneity, modernity and the fact that TLBO was developed by experts in the engineering optimization field present the fundamental reason and criteria behind the choice of this heuristic method.

## 2 TEACHING-LEARNING-BASED OPTIMIZATION

The Teaching-Learning-Based Optimization (TLBO) algorithm presents a new evolutionary algorithm (heuristic optimization method) which gathered a lot of attention in the research community. It is a modern heuristic method, and the first publications regarding TLBO appear starting in the year 2011 [1]. The operating principle of this method is developed on the influence of a teacher on learners. TLBO algorithm is divided into two characteristic phases: Teacher Phase, and Learner Phase, respectively. The average quality of the class increases depending on the teacher's quality. TLBO defines a good teacher as one who improves his students' knowledge up to his level. The TLBO algorithm structure is shown in figure 1.

The first phase is the Teacher phase. In this phase the teacher attempts to bring his students up to the same knowledge level. The ability to reach this level depends on the course students learning capabilities. This modification of old individuals, creating new individuals can be expressed as follows:

$$X_{New,i} = X_{Old,i} + r_i (X_{Teacher} - T_F X_{Mean}) \quad (1)$$

where  $T_F$  is the randomly determined learning factor, which can have a value of either 1 or 2. The best individual in the population is  $X_{Teacher}$ ,  $X_{Mean}$  is the current mean value of the course students,  $r$  is a uniform random number between 0 and 1.

The second phase is the Learner phase. In the learner phase, through interaction amongst themselves, learners increase their knowledge. Learning partners are chosen randomly ( $X_i$  learns from  $X_{ii}$ ). In the case that  $X_{ii}$  is better than  $X_i$ , that individual is modified through equation (2).

$$X_{new} = X_{old} + r \cdot (X_i - X_j) \quad (2)$$

If this is not the case however, the individual is modified as shown in equation (3).

$$X_{new} = X_{old} + r \cdot (X_j - X_i) \quad (3)$$

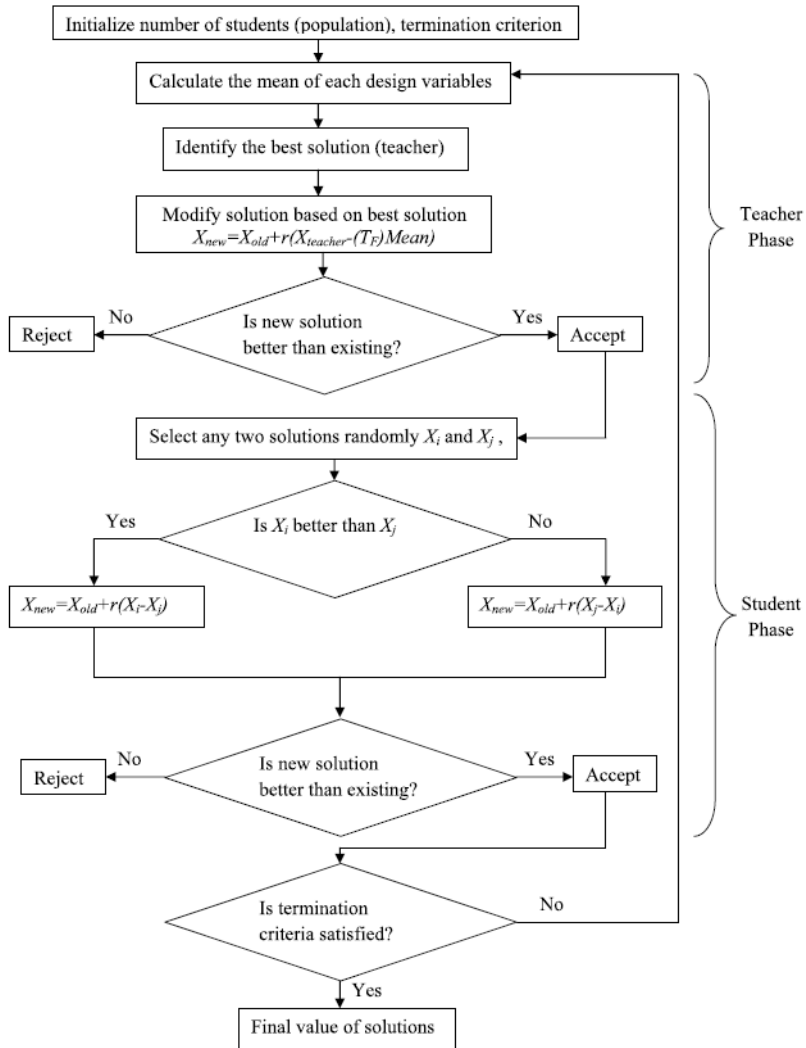


Figure 1. Structure of Teaching-learning-based optimization (TLBO) [1]

### 3 IMPLEMENTATION FOR TLBO OPTIMIZATION

When it comes to the process of implementation, and thereafter application of any optimization algorithm, the problem arises to achieve theoretical assumptions. There are numerous problems with implementation of methods. For TLBO some of these problems include synchronizing nomenclature, explicit definition of random values, should the random numbers differ for the teaching and learning phase, etc.

This research is oriented towards the real application of TLBO method for engineering problems, and in order to find the most influential segment the implementation of the algorithm during optimization software development was developed. For the purposes of this research an original software written in C++ was developed to use TLBO optimization. According to the instructions of the authors of this algorithm, the following steps were taken:

- Defining the optimization problem and initializing optimization parameters – population size, number of iterations, number of variables and their range, goal function, constraints, and optimization criteria.
- Generating the initial population – generating random values in the acceptable domain which will be the initial population.
- Teacher Phase -  $T_F$  is the same for the entire iteration, while  $r$  in this phase is different for each student individually in each iteration.
- Learner Phase – this phase is repeated the same number of times as there are students.
- The algorithm is stopped when the maximal number of iterations and post processes is achieved.

## 4 CONSTRAINED MACHINE DESIGN PROBLEMS FROM LITERATURE

### 4.1 Design of gear train

The objective for gear train design problem is minimize the weight of a gear train [1]. This problem has one discrete variable, and six continual variables, which makes it very complex. In addition the problem is constrained with four linear and seven nonlinear constraints. The problem can be presented by the following mathematical model.

Minimize:

$$f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \quad (4)$$

Subjec to:

$$g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0, \quad g_2(x) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0, \quad g_3(x) = \frac{11.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0, \quad g_4(x) = \frac{11.93x_5^3}{x_2x_3x_7^4} - 1 \leq 0,$$

$$g_5(x) = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9e6}}{110x_6^3} - 1 \leq 0, \quad g_6(x) = \frac{\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5e6}}{85x_7^3} - 1 \leq 0, \quad g_7(x) = \frac{x_2x_3}{40} - 1 \leq 0,$$

$$g_8(x) = \frac{5x_2}{x_1} - 1 \leq 0, \quad g_9(x) = \frac{x_1}{12x_2} - 1 \leq 0, \quad g_{10}(x) = \frac{1.5x_2 + 1.9}{x_4} - 1 \leq 0, \quad g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0,$$

$$2.6 \leq x_1 \leq 3.6, \quad 0.7 \leq x_2 \leq 0.8, \quad 17 \leq x_3 \leq 28, \quad 7.3 \leq x_4 \leq 8.3, \quad 7.8 \leq x_5 \leq 8.3,$$

$$2.9 \leq x_6 \leq 3.9, \quad 5 \leq x_7 \leq 5.5.$$

## 4.2 Design of welded beam

Problem of design of welded beam is frequently analyzed in reference [9]. Welded beam is designed for minimum cost subject to constraints on shear stress  $\tau$ , bending stress in the beam  $\sigma$ , buckling load on the bar  $P_c$ , and deflection of the beam  $\delta$ , and side constraint. The problem is shown in figure 2, with the variables which are used in optimization.

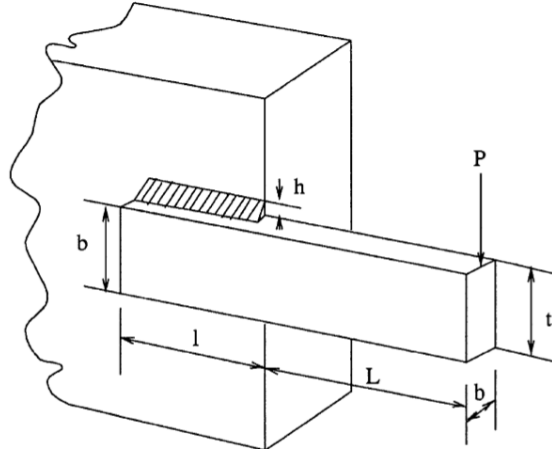


Figure 2. Welded beam design problem

The problem consists of four variables and seven complex constraints, which are presented in the following mathematical model:

Minimize:

$$f(x) = 1.10471x_1^2 \cdot x_2 + 0.04811x_3 \cdot x_4(14 + x_2) \quad (5)$$

Subject to:

$$\begin{aligned} g_1(x) &= \tau(x) - 13000 \leq 0, & g_2(x) &= \sigma(x) - 30000 \leq 0, & g_3(x) &= x_1 - x_4 \leq 0, \\ g_4(x) &= 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \leq 0, & g_5(x) &= 0.125 - x_1 \leq 0, \\ g_6(x) &= \delta(x) - 0.25 \leq 0, & g_7(x) &= 6000 - P_c(x) \leq 0, \end{aligned}$$

Where:

$$\begin{aligned} \tau(x) &= \sqrt{(\tau')^2 + 2\tau'\tau'' \frac{x_2}{2R} + (\tau'')^2}, & \tau' &= \frac{6000}{\sqrt{2}x_1x_2}, & \tau'' &= \frac{MR}{J} \\ M &= 6000 \left( 16 + \frac{x_2}{2} \right), & R &= \sqrt{\frac{x_2^2}{4} + \left( \frac{x_1 + x_3}{2} \right)^2}, & J &= 2 \left\{ \sqrt{2}x_1x_2 \left[ \frac{x_2^2}{12} + \left( \frac{x_1 + x_3}{2} \right)^2 \right] \right\}, \\ \sigma(x) &= \frac{504000}{x_4x_3^2}, & \delta(x) &= \frac{2.1952}{x_3^3x_4}, & P_c(x) &= 64746.022(1 - 0.0282346x_3)x_3x_4^3, \\ & & & & 0.1 \leq x_1 \leq 2, & 0.1 \leq x_2 \leq 10, & 0.1 \leq x_3 \leq 10, & 0.1 \leq x_4 \leq 2. \end{aligned}$$

### 4.3 Design of tension/compression spring

One of the older engineering optimization problems [9], is the weight minimization of a tension/compression spring (Figure 3), subject to constraints on minimum deflection, shear stress, surge frequency, limits on outside diameter and on design variables. The design variables are the mean coil diameter  $D(x_2)$ , the wire diameter  $d(x_1)$  and the number of active coils  $P(x_3)$ .

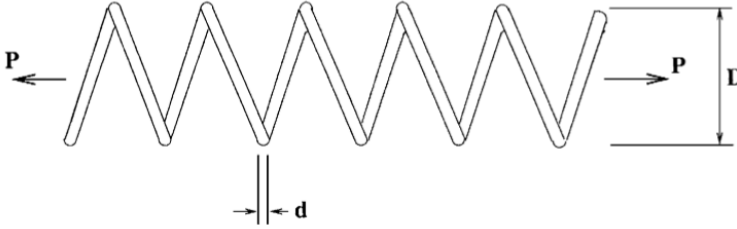


Figure 3. Tension/compression spring

The problem consists of three variables and four complex constraints, which are shown in the following mathematical model:

Minimize:

$$f(x) = (x_3 + 2)x_2 \cdot x_1^2 \quad (6)$$

Subject to:

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0, \quad g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \leq 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0, \quad g_4(x) = \frac{x_1 + x_2}{1.5} \leq 0, \quad x_1 = d, \quad x_2 = D, \quad x_3 = P,$$

$$0.05 \leq x_1 \leq 2, \quad 0.25 \leq x_2 \leq 1.3, \quad 2 \leq x_3 \leq 15.$$

### 4.4 Results and discussion

Table 1 shows achieved results from literature [1, 9] and experimental results of this research for implementing TLBO optimization method. Examples were solved with 30 repetitions in order to have the results validated, the evolutionary size was taken as 10 000 for the first case (TLBO 10) and for the second case 30 000 (TLBO 30), which is the most frequent case in most literature. It is noticeable that is a modern evolutionary method, as its use achieves extraordinary results in comparison to previous methods.

Results achieved by Rao [1], completely match the here achieved results, and they are off by very little. This can be the result of decimal tolerances, as well as the implementation not being identical, regardless of the strict following of the authors instructions.

Table 1. Result comparison

Problem		CPSO [9]	TLBO [1]	TLBO 10	TLBO 30
Gear Train	Best	NA	2996.348	2996.363	2996.348
	Mean	NA	2996.348	2996.414	2996.348
	Evaluations	NA	10 000	10 000	30 000
Welded Beam	Best	1.728	1.72485	1.72498	1.72485
	Mean	1.749	1.72845	1.72525	1.72485
	Evaluations	200 000	10 000	10 000	30 000
Tension / Compression Spring	Best	0.01267	0.01266	0.01267	0.01266
	Mean	0.0127	0.01267	0.01269	0.01267
	Evaluations	200 000	10 000	10 000	30 000

## 5 CONCLUSION

TLBO is a new approach in solving engineering optimization problems. The greatest advantage of this method is that it is a parameterless method. Implementation and practical use of almost all methods represents a serious and demanding problem. Theoretical assumptions do not always match real world application. Implemented TLBO was used on very demanding engineering optimization problems, in order to verify its quality in implementation. Due to numerous dilemmas and small discrepancies in presenting results, there are certain differences, however they are very small. It is necessary to state that the development of these optimization methods represents a perspective of solving complex optimization methods and their ever growing use in practical application, which can finally lead to large savings and improvement of these characteristics and improvement of existing ones.

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