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# INFLUENCE OF LEAKAGE CLEARANCE ON PRESSURE VARIATION IN GEROTOR PUMP CHAMBERS

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***Abstract:*** *Gerotors are used as pumps, motors, compressors, etc. and used in a wide range of industrial areas. Gerotors are, as pumps suitable for automotive industry as elements of lubricating systems, steering mechanisms and engines. The main component of gerotor pumps is a pair of toothed rotors. Usually, the outer rotor has a circular profile, while inner one has related coupled trochoid profile. In case of the theoretical ideal profile, contacts are simultaneous at all teeth. However, for a real profile, technological clearance and gaps are unavoidable. The existence of these gaps and clearances causes volumetric losses and consequently, variation of pressure in gerotor pump chambers. For the full understanding of real processes of fluid flow in gerotor pumps, the analytical model of fluid leaking through clearance between the profiles of teeth is presented in this paper. Due to the complexity of the fluid flow process, only volumetric losses caused by gaps between profiles of teeth and losses caused by an act of viscosity forces along the gap are analyzed. The effects of variation of geometrical parameters of profiles on the value of volumetric losses and variation of pressure in chambers of gerotor pumps are analyzed in this paper. Besides, the mutual comparison of pressure variation due to current flow obtained analytically in the pump with a theoretical ideal profile of teeth and with a real profile with gaps and clearances is done. Results and conclusions presented in this paper by highlighting real processes during exploitation bring very important perspectives for gerotor pump design that provides an effective constructional solution.*

***Keywords:*** *gerotor pump, trochoid, leakage clearance, pressure variation*

## 1. INTRODUCTION

The object of research presented in this paper is the gerotor pump used for the lubrication of the internal combustion motor. The basic components of gerotor pump are two toothed rotors, one outer usually with a circular profile and one inner with the corresponding coupled trochoid profile. In case of trochoid gear pairs with theoretical (ideal) profiles, the simultaneous contact of all teeth is achieved. In real constructions, simultaneous meshing of all the teeth at all times is not possible for the following reasons: real profiles are produced with technological tolerances, there are production and assembly errors, and due to the wear of profiles, there is a gap between the profiles. Despite the inevitable existence of gaps between the teeth, they can lead to fluid loss, reduce stability and increase noise and vibrations, especially at high speeds. Due to the presented reasons, the modeling of the mashing of the real profiles will be considered in this paper. In addition, it is assumed that all deviations from the theoretical measures are reflected in the equidistant modification of inner gear's trochoid profile.

In accordance with the subject and goals of this paper, the influence of the geometrical and kinematical parameters of mashed gear profiles on the volumetric losses and the pressure variation in the pump chambers will be done primarily.

Basic research in the area of trochoid profiles was performed by *Ansdal* [1]. On these bases, modern research related to the gerotor pump is based. *Manco* et al. [2-4] investigated the implementation of gerotor pumps in internal combustion engines as an element of its lubrication system. The same group of authors also worked on improving the profiles of gerotor pump teeth as well as on the automatic generation of the gear profile. *Maiti* and *Sinha* [5] analyzed the kinematic of modified epitrochoid profiles. *Shung* and *Pennock* [6] are evaluated in detail and extended this research to more contact aspects than only kinematics. *Raush* et al. [7] experimentally tested of damages and failures of gerotor pumps during exploitation. *Buono* [8] et al. performed a simulation of cavitation phenomena during exploitation of gerotors. *Ivanović* with a group of authors [9-12]

investigate processes that occur on the georotor teeth due to pressure during its exploitation. The same group of authors worked on the gerotor pumps models with technological gaps as well as with theoretical determination of pump flow.

In this paper, the model of the kinematic pair with fixed axis of gear shafts is considered. The input drive shaft is connected to the inner gear. For determination of current surface of the cross-section of the pump chamber, the method presented in reference [9] is used, while the minimal dimension of the gap is determined by the method presented in reference [11]. The determination of pressure variation in pump chambers is done by the method presented in reference [10] with certain modifications. The scientific significance of this paper is reflected in the setup and testing of the mathematical model of pressure variation in pump chambers under conditions very similar to the real ones. This precise simulation of exploitative conditions was done by introducing a technological gap into the mathematical model.

## 2. FLOW LOSSES AND PRESSURE VARIATION IN GEROTOR PUMP CHAMBERS WITH CLEARANCE

In the case of the considered gear pair of the trochoid pump, the theoretical profile of inner gear is in the form of an equidistant peritrochoid, while the outer gear is in form of a circular arc with radius, $r_c$. The profile of inner gear, which represents a realistic profile, will be generated in the form of an equidistant of the basis trochoid with an radius of equidistant, which is in relation to the theoretical one for assumed value of technological gap, $\varepsilon$. Due to the specific geometry of gear profiles and the exceptional complexity of the georotor pump operational process, in this paper, only losses due to the gap between gear profiles and losses due to the viscosity of working fluid (as a consequence of adhesion forces, fluid particles are paste to the side of gear tooth) are considered [2]. The volume loss analysis will be carried out for the pump model shown in Fig. 1. With the pump shown, both outer and inner gears rotate around theirs axis in the same direction, counterclockwise. When considering the pressure variation in pump chamber $K_i$, it is assumed that the flow $Q_{(in)I}$ that comes from the adjacent chamber is positive (inflows into the chamber $K_i$), while the flow $Q_{(out)I}$ is negative, since it is pushed from the chamber $K_i$ to the adjacent chamber.
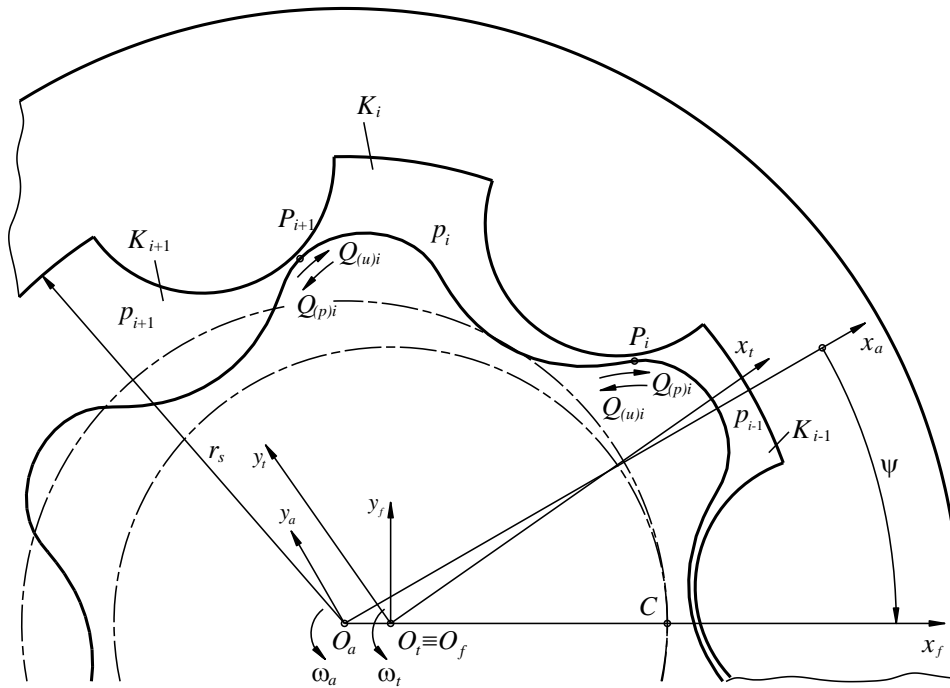


Fig.1: Model of pump with technological gap

The flowing of the fluid through the gap between the profiles of the teeth is caused by the difference of pressures between the two adjacent pump chambers. To calculate these losses, a hydraulic fluid flow model through rectangular notch with variable dimensions is adopted. Basic geometrical relations for determining the height of the gap between gear profiles are presented in Fig. 2.

Coordinate system $Oxy$ is introduced, whose coordinate start point $O$ is set at the point $P_{ai}$, the axis $x$ has the direction of tangent, the axis $y$ is has the direction of common normal of the profile, and are oriented as shown in Fig. 2. For calculating the height of the gap, the profile of inner gear is approximated by a circular arc of the radius $\rho_{ci}$ which is equal to the radius of curvature of the trochoid profile at the point $P_{ti}$. On the basis of this, an

equation can be written for calculating the height of the gap depending on the $x$ coordinate in following form [2]:

$$h_i(x) = (h_{\min})_i + r_c + \rho_{ci} - \sqrt{r_c^2 - x^2} \mp \sqrt{\rho_{ci}^2 - x^2} , \quad (1)$$

Where the radius of curvature $\rho_{ci}$ of equidistant trochoid profile is defined by the following equitation

$$\rho_{ci} = \frac{ez\left[1 + \lambda^2 - 2\lambda\cos(\tau_i - \psi_i)\right]^{3/2}}{z + \lambda^2 - \lambda(z+1)\cos(\tau_i - \psi_i)} - r_c^* , \quad (2)$$

Where $\psi_i$ is the angle which defines the current position of the point $P_{ti}$ [11]. In equitation (1) the sign "−" refers to the convex, and the sign "+" is refers to the concave zone of the profile. All other parameters that are used in this equitation are described in detail in reference [9, 11].

The basic relation for calculation of volume losses is principal equation for hydrodynamic lubrication, which can be derivate from the basic principles of fluid mechanic, or from Navier-Stocks equations [13]. The following assumptions have been made:

- the pressure is constant at the cross-section,
- the curvature of the surfaces are large in relation to the thickness of the lubrication layer,
- the lubricant is the Newtonian fluid,
- the fluid flow is laminar,
- the viscosity of the fluid is constant.



*Fig.2: Basic geometrical relations for determining the height of the gap*

Starting from the presented assumptions for calculation of total fluid flow through the gap with height $h(x)$, the following equitation can be used [13]:

$$Q = \frac{v_r b h}{2} - \frac{b h^3}{12\eta}\frac{dp}{dx} , \quad (3)$$

where

$\eta$ - Coefficient of fluid dynamic viscosity,

$v_r$ - Speed intensity of the relative movement of inner gear profile relative to the outer gear [14].

The first member of the right side of expression (3) reflects the effect of viscous forces on the flow, while the second member indicates the effect of the pressure force. Starting from the fact that flow has a constant value along the gap, equitation for calculation of the flow can be written for the minimal cross-section of the gap with height $h_{min}$. Accordingly, a variation of the flow in the pump chamber due to the action of the viscous forces along the gap can be determined by following relation:

$$Q_{i(v)} = \frac{(v_r)_i \, b \, (h_{\min})_i}{2} . \quad (4)$$

The fluid flow through the gap due to the action of the pressure forces is determined by the following expression:

$$Q_{i(\Delta p)} = \frac{b(h_{\min})_i^3}{12\eta}\frac{dp}{dx} . \quad (5)$$

To calculate the variation in the pressure of the fluid during its flow through the gap in direction of $x$ axis, the following assumption is made: in a limited segment defined by $[-a, a]$ around $h_{min}$, the cross-section of the gap between profiles of gears through which fluid flow can be modeled with rectangles of constant dimensions. For the limit value of $a$, the value of coordinate $x$ are used for which related gap height according to equation (1), which is not much larger than $h_{min}$, means $x = a \rightarrow h \approx h_{\min}$. In order to make a deviation from the accurate calculation of cross-section surface negligible, limits of the relative error are set in advance, in particular:

$$\frac{P_r - P_a}{P_a} 100 \le 2\,\%, \quad (6)$$

where $P_r$ is the value of the real surface of the cross-section of the gap and $P_a$ is the approximate value of surface obtained by approximating the geometrical form of the gap. Starting from the assumption made, the equation (5) can be written as follows:

$$Q_{i(\Delta p)} = \frac{b(h_{\min})_i^3}{24\eta}\frac{\Delta p}{a_i} , \quad (7)$$

where $\Delta p$ is the difference in the pressure between the two adjacent chambers of the pump [15].

Upon the presented analysis, the final form of equation for the calculation of flow rate through the gaps between related profiles of gears for the considered active chamber $K_i$ can be formed as follow:

$$Q_{i\Sigma} = sign(\psi)\left[\frac{(v_r)_{i+1}\, b\, (h_{\min})_{i+1}}{2} - \frac{(v_r)_i\, b\, (h_{\min})_i}{2}\right] - \\ - \frac{b(h_{\min})_i^3[p_i - p_{i-1}]}{24\eta a_i} - \frac{b(h_{\min})_{i+1}^3[p_i - p_{i+1}]}{24\eta a_{i+1}} . \quad (8)$$

Volumetric losses affect the pressure variation in pump working chambers. In a pump with fixed shaft axes, the fluid distribution is done through the holes with variable fluid flow area. In such constructions, we assume that fluid flow area is equal to instantaneous chamber area. This assumption leads to expression for calculation of the

pressure variation during the fluid flow in the chamber $K_i$ in the following form,

$$\Delta p_i = \frac{\rho_f b^2}{A_i^2} \left[ \frac{dA_i}{dt} \right]^2 . \quad (9)$$

In expression (9), $\rho_f$ is the fluid density and $dA_i/dt$ is the gradient of the instantaneous area $A_i$ of the chamber $K_i$ [5, 9]. Now, the equation for calculation of pressure in the intake chamber can be done as:

$$p_i = p_{in} - \Delta p_i \quad , \quad (10)$$

While in the output chamber is

$$p_i = p_{out} + \Delta p_i . \quad (11)$$

The current theoretical flow in the chamber $K_i$ represents the variation in the volume at the working chamber in the time and is calculated using the following general form [6]:

$$Q_i = \frac{dV_i}{dt} = C_p A_0 \left[ \frac{2\Delta p_i}{\rho_f} \right]^{\frac{1}{2}} , \quad (12)$$

where

$\Delta p_i$ - Pressure drop due to the fluid flow,

$A_0$ - cross-section surface of the distribution valve opening,

$\rho_f$ - fluid density.

During the intake phase, the volume of the chamber increases and the current theoretical flow is positive. Accordingly, the following expression for determining the theoretical flow for the chamber $K_i$ during intake phase:

$$Q_i = C_P A_0 \left[ \frac{2(p_{in} - p_i)}{\rho_f} \right]^{\frac{1}{2}} , \quad (13)$$

During the thrust phase, the volume of the chamber decreases and the current theoretical flow is negative. Therefore, the following expression for determining the theoretical flow for the chamber $K_i$ during thrust phase:

$$Q_i = -C_p A_0 \left[ \frac{2(p_i - p_{out})}{\rho_f} \right]^{\frac{1}{2}} . \quad (14)$$

On the basis of equitation (13) the expression for determining the pressure in the chamber $K_i$ during intake phase can be done as follows:

$$p_i = p_{in} - \frac{\rho_f [dV/dt]^2}{2 C_p^2 A_0^2} \quad (15)$$

Respectively, according to the equation (14), during thrust phase:

$$p_i = p_{out} + \frac{\rho_f [dV/dt]^2}{2 C_p^2 A_0^2} . \quad (16)$$

Starting from the equitation (9) and taking into account the flow losses defined by the expression (8), the following equation for the calculation of the pressure

variation in the chamber is obtained, depending on the height of the gap between gears profiles, can be done as follows:

$$\Delta p_{i\Sigma} = \frac{\rho_f}{A_i^2} \left[ \frac{dV_i}{dt} + Q_{i\Sigma} \right]^2 \text{ or } \Delta p_{i\Sigma} = \frac{\rho_f}{A_i^2} [Q_i + Q_{i\Sigma}]^2 . \quad (17)$$

For considered model of the pump, the following equation for calculation of current flow can be used in the following form:

$$Q_i = -2\pi n_t e^2 z b \left\{ 2\lambda \sin \frac{\pi}{z} \sin\left( \frac{2\pi i}{z} - \psi \right) - \frac{c}{z} \left[ 1 + \lambda^2 - 2\lambda \cos(\tau - \psi) \right]^{\frac{1}{2}} \Big|_{\tau_i}^{\tau_{i+1}} \right\} . \quad (18)$$

Based on the established relations between the geometric parameters and the functional characteristics of the gerotor pump, a computer program composed of several specific modules has been developed. The values of pressure variation in the pump chambers are calculated and graphic interpretation of the obtained results are done using the appropriate software package AutoLISP and Mathematica. The results are presented in next section of this paper.

## 3. TESTING OF MATHEMATICAL MODEL OF PRESSURE VARIATION IN PUMP CHAMBERS

The previously presented mathematical model was tested on two concrete examples of the gear pairs of the considered pump model, one is existing commercial, and the second gear pair with a profile obtained on the basis of the calculated results in order to improve existing design of the gerotor pump used as part of a system for lubrication of internal combustion engine [11]. The geometric parameters of the gear pairs are as follows: $z = 6$, $e = 3.56$ mm, $b = 16.46$ mm, $c = 2.75$, $r_s = 26.94$ mm. For a newly generated gear pair, parameters are as follow: $\lambda = 1.375$, $c = 2.75$ while for current commercial one parameters are as follow: $\lambda = 1.575$, $c = 3.95$. Other pump parameters are: $\Delta p = 0.6$ MPa, $\rho_f = 900$ kg/m³, $n_t = 1500$ rpm, $\omega_t = 2\pi n_t = 50\pi$ s⁻¹.

In order to better understand the influence of trochoidal coefficient $\lambda$ and the height of the gap on the pressure variation in the pump chambers, comparative diagrams are presented in Fig. 3. The analysis was performed for theoretically ideal profile and for a profile with tolerances, over a period of time corresponding to one phase of the working cycle of the gerotor pump. the graphic interpretation of pressure variation in pump chambers as a function of time is presented only for a transition zone between intake and thrust phase due to significant differences in pressures.

In the diagram shown, the symmetry of the pressure variation distribution, in the case of an ideal profile, is observed for chambers that are symmetrically arranged in relation to the transition of the intake and thrust phase. With the increase of the coefficient $\lambda$, the value of these variations increases. In the case of real profile meshing,

the value of these variation increases, but the trend of increase of those variations with an increase of coefficient λ remains.
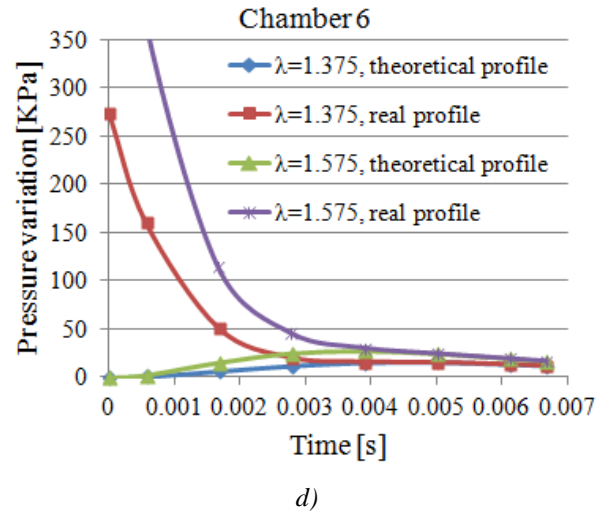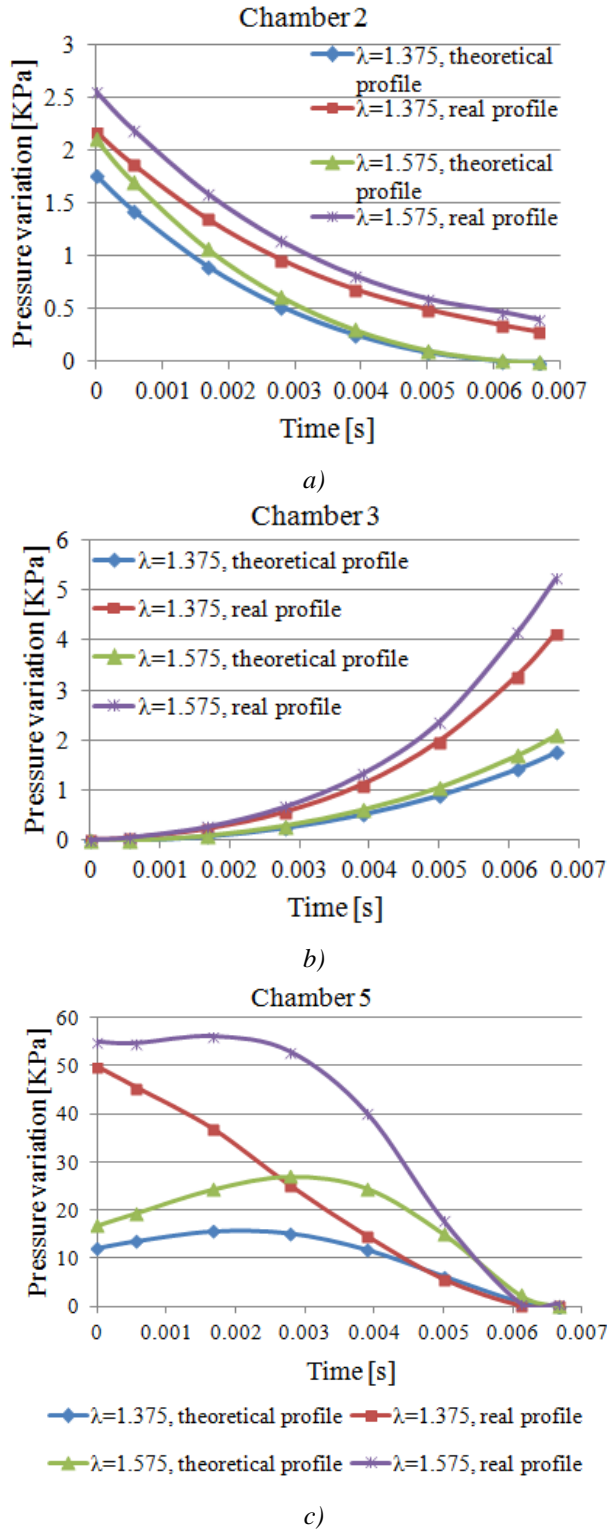
## Chamber 2



*a)*

## Chamber 3



*b)*

## Chamber 5



*c)*

## Chamber 6



*d)*

*Fig.3: Comparative diagrams of pressure variation in pump chambers between intake and output zone*

Based on the presented graphical interpretation, it can be concluded that the influence of the gap height on the pressure variation is greatest at the end of thrust (intake) zone and at entering the intake (thrust) phase. This can be explained by the greatest difference in the pressure between adjacent pump chambers. Highest values of pressure variations occur at the beginning of the considered phase period, in the chamber 6, in the commercial model. Based on the presented results the conclusion was made that more beneficial indicators are derived from the model ($\lambda = 1.375$) proposed as a solution with improved characteristics compared to the existing commercial ones ($\lambda = 1.575$).

After carrying out the theoretical analysis of the gerotor pump gear pair, it is necessary to verify and evaluate the formed models through the production of concrete gear pairs and the performance of the laboratory testing with the simulation of the working conditions of the pump.

## 4. CONCLUSION

Comparative analysis of pressure variation in pump chambers with different trochoidal coefficient $\lambda$ is presented in this paper. The analysis was performed both for theoretical and realistic profiles. It has been presented that in the case of gerotor pumps with the same number of chambers $z$ and same longitudinal base radius $r_s$, by selecting smaller value of trochoidal coefficient $\lambda$, smaller variation of pressure in pump chambers are obtained. The same conclusions are done on results obtained by analysis of ideal theoretical profile and in the real profiles with technological gaps and tolerances.

The scientific results of this paper establish new valuable guidelines and perspectives for constructors to creating more efficient construction solutions of the gerotor pumps. By implementing the results presented in this paper, it is possible to achieve more smoothly operation of georotor. The main guideline for future research on this topis will be focused on experimental testing of presented models and their comparative analysis with theoretical models.

## REFERENCES

[1] ANSDALE, R. F., LOCKLEY, D. J. (1970) *The Wankel RC Engine*, Iliffe Books Ltd., London

[2] MANCO, S., NERVEGNA, N., RUNDO, M., ARMENIO, G., PACHETTI, C., TRICHILO, R. (1998) *Gerotor Lubricating Oil Pump for IC Engines*, SAE Paper 982689, International Fall Fuels & Lubricants Meeting and Exposition, October 19-22, San Francisco, USA and SAE Transactions

[3] MANCO, G., MANCO, S., RUNDO, M., NERVEGNA, N. (2001) *Computerized generation of novel gearings for internal combustion engines lubricating pumps*, The International Journal of Fluid Power, Vol. 1 – No 1 - ISBN 1493-9776, pp 49-58

[4] MANCÒ, S., NERVEGNA, N., RUNDO, M. (2001) *Critical issues on performance of lubricating gerotor pumps at high rotational speed*, Proceedings, The 7th Scandinavian International Conference on Fluid Power, SICFP, Linköping, Sweden, Vol. 2, pp 23-38.

[5] MAITI, R. AND SINHA, G. L. (1998) *Kinematics of active contact in modified epitrochoid generated rotary piston machines*, Mechanism and Machine Theory, Vol. 23 No 1, pp 39-45.

[6] SHUNG J. B., PENNOCK G. R. (1994) *The direct contact problem in a Trochoid-Type Machines*, Mechanism and Machine Theory, Vol. 29, No 5, pp. 673-689.

[7] RAUSH, G., GAMEZ-MONTERO, P. J., CASTILLA, R., CODINA, E. (2017) *Experimental study on the impulsion port of a trochoid wheeled pump*, Flow Measurement and Instrumentation, Vol. 55, pp 13-22.

[8] BUONO, D., SCHIANO DI COLA, D., SENATORE, A., FROSINA, E., BUCCILLI G., HARRISON, J. (2016) *Modelling approach on a Gerotor pump working in cavitation conditions*, Energy Procedia, Vol. 101, pp 701-709.

[9] IVANOVIĆ, L., DEVEDŽIĆ, G., MIRIĆ, N., ĆUKOVIĆ, S. (2010) *Analysis of forces and moments in the gerotor pumps*, Proc. Instn Mech. Engrs, Part C: J. Mechanical Engineering Science, Vol. 224, No 10, pp 2257-2269 DOI: 10.1243/09544062JMES2041, ISSN 0954-4062.

[10] IVANOVIĆ, L., JOSIFOVIĆ, D., ILIĆ, A., STOJANOVIĆ , B. (2013) *Analytical model of the pressure variation in the gerotor pump chambers*, Technics Technologies Education Management / TTEM, Vol.8, No1, pp 323-331, ISSN 1840-1503.

[11] IVANOVIĆ, L., DEVEDŽIĆ, G., ĆUKOVIC, S., MIRIĆ, N. (2012) *Modeling of the Meshing of Trochoid Profiles With Clearances,* Journal of Mechanical Design, Vol.134, No 4, pp 041003-1 / 041003-9, ISSN 1050-0472, Doi 10.1115/1.4005621

[12] IVANOVIĆ, L., JOSIFOVIĆ, D., BLAGOJEVIĆ, M., STOJANOVIĆ, B., ILIĆ, A. (2012) *Determination of gerotor pump theoretical flow*, Proceedings, COMETa 2012 1st international scientific conference, Jahorina, B&H Republic of Srpska, 28th-30th November, pp. 243-250.

[13] TIPEI, N. (1962) *Theory of lubrication*, Stanford university press, Stanford, California

[14] IVANOVIĆ, L., JOSIFOVIĆ, D., ILIĆ, A, STOJANOVIĆ, B. (2011) *Tribological aspect of the kinematical analysis at trochoid gearing in contact*, Journal of the Balkan Tribological Association (JBTA), Vol. 17, No 1, pp. 37–47, ISSN 1310-4772.

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