DYNAMIC BEHAVIOR OF PLANETARY GEARBOX NEW CONCEPT

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Summary: *Noise and vibration have a very negative influence on mechanical gearbox functionality. That is the reason why a lot of attention is given to these analyses.*

A new concept of planetary gearbox is presented in this paper. Both a two dimensional model, and three dimensional model has been developed. Based on known dynamical models of mechanical gearboxes, the new dynamical model is defined for this specific concept of planetary gearbox. The paper concludes with the discussion and guidelines for further work.
Keywords: *planetary gearbox, dynamic analysis, dynamic model*

1. INTRODUCTION

Planetary gearboxes with their compact design are largely represented in operating systems of mobile machinery. Operating conditions for transmissions in mobile machinery vary within a wide range. Research of the gearbox dynamics in this case is of great importance. Examining the dynamics of planetary gearboxes leads to conclusions that could greatly assist the development of planetary reducers with regard to: improving their compact design, increasing reliability, increasing the lifetime of the drive, reducing vibration and reducing noise in working conditions, etc.

Due to the aforementioned reasons a lot of research is done in the field of gearbox dynamics. Analysis of the dynamic behavior of planetary reducers is possible with various computer software, which perform simulations [1], [2], [3]. Computer simulation could be verified by experimental methods [4], [5]. An even greater impact on planetary drive research is given by the possibility of performing physical experiments to verify the computer simulated dynamic analyses.

In this paper a new concept of planetary drive has been developed. Its dynamic model has been made, which has been solved in *MATLAB – SIMULINK*, [6]. The results of the simulation are also presented in the paper. The paper also presents the conclusions drawn from the simulation, and possible directions for future research.

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2. DYNAMIC MODEL OF NEW CONCEPT PLANETARY GEARBOX

Planetary gearbox of C conception has been developed in this paper. It consists of a pinion carrier (h), stationary central ring gear (e), dual pinion (f - g) and the movable central ring gears (b), (Figure 1).

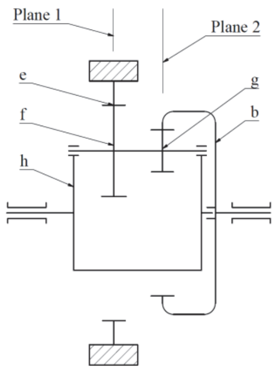


Fig. 1 *Schematics of the developed planetary gearbox*

The planetary gearbox in Figure 1 is designed for the parameters given in Table 1.

Table 1 *Parameters for the design of planetary gearbox*

Power	P_{in}	5 [kW]
Input rotation per minute	n_{in}	1200 [min^{-1}]
Transmission ratio	i_R	1:20

2.1 DYNAMIC MODEL SETUP

The dynamic model of the planetary reducer is set in such a way as to present the planetary reducer in two planes (Figure 1). Common elements to both planes are the pinion carrier (h), dual pinion (f - g) and the shaft that connects the dual pinion to the pinion carrier. The dynamic model has four degrees of freedom which defines the dynamic system of the planetary reducer: y_1 radial movement, the movement of the pinion carrier (h) around its axis Θ_h, moving dual pinion around its own axis Θ_f (it is equivalent to Θ_g, since it is a dual pinion setup) and moving of the portable central ring gear (b) around its axis Θ_b. The choice of the number of degrees of freedom best describes the operation of this planetary reducer. Contacts between gears which are coupled, are modeled as springs and dampers. Contact between the gear (e) and gear (f) is modeled as spring with stiffness c_1 and dumper with damping coefficient k_1, while the contact between the gear g and gear b is modeled as a spring with stiffness c_2, dumper with damping coefficient k_2. The dynamic model does not take into consideration the reduced mass of the system elements, however, in favor of more accurate calculations; the total mass of the system elements has been used here. The values of mass and moments of inertia were obtained from the CAD model of the design of planetary gearbox. The dynamic model of the planetary gearbox is shown in

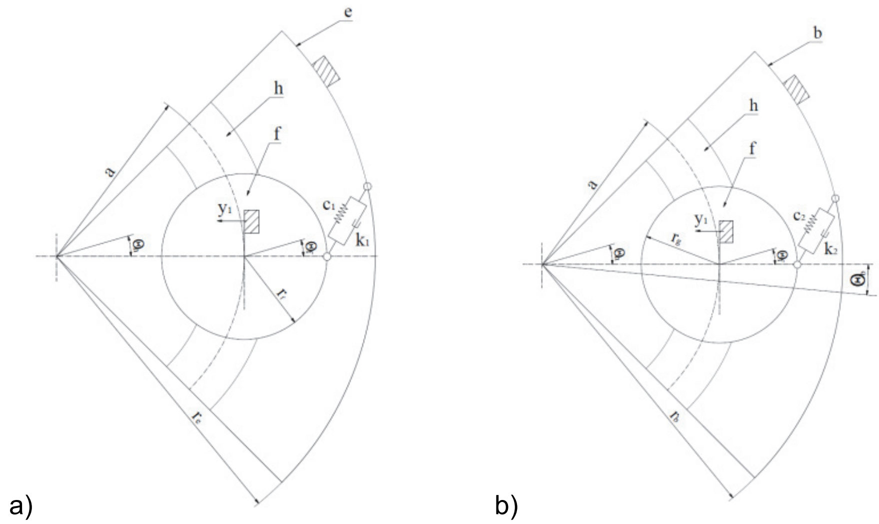


Fig. 2 *The dynamic model of the planetary gearbox a) plane 1; b) plane 2*

2.2 DEFINING THE EQUATIONS OF DYNAMIC MODEL

Defining the dynamic equations of planetary gearboxes is performed using Lagrange equations of the second kind. It has been adopted, because given the choice between it and Dalamber's principle or Hamilton's principle, Lagrange equations give the best depiction of a dynamic system. When setting up the dynamic equations via Lagrange equations of the second kind, the kinetic energy of the system is first calculated:

$$E_k = \frac{1}{2}\left[m_h\left(\dot{y}_1^2 + a^2\dot{\theta}_h^2\right) + J_{ch}\dot{\theta}_h^2\right] + \frac{1}{2}\left[m_f\left(\dot{y}_1^2 + a^2\dot{\theta}_h^2\right) + J_{cf}\left(\dot{\theta}_h + \dot{\theta}_f\right)^2\right] + \frac{1}{2}\left[m_g\left(\dot{y}_1^2 + a^2\dot{\theta}_h^2\right) + J_{cg}\left(\dot{\theta}_h + \dot{\theta}_f\right)^2\right] + \frac{1}{2}J_{cb}\dot{\theta}_b^2 \quad (1)$$

Following the kinetic energy, the potential energy of the system is calculated:

$$E_p = \frac{1}{2}c_1\left\{y_1^2 + \left[r_f\left(\theta_h + \theta_f\right) + a\theta_h\right]^2\right\} - c_1 y_1\left[r_f\left(\theta_h + \theta_f\right) + a\theta_h\right] + \frac{1}{2}c_2\left\{y_1^2 + \left[r_g\left(\theta_h + \theta_f\right) + a\theta_h - r_b\theta_b\right]^2\right\} - c_2 y_1\left[r_g\left(\theta_h + \theta_f\right) + a\theta_h - r_b\theta_b\right] \quad (2)$$

Finally, the function of system dissipation is calculated:

$$\phi = \frac{1}{2}k_1\left\{\dot{y}_1^2 + \left[r_f\left(\dot{\theta}_h + \dot{\theta}_f\right) + a\dot{\theta}_h\right]^2\right\} - k_1\dot{y}_1\left[r_f\left(\dot{\theta}_h + \dot{\theta}_f\right) + a\dot{\theta}_h\right] + \frac{1}{2}k_2\left\{\dot{y}_1^2 + \left[r_g\left(\dot{\theta}_h + \dot{\theta}_f\right) + a\dot{\theta}_h - r_b\dot{\theta}_b\right]^2\right\} - k_2\dot{y}_1\left[r_g\left(\dot{\theta}_h + \dot{\theta}_f\right) + a\dot{\theta}_h - r_b\dot{\theta}_b\right] \quad (3)$$

After calculating these functions the next step is the Lagrange equations of the second kind for the dynamic system according to the formula:

$$\frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}_i} - \frac{\partial E_k}{\partial q_i} = -\frac{\partial E_p}{\partial q_i} - \frac{\partial \phi}{\partial \dot{q}_i} + Q_i \quad (4)$$

System of differential equations of the dynamic model have been obtained:

$$\ddot{y}_1(m_h+m_f+m_g)+y_1(c_1+c_2)-c_1[r_f(\theta_h+\theta_f)+a\theta_h]-c_2[r_g(\theta_h+\theta_f)+a\theta_h-r_b\theta_b]+\dot{y}_1(k_1+k_2)-k_1[r_f(\dot{\theta}_h+\dot{\theta}_f)+a\dot{\theta}_h]-k_2[r_g(\dot{\theta}_h+\dot{\theta}_f)+a\dot{\theta}_h-r_b\dot{\theta}_b]=0 \tag{5}$$

$$(m_h a^2+J_{ch}+m_f a^2+J_{cf}+m_g a^2+J_{cg})\ddot{\theta}_h+(J_{cf}+J_{cg})\ddot{\theta}_f+c_1[\theta_h(r_f+a)+r_f\theta_f](r_f+a)-c_1y_1(r_f+a)+c_2[\theta_h(r_g+a)+r_g\theta_f-r_b\theta_b](r_g+a)-c_2y_1(r_g+a)+k_1[\dot{\theta}_h(r_f+a)+r_f\dot{\theta}_f](r_f+a)-k_1\dot{y}_1(r_f+a)+k_2[\dot{\theta}_h(r_g+a)+r_g\dot{\theta}_f-r_b\dot{\theta}_b](r_g+a)-k_2\dot{y}_1(r_g+a)=M_h \tag{6}$$

$$(J_{cf}+J_{cg})\ddot{\theta}_f+(J_{cf}+J_{cg})\ddot{\theta}_h+c_1[r_f(\theta_h+\theta_f)+a\theta_h]r_f-c_1y_1r_f+c_2[r_g(\theta_h+\theta_f)+a\theta_h-r_b\theta_b]r_g-c_2y_1r_g+r_gk_1[r_f(\dot{\theta}_h+\dot{\theta}_f)+a\dot{\theta}_h]r_f-k_1\dot{y}_1r_f+k_2[r_g(\dot{\theta}_h+\dot{\theta}_f)+a\dot{\theta}_h-r_b\dot{\theta}_b]r_g-k_2\dot{y}_1r_g=0 \tag{7}$$

$$J_{cb}\ddot{\theta}_b-c_2r_b[r_g(\theta_h+\theta_f)+a\theta_h-r_b\theta_b]+c_2y_1r_b-k_2r_b[r_g(\dot{\theta}_h+\dot{\theta}_f)+a\dot{\theta}_h-r_b\dot{\theta}_b]+k_2\dot{y}_1r_b=0 \tag{8}$$

2.3 SOLVING THE EQUATIONS OF DYNAMIC MODEL

Solving equation systems of the dynamic model, is performed with a simulation in *MATLAB-SIMULINK*. In order to solve the system of equations a solving scheme has been made (Figure 3), in the *SIMULINK* environment.

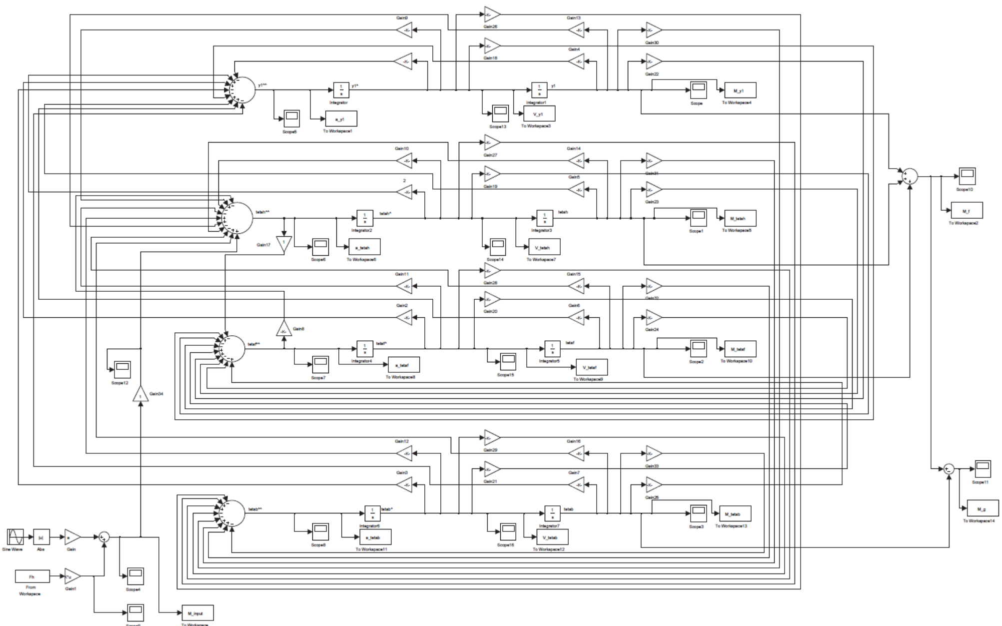


Fig. 3 *Solving scheme for SIMULINK*

3. SIMULATION RESULTS OF THE DYNAMIC MODEL

Simulation of the dynamic model has been performed in two periods of oscillation of the dynamic system. Impulse of the oscillating dynamic system was performed with the moment M_h, as an input impulse parameter. Moment M_h was developed as an absolute sine function (Figure 4), [7].

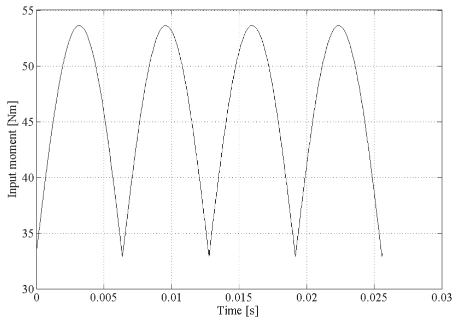


Fig. 4 *Impulse moment, M_h*

With the introduction of an impulse moment the simulation is started (Figure 3), which simulates the dynamic oscillation of the planetary gearbox system. As the output of diagrams are obtained: acceleration, velocity, displacement and total displacement of all four degrees of freedom in the dynamic system. Acceleration along the directions of degrees of freedom, at the beginning of the first period of oscillation, is with large variations (Figure 5), while at the end of the second period of the oscillations is in calm variations.

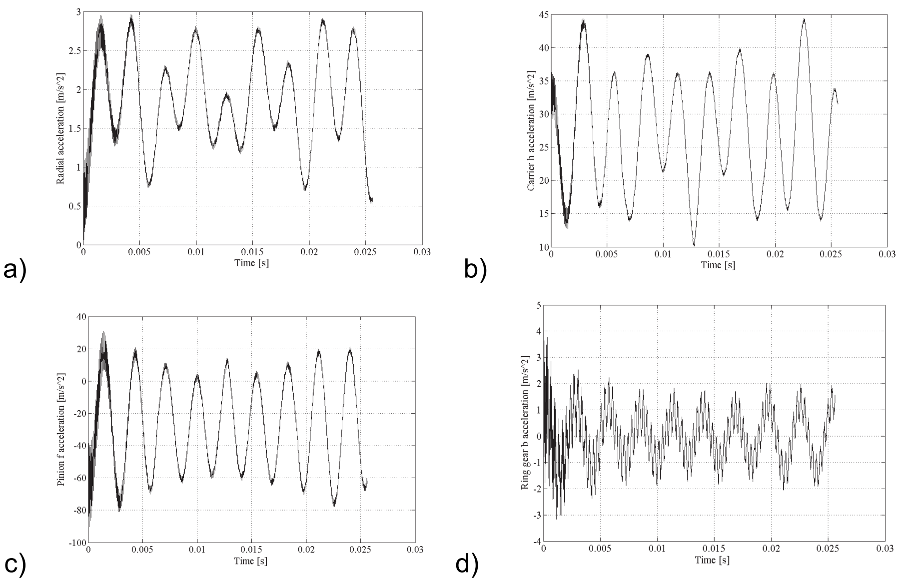


Fig. 5 *Acceleration along the axis of degrees of freedom at c_1=1,67x10^{10}[N/m]; c_2=1,56x10^{10}[N/m]; k_1 =3200[Ns/m]; k_2 =2400[Ns/m]; a) y_1 direction; b) Θ_h direction; c) Θ_f direction; d) Θ_b direction*

From the point of oscillation, accelerations have a similar behavior as speed and movement in all directions of degrees of freedom. As a final output diagrams are obtained total displacement of gears f and b (Figure 6).

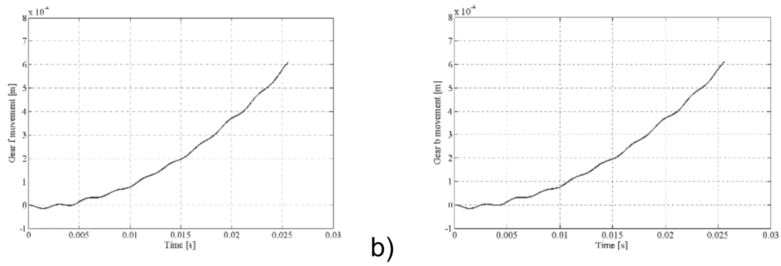
a) b)


Fig. 5 *Total movement at c_1=1,67x10^{10} [N/m]; c_2=1,56x10^{10} [N/m]; k_1 =3200 [Ns/m]; k_2 = 2400 [Ns/m]; a) gear f; b) gear b*

The total movement of gears f and b retain the characteristics of acceleration oscillation. At the beginning of the simulation all parameters have large amplitude oscillations, as time passes they are ultimately reduced.

4. CONCLUSION

When the simulation has been completed it can be concluded that the most critical is the first period of oscillation. After the first period of oscillation, the dynamic oscillations of the dynamical system are calming. Investigation of the dynamics of planetary gearbox with this aspect can greatly help to reduce vibration on startup of planetary gearbox. Further research into this problem could be experimental confirmation of the results of dynamic simulation.

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