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DYNAMIC BEHAVIOUR OF C CONCEPT PLANETARY REDUCER

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Abstract: *Appearance of vibrations has a negative influence on planetary reducer operation. Vibrations which appear at the start have the worst effect. Determination of the mechanism of vibration and their reduction to an acceptable level, are issues for a lot of modern research related to planetary reducers.*
Presented in this paper is a new design solution of a planetary C concept reducer. According to known dynamic models, for this particular reducer, an original dynamic model is developed. The original dynamic model describes dynamic parameters of the presented reducer. At the end of the paper, a discussion is given, and guidelines for further research possibilities

Key words: *planetary gearbox, dynamic model, dynamic behaviour*

1. INTRODUCTION

Planetary gearboxes with their compact design are largely represented in operating systems of mobile machinery. Operating conditions for transmissions in mobile machinery vary within a wide range. Research of the gearbox dynamics in this case is of great importance. Examining the dynamics of planetary gearboxes leads to conclusions that could greatly assist the development of planetary reducers with regard to: improving their compact design, increasing reliability, increasing the lifetime of the drive, reducing vibration and reducing noise in working conditions, etc.
Due to the aforementioned reasons, a lot of research is done in the field of gearbox dynamics. Analysis of the dynamic behavior of planetary reducers is possible with various computer software, which perform simulations [1], [2], [3]. Computer simulation could be verified by experimental methods [4], [5]. An even greater impact on planetary drive research is given by the possibility of performing physical experiments to verify the computer simulated dynamic analyses.
In this paper a new concept of planetary drive has been developed. Its dynamic model has been made, which has been solved in *MATLAB - SIMULINK*, [6]. The results of the simulation are also presented in the paper. The paper also presents the conclusions drawn from the simulation, and possible directions for future research.

2. DYNAMIC MODEL OF NEW CONCEPT PLANETARY GEARBOX

Planetary gearbox of C conception has been developed in this paper. It consists of a pinion carrier (h), a stationary central ring gear (e), dual pinion (f - g) and the movable central ring gears (b), (Figure 1).The planetary gearbox in Figure 1 is designed for the parameters given in Table 1.

Table 1. Parameters for the design of planetary gearbox

Power	P_{in}	5 [kW]
Input rot. per min.	n_{in}	1200 [min^{-1}]
Transmission ratio	i_R	1:20

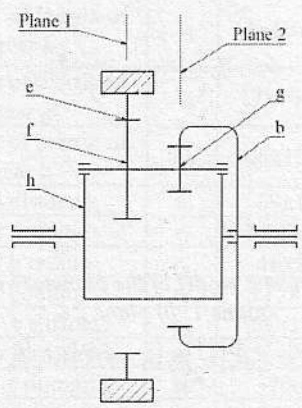

Fig. 1. Schematics of the developed planetary gearbox

2.1 Dynamic model setup

The dynamic model of the planetary reducer is set in such a way as to present the planetary reducer in two planes (Figure 1). Common elements to both planes are the pinion carrier (h), dual pinion (f - g) and the shaft that connects the dual pinion to the pinion carrier. The dynamic model has four degrees of freedom which defines the dynamic system of the planetary reducer: y_1 radial movement, the movement of the pinion carrier (h) around its axis Θ_h, moving dual pinion around its own axis Θ_f (it is equivalent to Θ_g, since it is a dual pinion setup) and moving of the portable central ring gear (b) around its axis Θ_b. The choice of the number of degrees of freedom best describes the operation of this planetary reducer. Contacts between gears which are coupled are modeled as springs and dampers. Contact between the gear (e) and gear (f) is modeled as spring with stiffness c_1 and dumper with damping coefficient k_1, while the contact between the gear g and gear b is modeled as a spring with stiffness c_2, dumper with damping coefficient k_2. The dynamic model does not take into consideration the reduced mass of the system elements, however, in favor of more accurate calculations; the total mass of the system elements has been used here. The values of mass and moments of inertia were obtained from the CAD model of the design of planetary gearbox. The dynamic model of the planetary gearbox is shown in Figure 2.

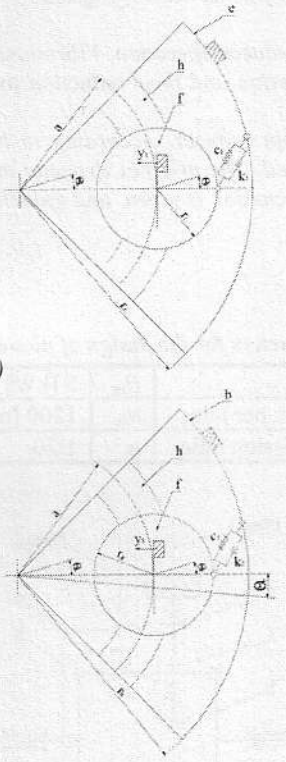

Fig. 2. The dynamic model of the planetary gearbox a) plane 1; b) plane 2

Degrees of freedom, accordin to which the dynamic model has ben made, are shown at Figure 3. Degrees of freedom are marked at three-dimensional figure of planetary reducer.

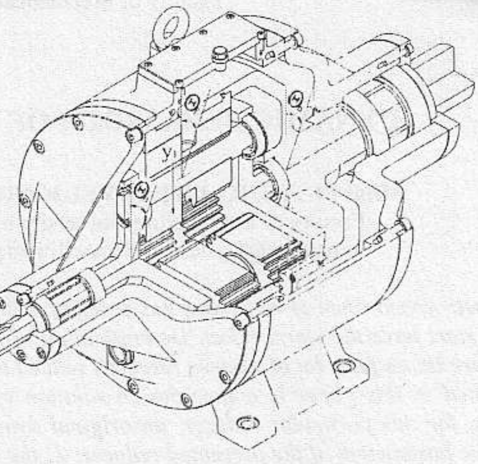

Fig. 3. Degrees of fredoom on planetary reducer

2.2 Dynamic model definition

Defining the dynamic equations of planetary gearboxes is performed using Lagrange equations of the second kind. It has been adopted, because given the choice between it and Dalamber's principle or even Hamilton's principle, Lagrange equations give the best depiction of a dynamic system. When setting up the dynamic equations via Lagrange equations of the second kind, the kinetic energy of the system is first calculated.

$$E_k=\frac{1}{2}\left[m_h\left(\dot{y}_1^2+a^2\dot{\theta}_h^2\right)+J_{ch}\dot{\theta}_h^2\right]+\frac{1}{2}\left[m_f\left(\dot{y}_1^2+a^2\dot{\theta}_h^2\right)+J_{cf}\left(\dot{\theta}_h+\dot{\theta}_f\right)^2\right]+\frac{1}{2}\left[m_g\left(\dot{y}_1^2+a^2\dot{\theta}_h^2\right)+J_{cg}\left(\dot{\theta}_h+\dot{\theta}_f\right)^2\right]+\frac{1}{2}J_{cf}\dot{\theta}_b^2 \quad (1)$$

Following the kinetic energy, the potential energy of the system is calculated:

$$E_p=\frac{1}{2}c_1\left\{y_1^2+\left[r_f\left(\theta_h+\theta_f\right)+a\theta_h\right]^2\right\}-c_1y_1\left[r_f\left(\theta_h+\theta_f\right)+a\theta_h\right]+\frac{1}{2}c_2\left\{y_1^2+\left[r_g\left(\theta_h+\theta_f\right)+a\theta_h-r_b\theta_h\right]^2\right\}-c_2y_1\left[r_g\left(\theta_h+\theta_f\right)+a\theta_h-r_b\theta_h\right] \quad (2)$$

Finally, the function of system dissipation is calculated:

$$\phi=\frac{1}{2}k_1\left\{\dot{y}_1^2+\left[r_f\left(\dot{\theta}_h+\dot{\theta}_f\right)+a\dot{\theta}_h\right]^2\right\}-k_1\dot{y}_1\left[r_f\left(\dot{\theta}_h+\dot{\theta}_f\right)+a\dot{\theta}_h\right]+\frac{1}{2}k_2\left\{\dot{y}_1^2+\left[r_g\left(\dot{\theta}_h+\dot{\theta}_f\right)+a\dot{\theta}_h-r_b\dot{\theta}_b\right]^2\right\}-k_2\dot{y}_1\left[r_g\left(\dot{\theta}_h+\dot{\theta}_f\right)+a\dot{\theta}_h-r_b\dot{\theta}_b\right] \quad (3)$$

After calculating these functions the next step is the Lagrange equations of the second kind for the dynamic system according to the formula:

$$\frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}_i}-\frac{\partial E_k}{\partial q_i}=-\frac{\partial E_p}{\partial q_i}-\frac{\partial \phi}{\partial \dot{q}_i}+Q_i \quad (4)$$

Writing the dynamic equations presents the final step before its putting in matrix form. System of dynamic equations has been putted in matrix form because of its easier solving:

$$\mathbf{M}\{\ddot{q}\}+\mathbf{B}\{\dot{q}\}+\mathbf{C}\{q\}=\{D\} \quad (5)$$

First member in expresion (5) is mass and moment of inertia matrix:

$$\mathbf{M}\{\ddot{q}\}=\begin{bmatrix} m_h+m_f+m_g & 0 & 0 & 0 \\ 0 & m_h a^2+J_{ch}+m_f a^2+J_{cf}+m_g a^2+J_{cg} & J_{cf}+J_{cg} & 0 \\ 0 & J_{cf}+J_{cg} & J_{cf}+J_{cg} & 0 \\ 0 & 0 & 0 & J_{cb} \end{bmatrix} \tag{6}$$

Second member in expresion (5), is dumping matrix:

$$\mathbf{B}\{\dot{q}\}=\begin{bmatrix} k_1+k_2 & 0 & 0 & 0 \\ 0 & k_1(r_f+a)^2+k_2(r_g+a)^2 & k_1 r_f(r_f+a)+k_2 r_g(r_g+a) & -k_2 r_b(r_g+a) \\ 0 & k_1 r_f(r_f+a)+k_2 r_g(r_g+a) & k_1 r_f^2+k_2 r_g^2 & -k_2 r_b r_g \\ 0 & -k_2 r_b(r_g+a) & -k_2 r_b r_g & k_2 r_b^2 \end{bmatrix} \tag{7}$$

Third member in expresion (5) is stiffneess matrix:

$$\mathbf{C}\{q\}=\begin{bmatrix} c_1+c_2 & 0 & 0 & 0 \\ 0 & c_1(r_f+a)^2+c_2(r_g+a)^2 & c_1 r_f(r_f+a)+c_2 r_g(r_g+a) & -c_2 r_b(r_g+a) \\ 0 & c_1 r_f(r_f+a)+c_2 r_g(r_g+a) & c_1 r_f^2+k_2 r_g^2 & -c_2 r_b r_g \\ 0 & -c_2 r_b(r_g+a) & -c_2 r_b r_g & c_2 r_b^2 \end{bmatrix} \tag{8}$$

Last matrix in expresion (5), is impulse matrix:

$$\{D\}=\begin{Bmatrix} 0 \\ M_h \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

By defining all of matrixes and equations it can be proceded to solving dynamic system.

2.3 Dynamic model solving

Solving equation systems of the dynamic model, is performed with a simulation in *MATLAB-SIMULINK*. In order to solve the system of equations a solving scheme has been made (Figure 4), in the *SIMULINK* environment. Before solving the system dynamic equations, parameters of the system must be taken from CAD model (Table 2):

Table 2. Dynamic model parameters

Parameter name	Sn.	Value
Sattelite carrier h mass	m_h	11,152[kg]
Gear f mass	m_f	3,376[kg]
Gear g mass	m_g	8,536[kg]
Moment of inertia of sattelite carrier h	J_{ch}	116x10-3[kgm2]
Moment of inertia of gear f	J_{cf}	3x10-3[kgm2]
Moment of inertia of gear g	J_{cg}	17x10-3[kgm2]
Moment of inertia of gear b	J_{cb}	306x10-3[kgm2]
Axes distance	a	69x10-3[m]
Gear f radius	r_f	52,5x10-3[m]
Gear g radius	r_g	48x10-3[m]
Gear b radius	r_b	117x10-3[m]
Pair f – e stiffness	c_1	1,67x1010[N/m]
Pair g – b stiffness	c_2	1,56x1010[N/m]
Pair f – e dumping	k_1	3200[Ns/m]
Pair g – b dumping	k_2	2400[Ns/m]

Dumping and stiffness coefficient values has been taken from literature, [3], [8].

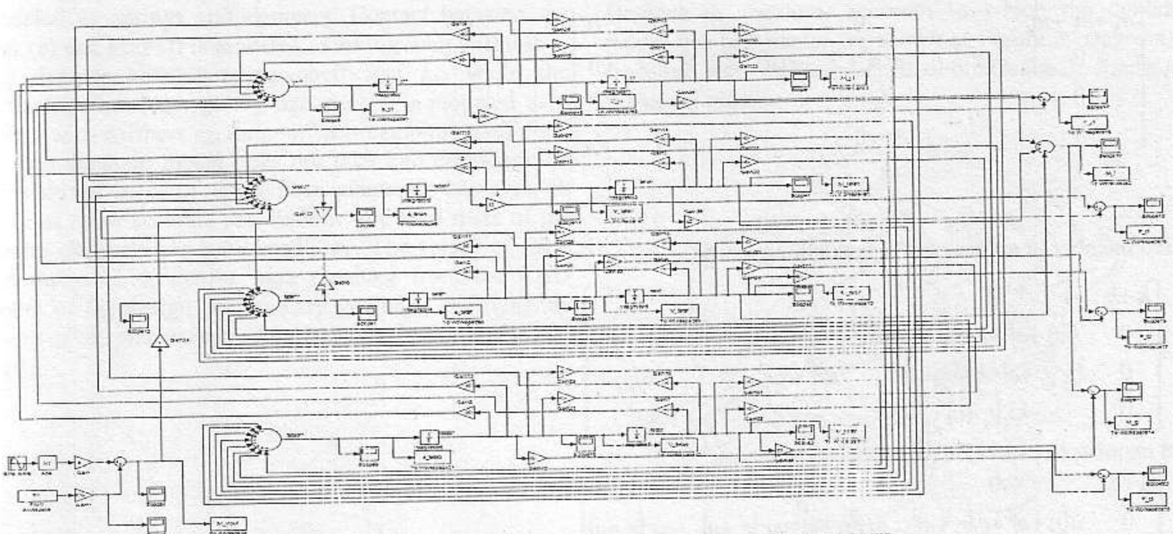

Fig. 4. Schematics of dynamic system solver in SIMULINK

3. RESULTS OF SIMULINK DYNAMIC SIMULATION

Simulation of the dynamic model has been performed in two periods of oscillation of the dynamic system. Impulse of the oscillating dynamic system was performed with the moment M_h, as an input impulse parameter. Moment M_h was developed as an absolute sine function (Figure 5), [7].

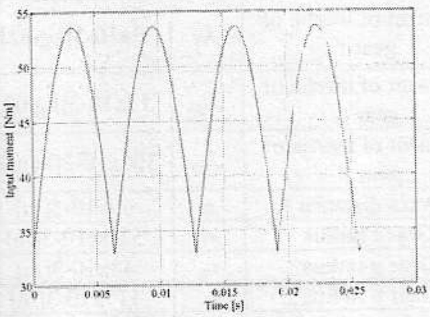

Fig. 5. Impulse moment, M_h

With the introduction of an impulse moment the simulation is started (Figure 5), which simulates the dynamic oscillation of the planetary gearbox system. As the output of diagrams are obtained: acceleration, velocity (Figure 6), displacement (Figure 7) and total displacement of all four degrees of freedom in the dynamic system. Acceleration along the directions of degrees of freedom, at the beginning of the first period of oscillation, is with large variations, while at the end of the second period of the oscillations is in calm variations. Final product of *MATLAB-SIMULINK* simulation are the dynamic forces (Figure8).

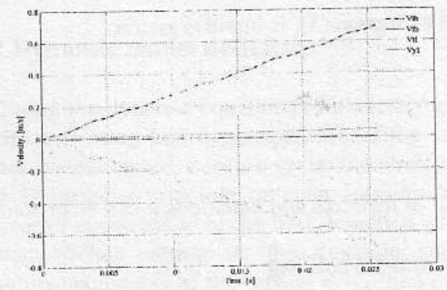

Fig. 6. Velocities of reducer elements at c_1=1,67x1010[N/m]; c_2=1,56x1010[N/m]; k_1=3200[Ns/m]; k_2 =2400[Ns/m]

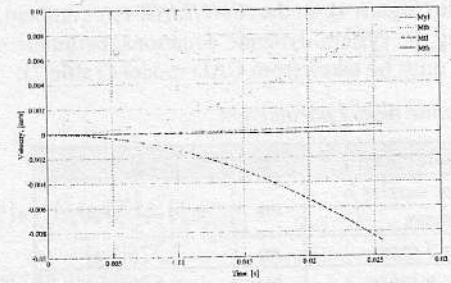

Fig. 7. Movements of reducer elements at c_1=1,67x1010[N/m]; c_2=1,56x1010[N/m]; k_1=3200[Ns/m]; k_2 =2400[Ns/m]

Calculation of dynamic forces has been performed according to formula:

$$F_{dyn} = c \cdot q + k \cdot \dot{q} \tag{10}$$

Dynamic force, which is calculated according to formula (9), presents complete dynamic force which is acting at particular moment at element of reducer. From figure 8 it can be seen that oscillations of dynamic force its far less than oscilations on velocities.

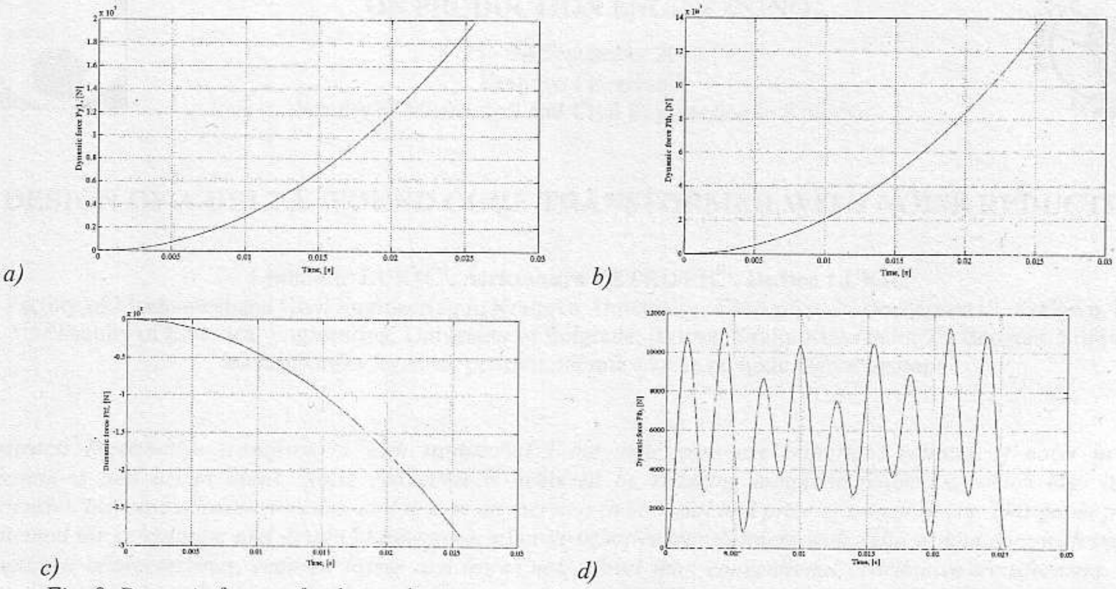

Fig. 8. Dynamic forces of reducer elements at c_1=1,67x1010[N/m]; c_2=1,56x1010[N/m]; k_1=3200[Ns/m]; k_2=2400[Ns/m], a) shaft of double satellite f-g; b) satelite carrier h; c) double satellite f-g and d) mov. central gear b

From figure 8 it can be seen that the bigest dynamic force acts at double satellite f-g. Double satellite only has the negative dynamic, because the direction of force depends of the direction of movement and velocity. The smallest dynamic force acts on movable centra gear b, but that dynamic force has the greatest oscillations (Figure 8d).

4. CONCLUSION

When the simulation has been completed, it can be concluded that the most critical is the first period of oscillation. After the first period of oscillation, the dynamic oscillations of the dynamical system are calming. This rule applies to the dynamic force, and acceleration, and velocity, and displacement. Research of the dynamics of planetary gear from this point of view can greatly help to reduce vibration at startup of planetary reducer. Further research on this issue are available on a theoretical and analytical and experimental design. At the level of theoretical screening can be done at: improving the model, the dynamic force that involves measuring of the dynamic force in real planetary gear unit, the determination of stiffness through simulation and many other parameters and their causes. On the experimental design can be made and measured values in order of confirmation of results obtained by simulation.

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