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DYNAMIC MODEL OF CYCLOIDAL SPEED REDUCER

ABSTRACT: Cycloidal speed reducer is a very complex mechanical system and for studying its dynamic behaviour must be considered all its specifics. On the level of internal dynamic forces very big impact have: manufacturing errors of cycloid disc and other elements of reducer, assembly errors, unequal load distribution, ...
On the basis of the models of external and internal gear trains, a dynamic model of a single - stage cycloidal speed reducer is developed in the paper. It is assumed that the values of the stiffness are constant and the excitation force time-variable function. The values of displacement, velocity, and dynamic force for single meshing are presented in this paper. The greatest impact on the dynamic force value has the coefficient of damping and coefficient of stiffness between cycloid disc and the ring gear.

KEYWORDS: cycloidal speed reducer, cycloid disc, dynamic model, dynamic force

INTRODUCTION

Thanks to the application of internal spur gears, planetary gearboxes are characterized by extreme compactness of the structure, high transmission ratios, as well as a good balance of the dynamic forces. Consequences of the smaller gear dimensions used in planetary speed reducer are mainly smaller rotational speeds and the sliding and rolling speed on the teeth. As a result, the dynamic loads and vibrations in the elements of the planetary gear trains are less. The sliding and rolling speeds on the teeth in the planetary gear are lower by approximately 30% to 40% compared to conventional speed reducer with parallel axes. Also, it should be noted significantly less rotating mass (up 75%), lower moment of inertia, the less impact force when starting and stopping gear, [4, 17].

Cycloidal speed reducer belongs to a group of modern planetary gear. Thanks to its good working characteristics (large gear ratio, long and reliable service life, compact design, high efficiency, low vibration, low noise, ...), cycloidal speed reducers have very widely application in modern industry.

In order to describe the dynamic behaviour of the cycloidal speed reducer, the first were analysed well known dynamic models of planetary gears classical concepts. The dynamic models of planetary speed reducers are presented in papers [9, 10]. The same author (*Kahraman*) with a group of co-authors analysed the influence of the ring gear rim thickness on planetary gear set behaviour, [11, 12]. Determination of gear mesh stiffness and internal dynamic forces at planetary gear drives is presented in papers [1, 2, 3, 15]. The elasto-dynamic model of internal

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gear planetary transmissions is described in paper [19]. Determination of planetary gear natural frequencies and vibration modes is presented in papers [7, 14].

In the papers [5, 13, 17] the various procedures for calculating of the cycloidal speed reducer efficiency have presented. Determination of stress and strain state of cycloidal speed reducer elements using the FEM in static and dynamic conditions is presented in papers [6, 8, 18]. The procedure for torsion stiffness calculation of cycloidal speed reducer is defined in paper [16].

A dynamic model of a single - stage cycloidal speed reducer is developed in this paper.

DYNAMIC MODEL OF ONE - STAGE CYCLOIDAL SPEED REDUCER

The main generators of internal dynamic forces in gear drives are: changes of deformations in meshing process, collisions of teeth in the meshing process, profile shape deviations, tooth wear, etc. Cycloidal speed reducer is a very complex mechanical system and for studying its dynamic behaviour it is necessary to take into accounts all of its peculiarities. The greatest impact on the internal dynamic forces at cycloidal speed reducer have: errors that occur during the production of cycloid disc's teeth, as well as other elements of cycloidal speed reducer, unequal distribution of load at cycloid disc teeth as well as at stationary central gear rollers and output rollers, elastic deformations of case and other elements,…

On the basis of a very careful analysis of dynamic models of external involute tooth gearing, as well as dynamic model of planetary gear trains, the dynamic model of single - stage cycloidal speed reducer was developed in this paper. This model is presented in Figure 1.

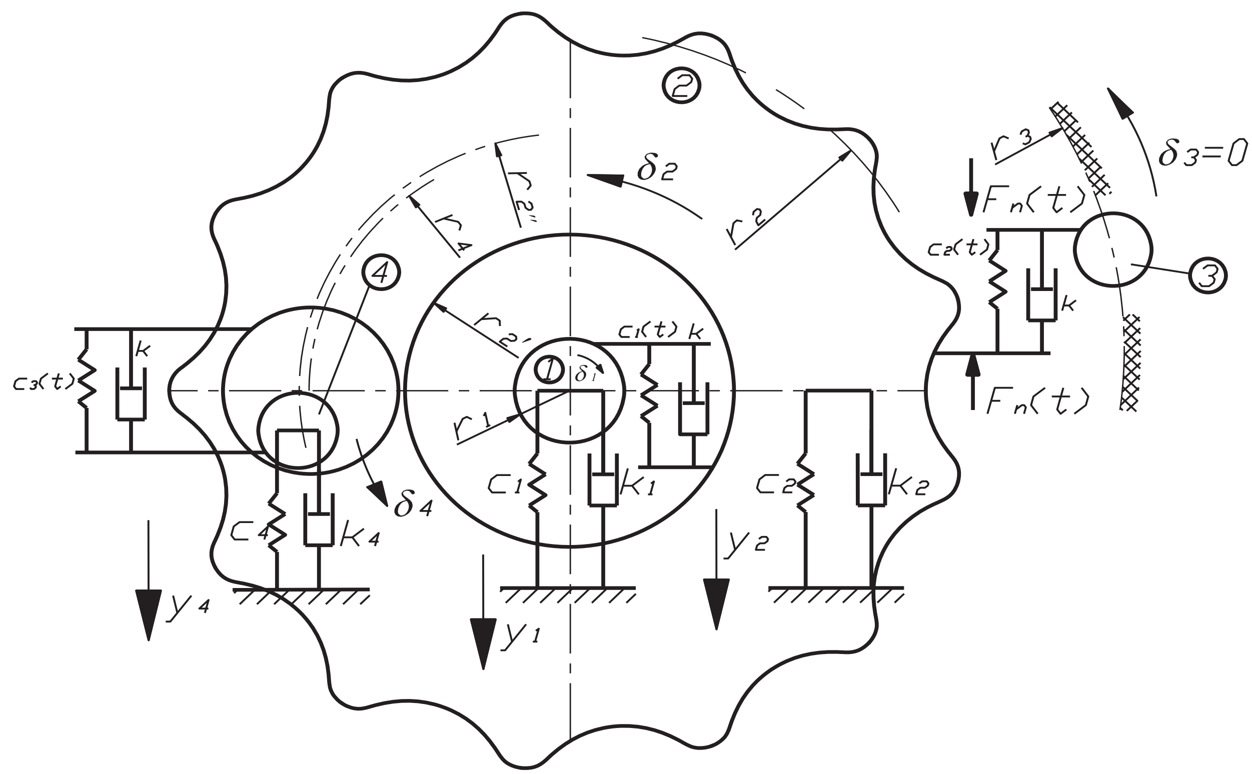

Figure 1 Dynamic model of a single-stage cycloidal speed reducer (1-input shaft with the eccentric cam, 2-cycloid disc, 3-stationary central gear roller, output roller)

The geometric dimensions from figure 1 are:

r_1 – external radius of the eccentric cam,
r_2 – radius of the pitch circle of the cycloid disc,
r_2' – radius of the central hole of the cycloid disc,
r_2'' – radius of the cycloid disc circle with the holes for the output rollers,
r_3 – radius of the pitch circle of the stationary central gear,
r_4 – radius of the output shaft flange with the holes for corresponding rollers.

The cycloidal speed reducer elements are connected in their supports in the following way:

- *Input shaft with eccentric cam*: elastic connect of the stiffness c_1 and a damper with the coefficient of damping k_1,
- *Cycloid disc:* elastic connection of the stiffness c_2 and a damper with coefficient of damping k_2,
- *Stationary central gear roller:* elastic connection of the stiffness c_3 and a damper with the coefficient of damping k_3,
- *Output roller:* elastic connection of the stiffness c_4 and a damper with the coefficient of damping k_4.

The contacts between the corresponding elements of the cycloidal speed reducer are described in the following way:

- *Input shaft with eccentric cam – cycloid disc:* elastic connection of the variable stiffness c_1(t) and damper with the coefficient of damping k,
- *Cycloid disc – stationary central gear roller:* elastic connection of the variable stiffness c_2(t) and a damper with the coefficient of damping k,
- *Cycloid disc – output roller:* elastic connection with the variable coefficient of stiffness c_3(t) and a damper with the coefficient of damping k.

The excitation force F_n(t) occurs in the contact between the cycloid disc tooth and the stationary central gear roller. The excitation force is calculated using the following expression, [1, 2, 3]:

$$F_n(t) = c \cdot w(t) \cdot b \tag{1}$$

where:

c - connected teeth stiffness,
w(t) - deformation of the cycloid disc tooth,
b - cycloid disc width.

Forasmuch as deformation`s magnitude periodical time`s function, the same case is for excitation force. The total displacement *x* in the contact between the cycloid disc tooth and the stationary central gear roller can be expressed as follows:

$$x = r_2 \cdot \delta_2 - y_2 \tag{2}$$

The result of the excitation force action is a dynamic force which loads the cycloid disc teeth and the stationary central gear rollers:

$$F_e = c \cdot x + b \cdot \dot{x} \tag{3}$$

The kinetic system energy is:

$$E_k = \frac{1}{2} m_1 \cdot \dot{y}_1^2 + \frac{1}{2} J_1 \cdot \dot{\delta}_1^2 + \frac{1}{2} m_2 \cdot \dot{y}_2^2 + \frac{1}{2} J_2 \cdot \dot{\delta}_2^2 + \frac{1}{2} m_4 \cdot \dot{y}_4^2 + \frac{1}{2} J_4 \cdot \dot{\delta}_4^2 \tag{4}$$

The potential system energy is:

$$E_p = \frac{1}{2} c_1 \cdot y_1^2 + \frac{1}{2} c_2 \cdot y_2^2 + \frac{1}{2} c_4 \cdot y_4^2 + \frac{1}{2} c_1(t) \cdot \left[\left(y_2 - r_2^{`} \cdot \delta_2 \right) - \left(y_1 + r_1 \cdot \delta_1 \right) \right]^2 + \frac{1}{2} c_2(t) \cdot \left[\left(y_2 - r_2 \cdot \delta_2 \right) \right]^2 + \\ + \frac{1}{2} c_3(t) \cdot \left[\left(y_4 + r_4 \cdot \delta_4 \right) - \left(y_2 + r_2^{``} \cdot \delta_2 \right) \right]^2 \tag{5}$$

The dissipation system function is:

$$\Phi = \frac{1}{2} k_1 \cdot \dot{y}_1^2 + \frac{1}{2} k_2 \cdot \dot{y}_2^2 + \frac{1}{2} k_4 \cdot \dot{y}_4^2 + \frac{1}{2} k \cdot \left[\left(\dot{y}_2 - r_2^{`} \cdot \dot{\delta}_2 \right) - \left(\dot{y}_1 + r_1 \cdot \dot{\delta}_1 \right) \right]^2 + \frac{1}{2} k \cdot \left[\left(\dot{y}_2 - r_2 \cdot \dot{\delta}_2 \right) \right]^2 + \\ + \frac{1}{2} k \cdot \left[\left(\dot{y}_4 + r_4 \cdot \dot{\delta}_4 \right) - \left(\dot{y}_2 + r_2^{``} \cdot \dot{\delta}_2 \right) \right]^2 \tag{6}$$

The virtual work of the conservative forces is:

$$\partial A = F_n(t) \cdot r_2 \cdot \partial \delta_2 \tag{7}$$

The conservative force is:

$$Q_{\delta_2} = F_n(t) \cdot r_2 \tag{8}$$

The signs m_1, m_2 and m_3 stand for masses of correspondent elements, and J_1, J_2 and J_3 stand for a rectangular moment of inertia of the same elements.

Stiffnesses in the corresponding contacts and the dynamic force are variable time functions. However, in order to enable the analysis of influence of deviation of geometric dimensions, it is adopted that the mentioned stiffnesses are constant and equal to some average values, and the excitation force stay a variable time function.

$$\begin{aligned} c_1(t) &= c_{01} = const. \\ c_2(t) &= c_{02} = const. \\ c_3(t) &= c_{03} = const. \end{aligned} \tag{9}$$

The differential equation of the system motion in matrix form is:

$$\begin{bmatrix} J_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_4 \end{bmatrix} \begin{Bmatrix} \ddot{\delta}_1 \\ \ddot{y}_1 \\ \ddot{\delta}_2 \\ \ddot{y}_2 \\ \ddot{\delta}_4 \\ \ddot{y}_4 \end{Bmatrix} + \begin{bmatrix} k \cdot r_1^2 & k \cdot r_1 & k \cdot r_1 \cdot r_2^{\prime} & -k \cdot r_1 & 0 & 0 \\ k \cdot r_1 & k_1 + k & k \cdot r_2^{\prime} & -k & 0 & 0 \\ k \cdot r_1 \cdot r_2^{\prime} & k \cdot r_2^{\prime} & k \cdot \left[\left(r_2^{\prime}\right)^2 + \left(r_2^{\prime\prime}\right)^2 + r_2^2 \right] & k \cdot \left(-r_2^{\prime} + r_2^{\prime\prime} - r_2\right) & -k \cdot r_4 \cdot r_2^{\prime\prime} & -k \cdot r_2^{\prime\prime} \\ k \cdot r_1 & k & k \cdot \left(r_2^{\prime} + r_2^{\prime\prime} + r_2\right) & k_2 - k & k \cdot r_4 & -k \\ 0 & 0 & -k \cdot r_4 \cdot r_2^{\prime\prime} & -k \cdot r_4 & k \cdot r_4^2 & k \cdot r_4 \\ 0 & 0 & -k \cdot r_2^{\prime\prime} & -k & k \cdot r_4 & k + k_4 \end{bmatrix} \begin{Bmatrix} \dot{\delta}_1 \\ \dot{y}_1 \\ \dot{\delta}_2 \\ \dot{y}_2 \\ \dot{\delta}_4 \\ \dot{y}_4 \end{Bmatrix} +$$

$$+ \begin{bmatrix} c_{01} \cdot r_1^2 & c_{01} \cdot r_1 & c_{01} \cdot r_1 \cdot r_2^{\prime} & -c_{01} \cdot r_1 & 0 & 0 \\ c_{01} \cdot r_1 & c_1 + c_{01} & c_{01} \cdot r_2^{\prime} & -c_{01} & 0 & 0 \\ c_{01} \cdot r_1 \cdot r_2^{\prime} & c_{01} \cdot r_2^{\prime} & c_{01} \cdot \left(r_2^{\prime}\right)^2 + c_{02} \cdot r_2^2 + c_{03} \cdot \left(r_2^{\prime\prime}\right)^2 & -c_{01} \cdot r_2^{\prime} + c_{03} \cdot r_2^{\prime\prime} - c_{02} \cdot r_2 & -c_{03} \cdot r_4 \cdot r_2^{\prime\prime} & -c_{03} \cdot r_2^{\prime\prime} \\ c_{01} \cdot r_1 & c_{01} & c_{01} \cdot r_2^{\prime} + c_{03} \cdot r_2^{\prime\prime} + c_{02} \cdot r_2 & -c_{01} + c_2 - c_{02} + c_{03} & -c_{03} \cdot r_4 & -c_{03} \\ 0 & 0 & -c_{03} \cdot r_4 \cdot r_2^{\prime\prime} & -c_{03} \cdot r_4 & c_{03} \cdot r_4^2 & c_{03} \cdot r_4 \\ 0 & 0 & -c_{03} \cdot r_2^{\prime\prime} & -c_{03} & c_{03} \cdot r_4 & c_{03} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ y_1 \\ \delta_2 \\ y_2 \\ \delta_4 \\ y_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F_n(t) \cdot r_2 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

DYNAMIC BEHAVIOUR OF ONE CONCRETE ONE - STAGE CYCLOIDAL SPEED REDUCER

The differential equation of the system motion was solved using MATLAB - SIMULINK for one concrete one - stage cycloidal speed reducer with following working parameters:

- Input power: P_{in} = 0,25 kW;
- Input rotations per minute: n_{in} = 1390 min^{-1};
- Reduction ratio: u = 11.

The input system parameter is the excitation force F_n(t), and the output parameters are: total displacement (x) in the point of contact between a cycloid disc tooth and a stationary central gear roller, the adequate velocity ($\dot{x}$) and the dynamic force F_e.

The values of the coefficients of damping as well as adequate stiffnesses are adopted from literature [1, 2, 3, 15]: k = 5500 Ns/m, k_1 = 2700 Ns/m, k_2 = 200 Ns/m, k_3 = 1800 Ns/m, k_4 = 1000 Ns/m, c_{01} = 1,4·10^8 N/mm, c_{02} = 1,5·10^7 N/mm, c_{03} = 1,6·10^8 N/mm, c_1 = 1,9·10^9 N/mm, c_2 = 1,7·10^8 N/mm, c_3 = 2,1·10^9 N/mm, c_4 = 1,6·10^8 N/mm.

The masses of elements, the rectangular moments of inertia and the adequate radii have the following values: m_1 = 0,17 kg, m_2 = 0,74 kg, m_4 = 0,70 kg, J_1 = 26 kgmm^2, J_2 =1422,3 kgmm^2, J_4 = 762,5 kgmm^2, r_1 = 17,5 mm, r_2 = 62 mm, $r_2^{\prime} = 20\,mm$, $r_2^{\prime\prime} = 39\,mm$, r_4 = 39 mm.

Diagram of excitation force is presented in Figure 2, and the results of calculation for total displacement, velocity and dynamic force are presented in Figures 3, 4 and 5.

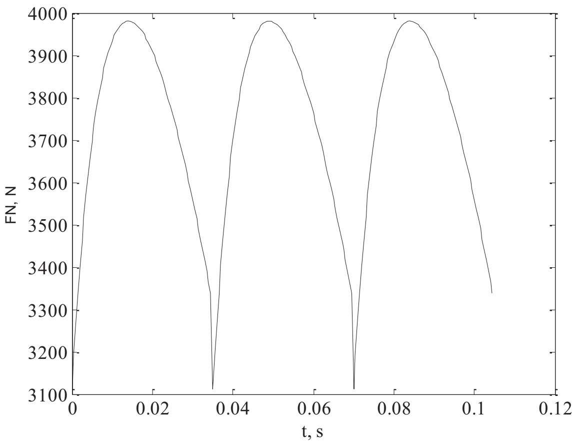

Figure 2 Diagram of excitation force Fn(t)

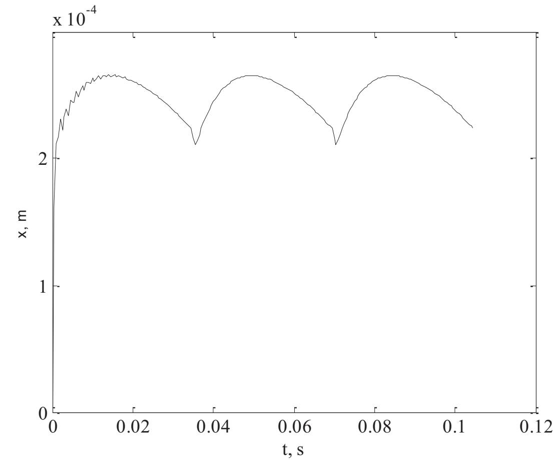

Figure 3 Diagram of the total displacement **x**

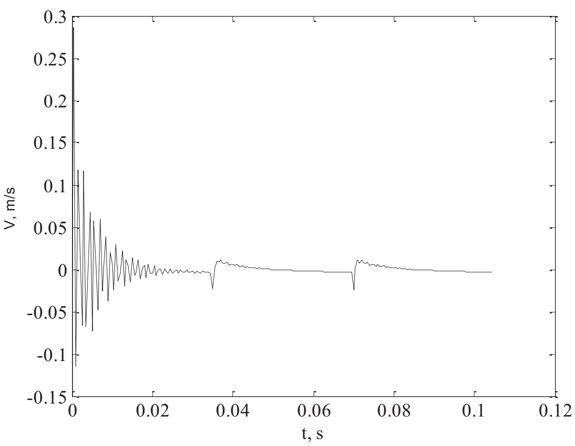

Figure 4 Diagram of the velocity $\dot{x}$

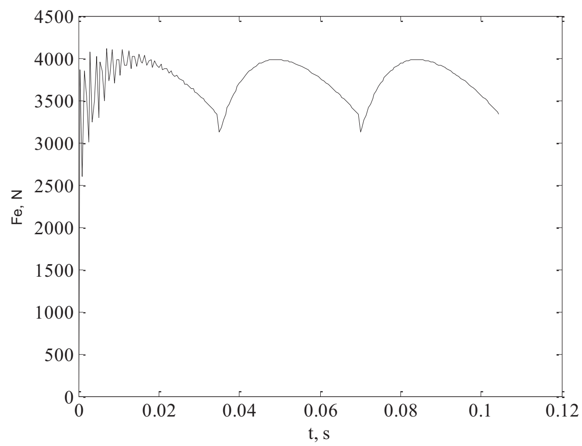

Figure 5 Diagram of the dynamic force Fe

The presented results refer to the most critical case - the case of single meshing (one cycloid disc tooth and one stationary central gear roller are in the contact).

CONCLUSIONS

When the values for the coefficients of damping and adequate stiffnesses are selected from a recommended range, vibrations with damping in time disappear very quickly and only forced vibrations remain.

Forasmuch as the excitation force is a periodical time function, the same goes for the other parameters (total displacement, velocity and the dynamic force).

The biggest influence on dynamic behaviour of one - stage cycloid speed reducer has the coefficient of the damping during the contact between the cycloid disc tooth and the stationary central gear roller.

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