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A Fuzzy Multicriteria Method for E-learning Quality Evaluation*

MILADIN STEFANOVIC[1], DANIJELA TADIC[1], SLAVKO ARSOVSKI[1], ZORA ARSOVSKI[2], ALEKSANDAR ALEKSIC[1]
[1] *Faculty of Mechanical Engineering, University of Kragujevac, Sestre Janjic 6, 34000 Kragujevac, Serbia. E-mail: miladin@kg.ac.rs galovic@kg.ac.rs cqm@kg.ac.rs aaleksic@kg.ac.rs*
[2] *Faculty of Economics, University of Kragujevac, Djure Pucara bb. 34000 Kragujevac, Serbia. E-mail: zora@kg.ac.rs*

Quality is a phenomenon which is in the core of any type of e-learning development. There is a demand for e-training systems evaluation in order to benchmark them and improve factors that will lead to an effective high quality e-training system. This paper introduces a multi-criteria method for e-learning quality evaluation. To evaluate an e-learning system, we decomposed the e-learning process into sub-processes and activities. The quality of each activity was evaluated both numerically and using linguistic expressions. Following this methodology a number of issues were addressed, such as trainee satisfaction, relative importance of decision criteria, measurement and analysis of trainee on-the-job performances, improvement of the relationships with management and other stakeholders. Quantitative presentations of the e-training process quality enable comparison of different e-training processes and their ranking according to the level of quality.

Keywords: e-training process; fuzzy data; evaluation

1. INTRODUCTION

QUALITY IMPROVEMENT—an explicit goal of many European resolutions—may be seen as a cornerstone of a programme for further development of e-learning. There is no doubt that quality is the most critical factor determining future of e-learning. There are numbers of methods and instruments in the field of quality [1]: quality management, quality assessment, evaluation as well as other approaches to e-learning. Different authors have considered issues such as quality, evaluation and e-learning outcomes quality [2, 3, 4]. Quality in e-learning has two completely different meanings. The first is associated with an increase of educational opportunities and development of information society and economics. The second is associated with improvement of the quality of e-learning itself. In this paper we address quality of e-learning itself and the need to evaluate and measure the quality of e-learning processes. It is necessary to be able to compare and contrast the level of quality of different e-learning processes as well as the quality of traditional training and e-learning and to evaluate their advantages [5].

In the evaluation of e-training, it is important to define necessary quality aspects as well as parts of interests. The model for assessing quality in e-learning—E-learning quality (ELQ)—comprises ten quality aspects which are central to such assessments [6]:

- material/content;
- structure/virtual environment;
- communication, cooperation and interactivity;
- student assessment;
- flexibility and adaptability;
- support (student and staff);
- staff qualifications and experience;
- vision and institutional leadership;
- resource allocation and the holistic and process aspect.

In evaluation of training it is necessary to define parts of interests, isolate typical training operational elements: training programme as a whole, training projects, training staff, training manuals and materials, curriculum, individual courses, students, exercises and tests and special team-based training.

With genuine problems, such as we have been considering, there are many imprecise data and it is difficult to provide quality analysis. A fuzzy multi-criteria evaluation is selected to provide evaluation because fuzzy system models [7]:

- are easy to understand;
- flexible; can capture most nonlinear functions of arbitrary complexity;
- tolerant of imprecise data;
- can be built on top of the experience of experts;
- can be blended with conventional control techniques;
- are based on natural languages.

In this paper we will give a proposal for evaluation of e-learning process (with core sub-processes and

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activities) as a whole using fuzzy multicriteria evaluation (and to include both crisp and uncertain values). The multi-criteria evaluation of the web-based, e-learning system and training systems have been addressed by different authors [8, 9], but they were concerned just with learner satisfaction. Since training refers to the acquisition of knowledge, skills and competencies as a result of the teaching of vocational or practical skills and knowledge related to specific useful competencies, the suggested approach will include other important issues such as measurement and analysis of the trainees' on-the-job performances and improvement of the relationships with management and other stakeholders.

To accomplish this task we presuppose that:

- each activity and each sub-process could be considered separately;
- the number of activities and sub-processes is finite;
- the evaluation of quality of e-learning process is a multi-criteria optimization task;
- solution of the treated problem can be found by using real numbers.

The main idea of this paper is to provide a method for evaluation and ranking of different e-learning processes.

2. QUALITY OF E-TRAINING PROCESS

The e-training systems have emerged as a new means of skill training and knowledge acquisition, encouraging both education and industry to invest resources in their adoption [9]. One of the important indicators of training systems is quality. Structure of training process management has a hierarchical structure [10, 11]. The goal of management on the first, highest level is to achieve the highest possible quality of training in the shortest period of time. The fulfilment of defined goals could be achieved through fulfilment of goals on the lower hierarchical levels. Hierarchical structure of e-learning quality goals is presented in Fig. 1. The first step in the evaluation of e-learning or e-training process quality is decomposition of the process into sub-processes (simpler and less complex), further more sub-processes are decomposed into activities.

According to Fig. 1, an e-training process could be decomposed into the following sub-processes:

- definition of training needs,
- design and planning of training,
- providing training;
- evaluation of training outcomes.

In order to evaluate quality of the complete e-training process, it is necessary to evaluate the quality of each sub-process. Further, each sub-process could be decomposed into activities and the evaluation of each sub-process quality is the function of specific activities quality.

We used Qe to mark the quality of e-training process, and we assumed that an e-training process could be decomposed into N different sub-processes. The quality of each sub-process n (n = 1, . . . , N) is marked as QP_n (n = 1 , . . . , n, . . . , N).

Generally speaking, each sub-process n (n = 1, . . . , N) is further decomposed into activities. Total number of the activities of sub-processes n (n = 1, . . . , N) is marked as J^n $(n = 1, \ldots, N)$. QP_j^n $(j = 1, \ldots, J^n;\ n = 1, \ldots, N)$ marks the quality of (j^{th}) activity $(j = 1, \ldots, J^n)$ which is contained in a sub-process n (n = 1, . . . , N).

A mathematical model for the evaluation of e-training process quality is developed within the following assumptions:

1. The quality of the n-th sub-process (n = 1, . . . , N) of e-learning process, as well as the quality of activity j (j = 1, . . . , Jn) could be described on crisp or imprecise variables.
2. Cardinal values are obtained by measuring. Measurement is formally represented by the union of indexes: $\mu = \{1, \ldots, m, \ldots, M\}$.

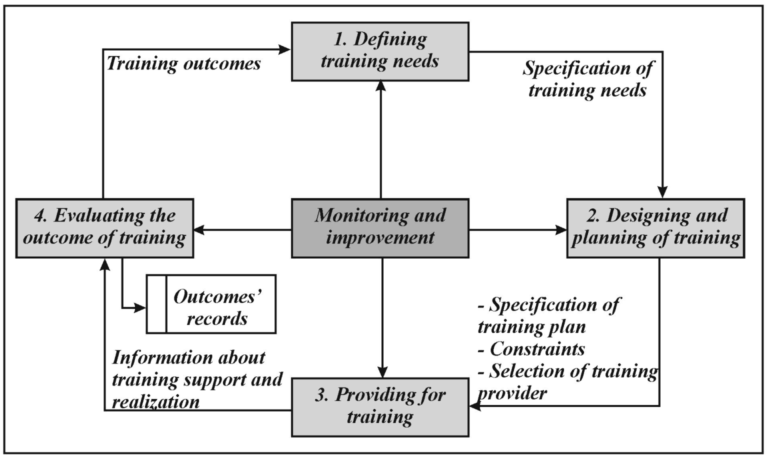

Fig. 1. Hierarchal structure of e-training process management.

where:

- m stands for the m-th measurement in a sub-process n (n = 1, . . . , N), or activity j (j = 1, . . . , Jn)
- M is the total number of measurements; this number is defined by managers of the project based on their experience.

3. The evaluation qualities of sub-processes n (n = 1, . . . , N), or activities, j (j = 1, . . . , Jn) which could be measured, are described by linguistic expressions. A set of linguistic expressions for description of e-learning process components quality could be defined.

There is the assumption that E experts, independent of each other, evaluate the quality of each component of e-learning process. Experts are formally represented with a number of indexes: $\varepsilon = \{1, \ldots, e, \ldots, E\}$

where:

- e stands for an expert who takes part in the evaluation of the level of e-learning process quality,
- Total number of experts is marked with E.

4. As is known, attributes that describe sub-processes and activities of an e-learning process can be either of benefit or cost type. Yoon and Hwang [12] define the two criteria types in the following way:

- Benefit of sub-processes and activities of an e-learning process are positively correlated with utility or the preferences of a decision maker, which means: if their values increase, so does the utility for a decision maker,
- Cost of sub-processes and activities are negatively correlated with utility or the preferences of a decision maker, which means: if they increase, so does the utility for a decision maker.

In this paper we consider just one attribute of sub-processes and activities—quality. According to the classification given in [12], quality is of benefit-type.

5. In general, the relative importance of each sub-process n(n = 1, . . . , N), as well as the relative importance of each activity j (j = 1, . . . , Jn) is different. There are a number of techniques to assess their weight [13]. This paper illustrates the use of comparison pair matrix of the relative sub-process importance and comparison pair matrix of the activities relative importance. It is assumed that elements of this matrix are linguistic expressions.

3. MODELLING OF UNCERTAINTIES

The following uncertainties are treated and modelled on the basis of the fuzzy set theory [14]: sub-processes and activities of e-learning process values and their weights. All uncertainties which appear in the considered example are described with discrete fuzzy numbers. We used discrete membership function in order to avoid analytical considerations and to apply a 'digital way of thinking' [15].

3.1 Modelling of sub-processes and activities quality within e-learning process

The quality of a number of processes, sub-processes and activities of an e-learning process could be described with linguistic expressions. For example, the following linguistic expressions for evaluation of e-learning process quality and their components can be used: 'unsatisfactory', 'poor', 'satisfactory', 'good' and 'excellent'.

In this paper, these linguistic expressions are modelled by discrete fuzzy numbers. The membership functions of the corresponding discrete fuzzy numbers are given on a scale interval [1,9]. Value 1 marks the lowest quality of the considered component (sub-process, activity), whereas value 9 denotes the highest one. The membership values are obtained by the subjective judgments of experts. In this paper, each considered discrete fuzzy number has the discrimination step $\Delta = 0.25$.

The discrete fuzzy numbers describing the values of sub-processes and activities in e-learning processes are given in the following way:

$$\text{'unsatisfactory'} = \tilde{L}_1 = \{(1,1),(1.5,0.75),(2,0.5),(2.5,0.25)\} \tag{1}$$

$$\text{'poor'} = \tilde{L}_2 = \left\{ \begin{array}{l} (1.5,0.25),(2,0.5),(2.5,0.75),(3,1), \\ (3.5,0.75),(4,0.5),(4.5,0.25) \end{array} \right\} \tag{2}$$

$$\text{'satisfactory'} = \tilde{L}_3 = \left\{ \begin{array}{l} (3.5,0.25),(4,0.5),(4.5,0.75),(5,1), \\ (5.5,0.75),(6,0.5),(6.5,0.25) \end{array} \right\} \tag{3}$$

$$\text{'good'} = \tilde{L}_4 = \left\{ \begin{array}{l} (5.5,0.25),(6,0.5),(6.5,0.75),(7,1), \\ (7.5,0.75),(8,0.5),(8.5,0.25) \end{array} \right\} \tag{4}$$

$$\text{'excellent'} = \tilde{L}_5 = \{(9,1),(8.5,0.75),(8,0.5),(7.5,0.25)\} \tag{5}$$

3.2 Calculation of sub-processes and activities weights within e-learning process

All the sub-processes and activities in an e-learning process usually do not have the same relative importance. Also, they can be considered as unchangeable during the considered period of time. They involve a high degree of subjective judgment and individual preferences of decision makers. We think that the judgment of each pair of treated sub-processes and activities in e-learning processes best suits human-decision nature (with analogy to Analytic Hierarchy Process—AHP method [17]. It appears that the weight determination of sub-processes and activities in e-learning processes are more reliable when gained by using pairwise comparisons than when obtained directly, because it is easier to make a comparison between two sub-processes, i.e. two activities within an e-learning process than to make an overall weight assignment. Also, decision makers express their judgements far better by using linguistic expressions than by representing them in terms of precise numbers. It feels more confident to give interval judgments than fixed value judgments. According to the introduced assumptions, the elements of these matrixes are defined as the relatively important sub-process n over the sub-process n' ($n, n' = 1, \ldots, N$; $n \neq n'$), i.e. the relative importance of the activity j over j' ($j, j' = 1, \ldots, J^n$; $j \neq j'$).

In this paper, it is assumed that the relative importance of a pair of sub-processes (n, n', $n = 1, \ldots, N$; $n \neq n'$) as well as the relative importance of a pair of activities (j, j'; $j = 1, \ldots, J^n$; $n = 1, \ldots, N$; $j \neq j'$) is subjectively assessed by a weighting coefficient, which is supposed to be a vague linguistic expression. A discrete fuzzy number is associated with each vague linguistic expression. These linguistic expressions are modeled by discrete fuzzy numbers $\tilde{X}_1$, $\tilde{X}_2$, $\tilde{X}_3$, respectively. These discrete fuzzy numbers are defined on the almost standard integer scale [1,9]. Value 1 defines that relative importance of the sub-process n ($n = 1, \ldots, N$), or the activityj, ($j = 1, \ldots, J^n$) versus n' ($n' = 1, \ldots, N$), or activity j', ($j' = 1, \ldots, J^n$) is the lowest and number 9 denotes the highest importance of the sub-process n ($n = 1, \ldots, N$), or activity j, ($j = 1, \ldots, J^n$) versus n' ($n' = 1, \ldots, N$), or activity j', ($j' = 1^n, \ldots, J^n$).

In this paper, each considered discrete fuzzy number has the discretization step $\Delta = 0.25$. The values of membership functions are equal: $\alpha_1 = 0.25$, $\alpha_3 = 0.5$, $\alpha_3 = 0.75$, $\alpha_4 = 1$.

According to these assumptions, it means that:

$$\text{'less important'} = \tilde{X}_1 = \{(1,1), (1.5, 0.75), (2, 0.5), (2.5, 0.25)\} \quad (6)$$

$$\text{'important'} = \tilde{X}_2 = \left\{ \begin{array}{l} (3.5, 0.5), (4, 0.5), (4.5, 0.75), (5, 1), \\ (5.5, 0.75), (6, 0.5), (6.5, 0.25) \end{array} \right\} \quad (7)$$

$$\text{'very important'} = \tilde{X}_3 = \{(9,1), (8.5, 0.75), (8, 0.5), (7.5, 0.25)\} \quad (8)$$

Calculation of the sub-process or activities weight vector is based on the concept of equal possibilities [13]. For each level of membership function of treated discrete fuzzy numbers $\tilde{X}_1$, $\tilde{X}_2$, $\tilde{X}_3$ the comparison pair matrix of relative sub-processes or activities importance is formed.

The sub-processes or activity weighted vector for level α is calculated by applying the eigenvector method [16].

4. EVALUATION OF E-LEARNING PROCESS QUALITY

The algorithm for evaluation of an e-learning process has the following seven steps.

4.1 Step 1

In this step we define the importance of the influence e-learning sub-processes and activities have on the evaluation of e-learning sub-processes quality and activities quality. For each level of membership functions of discrete fuzzy numbers $\tilde{X}_1$, $\tilde{X}_2$, $\tilde{X}_3$, we define a pairwise comparison matrix of the sub-processes relaive importance, $(\tilde{w})^\alpha$ and pairwise comparison matrix of activities relative importance, $(\tilde{w}^n)^\alpha$ in e-learning processes. By applying engine vector method [17], for each considered α level of membership functions, weights vector of sub-processes and weights vector of activities in e-learning processes are calculated. On the other hand, we calculate weights vector of sub-processes, $\tilde{w}$ and weights vector of activities, $\tilde{w}^n$ in e-learning processes. The elements of these two vectors are discrete fuzzy numbers:

(a) the vector of importance of sub-processes of e-learning process is:

$$\tilde{w} = [\tilde{w}_1, \ldots, \tilde{w}_n, \ldots, \tilde{w}_N], n = 1, \ldots, N \quad (9)$$

(b) the vector of importance of activities n is:

$$\tilde{w}^n = \left[\tilde{w}_1^n, \ldots, \tilde{w}_j^n, \ldots, \tilde{w}_{J^n}^n\right], \quad (n = 1, \ldots, N; j = 1, \ldots, J^n) \quad (10)$$

4.2 Step 2

The problem of normalization of cardinal values through which the various sub-processes and activities dimensions are transformed into non-dimensional criteria. The normalization is necessary for different values of these variables to become comparable.

The procedure of optimization criteria normalization is conducted. There are many methods of normalization in the literature [16]. In this paper, linear normalization is used:

(a) for a benefit-type:

$$(QP_j^n)'_m = \frac{(QP_j^n)_m}{\left(QP_j^n\right)^{max}} \quad (11)$$

for activity $j (j = 1, \ldots, J^n)$ and measurement m ($m = 1, \ldots, M$)

$$(QP_n)'_m = \frac{(QP_n)_m}{(QP_n)^{max}} \quad (12)$$

for sub-process $(n = 1, \ldots, N)$ and measurement m $(m = 1, \ldots, M)$

(b) for a cost-type:

$$(QP_j^n)'_m = 1 - \frac{(QP_j^n)_m - \left(QP_j^n\right)^{min}}{\left(QP_j^n\right)^{max}} \quad (13)$$

for activity j $(j = 1, \ldots, J^n)$ and measurement m $(m = 1, \ldots, M)$

$$(QP_n)'_m = 1 - \frac{(QP_n)_m - (QP_n)^{min}}{(QP_n)^{max}} \quad (14)$$

for sub-process n $(n = 1, \ldots, N)$ and measurement m $(m = 1, \ldots, M)$

where: $(QP_j^n)^{max} = \max_m (QP_j^n)_m$ and $(QP_j^n)^{min} = \min_m (QP_j^n)_m$

4.3 Step 3

Transform all the quality values of linguistic sub-processes and activities into degrees of belief $(b_n)_e$, i.e. $(b_j^n)_e$ $(n = 1, \ldots, N;\ j = 1, \ldots, J^n;\ e = 1, \ldots, E)$ expressed on a common scale [0,1] by applying a fuzzy set comparison method [12]:

(a) for a benefit type sub-process and activity, n $(n = 1, \ldots, N)$, i.e. j $(j = 1, \ldots, J^n)$, find the degree of belief $(b_n)_e$, i.e. $(b_j^n)_e$ $(n = 1, \ldots, N;\ j = 1, \ldots, J^n;\ e = 1, \ldots, E)$ that $(b_n)_e$, i.e. $(b_j^n)_e$ $(n = 1, \ldots, N;\ j = 1, \ldots, J^n;\ e = 1, \ldots, E)$ is greater than or equal to all other $(b_n)_{e'}$, i.e. $(b_j^n)_{e'}$ $(n = 1, \ldots, N;\ j = 1, \ldots, J^n;\ e = 1, \ldots, E;\ e \neq e')$,

(b) for a cost type sub-process and activity, n $(n = 1, \ldots, N)$, i.e. j $(j = 1, \ldots, J^n)$, find the degree of belief $(b_n)_e$, i.e. $(b_j^n)_e$ $(n = 1, \ldots, N;\ j = 1, \ldots, J^n;\ e = 1, \ldots, E)$ that $(b_n)_e$, i.e. $(b_j^n)_e$ $(n = 1, \ldots, N;\ j = 1, \ldots, J^n;\ e = 1, \ldots, E)$ is less than or equal to all other $(b_n)_{e'}$, i.e. $(b_j^n)_{e'}$ $(n = 1, \ldots, N;\ j = 1, \ldots, J^n;\ e = 1, \ldots, E;\ e \neq e')$.

4.4 Step 4

The value of the activity quality, or the quality of sub-process which are accompanied with cardinal values, with respect to their relative importance, could be calculated according to following:

$$\text{(a)} \quad E\tilde{Q}P_j^n = \frac{1}{M} \sum_{m=1}^{M} \tilde{w}_j^n \cdot \left(QP_j^n\right)'_m \quad (15)$$

for activity j $(j = 1, \ldots, J^n)$; $m = 1, \ldots, M$

$$\text{(b1)} \quad E\tilde{Q}P_n = \frac{1}{J^n} \cdot \frac{1}{M} \cdot \sum_{j=1}^{J^n} \sum_{m=1}^{M} \tilde{w}_j^n \cdot \left(OP_j^n\right)'_m \quad (16)$$

for a sub-process n $(n = 1, \ldots, N)$ which consists of J^n activities

$$\text{(b2)} \quad E\tilde{Q}P_n = \frac{1}{M} \sum_{m=1}^{M} \tilde{w}_n \cdot (QP_n)'_m \quad (17)$$

for sub-process n $(n = 1, \ldots, N)$.

4.5 Step 5

Respecting the importance, value of activities quality, or sub-processes quality, all of which are described with linguistic expressions, the calculation follows:

$$\text{(a)} \quad E\tilde{Q}P_j^n = \frac{1}{E} \sum_{e=1}^{E} \tilde{w}_j^n \cdot \left(b_j^n\right)'_e \quad (18)$$

for the activity j $(j = 1, \ldots, J^n)$; $e = 1, \ldots, E$

$$\text{(b1)} \quad E\tilde{Q}P_n = \frac{1}{J^n} \cdot \frac{1}{E} \cdot \sum_{j=1}^{J^n} \sum_{e=1}^{E} \tilde{w}_j^n \cdot \left(b_j^n\right)'_e \quad (19)$$

for the sub-process n $(n = 1, \ldots, N)$ which consists of J^n activities

$$\text{(b2)} \quad E\tilde{Q}P_n = \frac{1}{E} \sum_{e=1}^{E} \tilde{w}_n \cdot (b_n)'_e \quad (20)$$

for the sub-process n $(n = 1, \ldots, N)$.

4.6 Step 6

The value of e-learning process quality is calculated according to:

$$E\tilde{Q}e = \frac{1}{N} \cdot \sum_{n=1}^{N} E\tilde{Q}P_n \quad (21)$$

4.7 Step 7

Values of the quality of activities, sub-processes and e-learning process are described by discrete fuzzy numbers. Representative scalars obtained by discrete fuzzy numbers, in this paper, are determined by applying the moment method [14].

5. ILLUSTRATIVE EXAMPLE

After decomposition of the training process into its sub-processes QP_1—Defining the training needs, QP_2—Designing and planning training, QP_3—Providing for training and QP_4—Evaluating the outcome of training, the selected sub-process will be further decomposed into activities [18, 19].

The sub-process 4 (Fig. 1), evaluating the training outcome, consists of four activities defined as (Fig. 2):

- QP_1^4—Measurement of the trainees' satisfaction;
- QP_2^4—Analysis of the training process;
- QP_3^4—Improvement of the relationships with management and other stakeholders;
- QP_4^4—Measurement and analysis of the trainees' on-the-job performances.

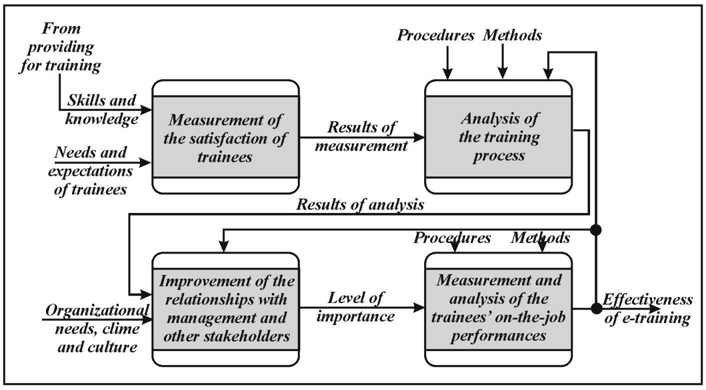

Fig. 2. Decomposition of sub-process 4—measurement, analysis and improvement of training.

Activity, QP_1^4—Measurement of trainee satisfaction is evaluated with marks between 6 and 10, mark 6 denoting the lowest satisfaction, and mark 10 denoting the highest satisfaction. For the purpose of this paper, we selected training for Control Engineering. This consists of theoretical material organized in Moodle Learning Management System (LMS), preparation for laboratory exercises using virtual instrumentations and remote control of real laboratory equipment on the Internet (web laboratory). Each lesson is accompanied with a specific laboratory exercise where trainees have remote access to laboratory equipment with video feedback in order to demonstrate and learn different techniques such as: developing skills in the field of design of mechatronics systems, implementation of different control techniques, real time programming and using different functional elements, sensors, computer hardware and software. Having completed the training, the trainees evaluated their satisfaction with training (there were three groups of trainees; in total 22 people (Table 1) following next aspects: material/content; structure/virtual environment; communication, cooperation and interactivity; student assessment; flexibility and adaptability; support. The theoretical [20] and practical [21, 22] parts of the training are easily accessible. The theoretical part consists of o15 lessons presented on Learning Management System. The practical part consists of laboratory exercises over Internet (Remote control of real laboratory equipment such as: temperature control of auditorium, coupled water tanks, gantry crane. . .).

Quality of other QP_4 sub-processes as well as quality of other processes of the considered e-training process is described by linguistic expressions (Section 3). Ten experts took part in evaluation of these activities and sub-processes quality. Results of their evaluations are presented in Table 2 and Table 3.

The selected experts described the quality using linguistic expressions which are modelled by discrete fuzzy numbers. Experts were selected among

Table 1. Evaluation of quality QP_1^4— Measurement of satisfaction of trainees

Trainee	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**	**T8**	**T9**	**T10**	**T11**
Mark	10	8	9	10	8	8	8	9	10	10	7
Trainee	**T12**	**T13**	**T14**	**T15**	**T16**	**T17**	**T18**	**T19**	**T20**	**T21**	**T22**
Mark	8	8	8	9	8	8	8	8	9	10	10

Table 2. Evaluation of QP_4 sub-processes quality— Evaluating training outcome (according to equations 1–5)

Evaluation of quality/Expert	**E1**	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**	**E8**	**E9**	**E10**
QP_2^4— *Analysis of the training process*	$\tilde{L}_3$	$\tilde{L}_1$	$\tilde{L}_2$	$\tilde{L}_4$	$\tilde{L}_3$	$\tilde{L}_4$	$\tilde{L}_4$	$\tilde{L}_3$	$\tilde{L}_2$	$\tilde{L}_3$
QP_3^4—*Improvement of the relationships with management and other stakeholders*	$\tilde{L}_2$	$\tilde{L}_1$	$\tilde{L}_2$	$\tilde{L}_1$	$\tilde{L}_3$	$\tilde{L}_1$	$\tilde{L}_2$	$\tilde{L}_1$	$\tilde{L}_2$	$\tilde{L}_3$
QP_4^4—*Measurement and analysis of the trainees' on-the-job performances*	$\tilde{L}_2$	$\tilde{L}_1$	$\tilde{L}_2$	$\tilde{L}_1$	$\tilde{L}_3$	$\tilde{L}_2$	$\tilde{L}_4$	$\tilde{L}_3$	$\tilde{L}_2$	$\tilde{L}_1$

Table 3. Evaluation of QP_1, QP_2, QP_3 processes quality (according to equations 1–5)

Evaluation of quality/Expert	**E1**	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**	**E8**	**E9**	**E10**
QP_1— *Defining the training needs*	$\tilde{L}_5$	$\tilde{L}_4$	$\tilde{L}_4$	$\tilde{L}_3$	$\tilde{L}_3$	$\tilde{L}_3$	$\tilde{L}_4$	$\tilde{L}_3$	$\tilde{L}_5$	$\tilde{L}_4$
QP_2—*Designing and planning training*	$\tilde{L}_5$	$\tilde{L}_4$	$\tilde{L}_4$	$\tilde{L}_5$	$\tilde{L}_3$	$\tilde{L}_5$	$\tilde{L}_4$	$\tilde{L}_3$	$\tilde{L}_5$	$\tilde{L}_4$
QP_3—*Providing training*	$\tilde{L}_2$	$\tilde{L}_4$	$\tilde{L}_4$	$\tilde{L}_2$	$\tilde{L}_3$	$\tilde{L}_3$	$\tilde{L}_4$	$\tilde{L}_3$	$\tilde{L}_2$	$\tilde{L}_4$

managers from enterprises (2 persons), people employed in the measurement of trainee on-the-job performances (2), university professors (3), experts for e-learning (2) and e-training system administrator (1) in order to cover all processes and sub-processes of e-training. These experts were allowed to use linguistic expressions to evaluate sub-processes and activities. In the process of evaluation they have the following aspects for discussion:

- material/content,
- structure/virtual environment,
- communication,
- cooperation and interactivity,
- student assessment,
- flexibility and adaptability,
- support (student and staff),
- staff qualifications and experience,
- vision and institutional leadership,
- resource allocation.

5.1 Determining values of data entry

5.1.1 Step 1

The matrix of relative relation of importance of sub-processes of considered e-learning process is defined as:

$$\begin{bmatrix} - & \text{"less impor tan t"} & \text{"1/"impor tan t"} & \text{"less impor tan t"} \\ & & \text{"1/"impor tan t"} & \text{"less impor tan t"} \\ & & - & \text{"impor tan t"} \\ & & & - \end{bmatrix} \tag{22}$$

In this example, the vector of the considered sub-processes importance is:

$$\tilde{w} = [\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{w}_4], \text{so that :}$$

$$\tilde{w}_1 = \left\{ \begin{array}{l} (0.236, 0.25), (0.199, 0.5), (0.162, 0.75), \\ (0.125, 1), (0.143, 0.75), (0.155, 0.5), \\ (0.163, 0.25) \end{array} \right\} \tag{23}$$

$$\tilde{w}_2 = \left\{ \begin{array}{l} (0.149, 0.25), (0.14, 0.5), (0.133, 0.75), \\ (0.125, 1), (0.117, 0.75), (0.11, 0.5), \\ (0.103, 0.25) \end{array} \right\} \tag{24}$$

$$\tilde{w}_3 = \left\{ \begin{array}{l} (0.521, 0.25), (0.562, 0.5), (0.597, 0.75), \\ (0.625, 1), (0.644, 0.75), (0.658, 0.5), \\ (0.669, 0.25) \end{array} \right\} \tag{25}$$

$$\tilde{w}_4 = \left\{ \begin{array}{l} (0.094, 0.25), (0.099, 0.5), (0.108, 0.75), \\ (0.125, 1), (0.096, 0.75), (0.078, 0.5), \\ (0.065, 0.25) \end{array} \right\} \tag{26}$$

Matrix of relative relation of importance of activities of sub-process is defined as:

$$\begin{bmatrix} - & \text{1/"less impor tan t"} & \text{"1/"impor tan t"} & \text{1/"very impor tan t"} \\ & & \text{1/"less impor tan t"} & \text{1/"impor tan t"} \\ & & - & \text{1/less "impor tan t"} \\ & & & - \end{bmatrix} \tag{27}$$

The vector of QP_4 sub-process activities importance is:

$$\tilde{w}_j^4 = \left[\tilde{w}_1^4, \tilde{w}_2^4, \tilde{w}_3^4, \tilde{w}_4^4\right], \text{so :}$$

$$\tilde{w}_1^4 = \left\{ \begin{array}{l} (0.066, 0.25), (0.067, 0.5), (0.069, 0.75), \\ (0.048, 1), (0.063, 0.75), (0.057, 0.5), \\ (0.052, 0.25) \end{array} \right\} \tag{28}$$

$$\tilde{w}_2^4 = \left\{ \begin{array}{l} (0.139, 0.25), (0.133, 0.5), (0.132, 0.75), \\ (0.177, 1), (0.123, 0.75), (0.113, 0.5), \\ (0.106, 0.25) \end{array} \right\} \tag{29}$$

$$\tilde{w}_3^4 = \left\{ \begin{array}{l} (0.26, 0.25), (0.267, 0.5), (0.279, 0.75), \\ (0.81, 1), (0.285, 0.75), (0.2777, 0.5), \\ (0.271, 0.25) \end{array} \right\} \tag{30}$$

$$\tilde{w}_4^4 = \left\{ \begin{array}{l} (0.535, 0.25), (0.533, 0.5), (0.521, 0.75), \\ (0.494, 1), (0.529, 0.75), (0.554, 0.5), \\ (0.571, 0.25) \end{array} \right\} \tag{31}$$

5.1.2 Step 2

Implementing process of normalization (Table 4).

5.1.3 Step 3

Procedure of transformation on values of the considered activities (Table 5).

The procedure of transformation on values of considered sub-processes of e-learning process (Table 6).

5.1.4 Step 4

Evaluation of the quality for the activities accompanied with cardinal values, with respect to

Table 4. Normalization of values for activity measurement of trainee satisfaction

$(QP_1^4)_m$	Frequency	$(QP_1^4)'_m$
10	6	1
9	4	0.9
8	11	0.8
7	1	0.7
M =	22	

Table 5. Procedure of transformation on values of QP_2^4, QP_3^4 and QP_4^4

$(QP_2^4)_e$	Freq.	$(QP_2^4)'_e$	$(QP_3^4)_e$	Freq.	$(QP_3^4)'_e$	$(QP_4^4)_e$	Freq.	$(QP_4^4)'_e$
$\tilde{L}_1$	1	0.09	$\tilde{L}_1$	4	0.09	$\tilde{L}_1$	3	0.09
$\tilde{L}_2$	2	0.96	$\tilde{L}_2$	4	0.96	$\tilde{L}_2$	5	0.96
$\tilde{L}_3$	4	0.98	$\tilde{L}_3$	2	1	$\tilde{L}_3$	2	1
$\tilde{L}_4$	3	1						
E =	10		E =	10		E =	10	

Table 6. Procedure of transformation on values of considered sub-processes QP_1, QP_2 andQP_3

$(QP_1)_e$	Freq.	$(QP_1)'_e$	$(QP_2)_e$	Freq.	$(QP_2)'_e$	$(QP_3)_e$	Freq.	$(QP_3)'_e$
$\tilde{L}_3$	4	0.05	$\tilde{L}_3$	2	0.05	$\tilde{L}_3$	3	0.05
$\tilde{L}_4$	4	0.98	$\tilde{L}_4$	4	0.98	$\tilde{L}_4$	3	0.98
$\tilde{L}_5$	2	1	$\tilde{L}_5$	4	1	$\tilde{L}_5$	4	1
E =	10		E =	10		E =	10	

their relative importance are obtained according to the expression (15):

$$E\tilde{Q}P_1^4 = \left\{ \begin{array}{l} (0.057, 0.25), (0.058, 0.5), \\ (0.059, 0.75), (0.0417, 1), \\ (0.055, 0.75), (0.076, 0.5), \\ (0.045, 0.25) \end{array} \right\} \quad (32)$$

$$E\tilde{Q}P_2^4 = \left\{ \begin{array}{l} (0.125, 0.25), (0.119, 0.5), \\ (0.118, 0.75), (0.159, 1), \\ (0.11, 0.75), (0.101, 0.5), \\ (0.095, 0.25) \end{array} \right\} \quad (33)$$

$$E\tilde{Q}P_3^4 = \left\{ \begin{array}{l} (0.255, 0.25), (0.262, 0.5), \\ (0.273, 0.75), (0.275, 1), \\ (0.279, 0.75), (0.271, 0.5), \\ (0.266, 0.25) \end{array} \right\} \quad (34)$$

$$E\tilde{Q}P_4^4 = \left\{ \begin{array}{l} (0.908, 0.25), (0.912, 0.5), \\ (0.909, 0.75), (0.911, 1), \\ (0.91, 0.75), (0.936, 0.5), \\ (0.909, 0.25) \end{array} \right\} \quad (35)$$

The value of QP_4 sub-process quality respects its relative importance and is determined according to expression (16):

$$E\tilde{Q}P_4 = \left\{ \begin{array}{l} (0.085, 0.25), (0.09, 0.5), \\ (0.089, 0.75), (0.114, 1), \\ (0.087, 0.75), (0.073, 0.5), \\ (0.059, 0.25) \end{array} \right\} \quad (36)$$

5.1.5 *Step 5*

Values of the sub-processes quality, linguistically expressed values, are determined according to expression (20):

$$E\tilde{Q}P_1 = \left\{ \begin{array}{l} (0.144, 0.25), (0.122, 0.5), \\ (0.099, 0.75), (0.076, 1), \\ (0.088, 0.75), (0.095, 0.5), \\ (0.1, 0.25) \end{array} \right\} \quad (37)$$

$$E\tilde{Q}P_2 = \left\{ \begin{array}{l} (0.144, 0.25), (0.112, 0.5), \\ (0.107, 0.75), (0.1, 1), \\ (0.094, 0.75), (0.089, 0.5), \\ (0.083, 0.25) \end{array} \right\} \quad (38)$$

$$E\tilde{Q}P_3 = \left\{ \begin{array}{l} (0.369, 0.25), (0.398, 0.5), \\ (0.423, 0.75), (0.443, 1), \\ (0.457, 0.75), (0.467, 0.5), \\ (0.474, 0.25) \end{array} \right\} \quad (39)$$

5.1.6 *Step 6*

The value of the considered e-learning process quality is determined according to expression (21):

$$E\tilde{Q}e = \left\{ \begin{array}{l} (0.185, 0.25), (0.181, 0.5), \\ (0.182, 0.75), (0.183, 1), \\ (0.182, 0.75), (0.181, 0.5), \\ (0.179, 0.25) \end{array} \right\} \quad (40)$$

5.1.7 *Step 7*

Representative scalars of discrete fuzzy numbers that describe marks of activities, sub-processes as well as e-learning processes are determinated using methods of moments (Table 7 and 8).

Mark of e-learning process is defuzz $(E\tilde{Q}e) = 0.182$.

Finally, we can express quality of the process by a single mark. The importance of this evaluation using fuzzy algorithm is the fact that we are in the position to assign a single grade to a specific e-training process. Further, this is important because the quality of each e-training process could be evaluated, quantified and different training processes could be compared according to their marks. This mark relies on evaluation of trainees and managers, but the important contribution of this approach is the fact that they do not need to use just numeric quantification for evaluation; they could also use linguistic expressions. The structure of the training process for both experts that take part in the evaluation process and trainees should be the same, as shown in Fig. 2. The final evalua-

Table 7. Representative scalars of discrete fuzzy numbers that describe marks of activities

Activities
Defuzz $(\tilde{EQP})_j^4$
QP_1^4
0.055
QP_2^4
0.124
QP_3^4
0.271
QP_4^4
0.463

Table 8. Representative scalars of discrete fuzzy numbers that describe marks of sub-processes

Sub-processes
Defuzz $(\tilde{EQP})_n$
QP_1
0.096
QP_2
0.102
QP_3
0.437
QP_4
0.093

tion and mark for e-training programmes could be useful for potential trainees (they could rank the quality of e-training), human resource managers in enterprises (because they will be able to select e-training programmes to their workers) and to e-training providers (since they will be able to compare and contrast training programmes, to benchmark them and improve the selected ones).

6. CONCLUSIONS

It is clear that the quality of e-learning is a very important issue. It is also evident that the quality of any process could be assessed and improved only if we have the process measurement and evaluation tool. In this paper we suggest a method for evaluation of e-learning process using multi-criteria fuzzy approach. Each e-learning process can be evaluated with a specific number so they could be compared or ranked. The novelty is that this model deals with fuzzy multi-criteria evaluation of processes and sub-processes instead of user satisfaction, for instance [8, 9]. User satisfaction is an important factor in evaluation (we account it using defined list of aspects) but there are other issues incorporated in the presented approach: Improvement of the relationships with management and other stakeholders, measurement and analysis of trainee on-the-job performances and analysis of the training process. The assessment of alternative training requires the opinion of several experts. Every expert's feeling or perception about a score of 80 is not the same, so it is hard and unrealistic to let an expert express his/her own opinion in a crisp value. This model enables experts to give their opinions in linguistic variables because a range value can express opinions and feelings more accurately. Even though there is a fact that effectiveness and quality are subjective and relative by nature.

The developed model is flexible according to the possibility of number change, the kind of optimization criteria change and importance of optimization criteria change. The proposed fuzzy model also represents a suitable basis for software development.

The following conclusion is drawn:

(1) It is possible to formally describe the quality of an e-learning process.
(2) Uncertainties existing in the model can be described by discrete fuzzy numbers.
(3) All the changes, as changes in the number of activities and sub-processes or its importance, can be easily incorporated into the model.
(4) The developed methodology offers possibilities to get the answer through simulation that the result will change if the input data change.
(5) The developed methodology is illustrated by a numerical example.

The limitations of this model are based on two main factors: the accuracy in training evaluation by the trainees and the level of expertise of experts who defined the relative relation of the sub-process activities importance. But since the mathematical model is presented, it could be used with any inputs from trainees and any new definition of relative level of importance introduced by any expert(s). In this paper we presented evaluation of e-training process that was decomposed on four main sub-processes; in addition, the fourth sub-process (Evaluating the training outcome) was further decomposed on activities. By using this approach, it is possible to evaluate the quality of each activity, sub-process and process. Additionally, this model could be further developed or expanded on the specific elements of the training and learning processes. Since this model is very flexible, it is very easy to expand presented pattern on additional sub-processes and activities. Finally, the model was tested comparing results of evaluation using the suggested approach and other approaches which use multi-criteria evaluation, which proved this model to be broader in its scope and cover all important sub-processes and activities. The developed and presented model is useful because it enables benchmarking and ranking of achieved quality of specific activities, sub-processes and processes.

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M. Stefanovic received his Ph.D. in Department of Industrial Engineering, Faculty of Mechanical Engineering University of Kragujevac, Serbia. He is currently Associate professor in Department of Industrial Engineering. His current research interests include web services, information systems and e-learning. He is a member of IFIP Council TC3 - Education.

D. Tadic received her Ph.D. in Faculty of Mechanical Engineering University of Belgrade, Serbia. She is currently Associate professor in Department of Production Engineering and Industrial Engineering. Her current research interests include operational research and fuzzy logic.

S. Arsovski received his Ph.D. in Faculty of Mechanical Engineering University of Kragujevac, Serbia. He is currently full professor in Department of Production Engineering and Industrial Engineering. His current research interests include integration management systems and quality research.

Z. Arsovski received his Ph.D. in Faculty of Mechanical Engineering University of Kragujevac, Serbia. She is currently full professor in Faculty of Economics, University of Kragujevac. Her current research interests include information systems, management information systems and e-learning.

A. Aleksic is Ph.D. student in Mechanical Engineering University of Kragujevac, Serbia.