A METHOD FOR DETERMINATION OF KINETIC FRICTION COEFFICIENT UNDER DYNAMIC LOADING CONDITIONS

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Abstract. In general, the kinetic friction coefficient can, under any contact loading condition, be determined using methods completely different from the existing methods based on the measurements of contact load and friction force. The proposed method refers to the determination of kinetic friction coefficient using the dynamic equation of motion for a rotating body, where the active force acts on the rotating body only at the initial moment of motion, while body masses, which are concentrically and eccentrically distributed in relation to the axis of rotation, provide a static and dynamic component of a desired 'sleeve-bearing' contact load. If the body angle of rotation change is experimentally determined as a function of time, then, based on the dynamic equation of motion, it is possible to determine the current friction coefficient values for the entire time period from the start of the motion to the rotation stop. It can be said that acceleration is a physical and energy indicator for friction and energy dissipation in tribomechanical systems, and that it defines the complete dynamics of the friction process itself.

Key words: Kinetic friction coefficient, dynamic contact load, dynamic equation of motion, angular acceleration.

1. INTRODUCTION

The first published theoretical research related to the determination of kinetic friction coefficient *via* the dynamic equation of motion for a body moving down an inclined plane was published by Euler in 1748 [1]. In a paper “On the friction of solid bodies” [2], published in 1750, Euler analyzed the motion of a body down an inclined plane and expressed the coefficient of friction as a function of time. His approach enables the determination of the kinetic coefficient of friction based on experimental measurements. Unfortunately, it is safe to say that this method has not experienced a wider expansion in the scientific field and, especially in the field of design of contemporary tribodiagnostic equipment.

Papers based on (or related to) Euler's research are mainly published in journals regarding education in the field of physics. Examples of this are papers based on analysis of the motion of a body on an inclined plane, that start from the equation of motion and establish relations between the coefficient of rolling friction and acceleration [3, 4] or follow nonlinear changes in the coefficient of rolling friction as a function of energy 'losses' [5].

In the education field, J. Alam *et al.* [6] deals with the dynamics of rotational motion, developing a simple method for experimental determination of friction losses based on the moment of inertia of a disk that is alternately rotating clockwise and counterclockwise. Applying the relations between translational and rotational motion, they indicated a linear dependence of the friction losses, *i.e.* the moment of friction, on the angular velocity. R. Drosd and L. Minkin [7] also used a simple laboratory device to study the effects of friction between two discs coming into contact while one of the discs rotates. The kinetic friction coefficient was determined based on a few basic parameters, namely, it was concluded to be dependent on the stationary disk radius, gravitational acceleration, and angular acceleration of the disk.

The specifics of motion of a disk rotating on a flat surface, in the field of education also known as 'Euler's disc', have often been examined in a literature. In addition to a review of papers on this topic, C. Le Saux *et al.* [8] proposed a numerical model of such motion, using Euler parameters, *i.e.* quaternions for parametrization of the variable orientation of the disc, taking into consideration limitations of contact parameters and different types of friction models.

In the field of applied research, Y. Fujii *et al.* [9] proposed a method for the determination of the dynamic coefficient of friction in a linear bearing, which is based on a modification of the *Levitation Mass Method* (LMM) and the application of a laser interferometer. Unlike conventional methods for determining bearing friction, which employ force transducers [10] or standard-defined inclined plane-based tests [11], Y. Fujii *et al.* [9] directly measure the force acting on the moving part of the bearing, expressing it as a product of mass and acceleration of the center of mass.

T. Jankowiak *et al.* [12] provide an optimal analysis for an accurate assessment of the dynamic coefficient of friction using a tribometric device. To better understand this method of precise coefficient of friction definition under dynamic loading, a three-dimensional *finite element* model (FE) has been developed. Based on the FE analysis, a new methodology for the determination of the dynamic coefficient of friction by introducing a correction factor has been proposed. This correction factor depends on the initial velocity and pressure for any coefficient of friction value.

F. Marques *et al.* [13] examined and compared several friction force models concerning different friction phenomena in the context of multibody system dynamics. On one hand, static friction models are generally simpler and they describe the stationary behavior of the friction force. However, most of them display an inability to properly 'capture' the effects of friction. Several static models present a discontinuity of the friction force at zero velocity, which can cause numerical instability during dynamic simulation. On the other hand, dynamic models use extra state variables in order to account for additional physical friction phenomena. These models are more complex and require the determination of a larger number of parameters. Friction models show a more significant difference in acceleration, mainly with the presence of changes in the velocity direction. In general, for detailed modeling of the friction phenomenon, it is necessary to use a friction model with a large number of parameters. In the most cases, these parameters need to be experimentally determined.

One of a few papers based on Euler's idea to determine the kinetic friction coefficient using the differential equation of motion was published in a thematic journal in the field of tribology [14]. In order to study the friction between grains of granular material at motion, Mihajlovic *et al.* [14] developed a method and physical model of a vibrating platform. They have theoretically and experimentally proven that the coefficient of friction between sand grains and sieve can be reliably determined using the dynamic equation of motion in real operating conditions of the vibrating platform. The results of this research indicate that the coefficient of friction can, even in this otherwise very complex process, be determined based on the differential equation of motion for a granule of sand. The obtained results are very compatible with the results obtained by standard and considerably more complex methods. This research is based on Euler's research but at the same time, it generalizes the application of the method for significantly more complex tribological processes compared to the study of friction for a body moving down an inclined plane.

Following Euler's idea, a large number of tribometers intended for the determination of kinetic friction coefficient were realized within the Center for Revitalization of Industrial Systems at the Faculty of Engineering, University of Kragujevac. Results of the published research [14–17] indicate a great potential for this method in terms of an essential understanding of dynamic processes, friction phenomena, and a wide range of possible applications. The method of determination of kinetic friction coefficient based on differential equations of motion is, in essence, related to the measurement of three base units of the SI system (mass, time, and length), which is a significant advantage in a theoretical, experimental, and technological sense.

2. THEORETICAL BASIS OF THE METHOD

The method for the determination of kinetic friction coefficient under dynamic loading conditions is, in general, based on the differential equation of body motion. The motion of a body rotating around a fixed axis (Fig. 1) is described by a differential equation:

$$I \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = \sum M_i = M_a - M_t - M_w \tag{1}$$

where: I – mass moment of inertia; ω – angular velocity; M_a – active moment initiating motion; M_t – resulting moment of the integral sum of the elementary resistant moments of friction over the contact surface; M_w – resulting moment of air resistance.

If the body's angle of rotation change is experimentally determined as a function of time, then, based on the dynamic equation of motion, it is possible to determine the current friction coefficient values for the entire time period from motion initiation to the moment body stops to rotate.

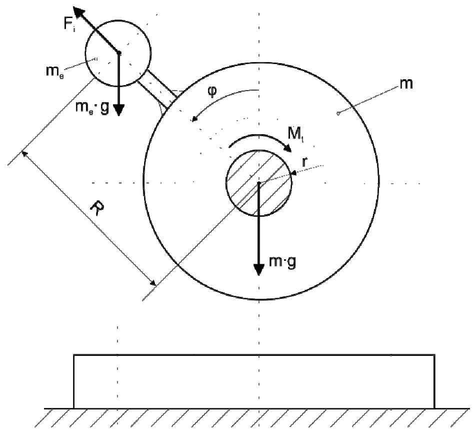


Fig. 1 – Schematic representation of the distribution of active and resistive forces when the body is rotating around a fixed axis.

After the action of an impulse of a force which, in a short time interval Δ_{t1} initiates motion *via* the active moment M_a from the differential equation (1), for a time interval $\Delta_{t1} < t$ it follows that $M_a = 0$, which means that in the stated time interval ($\Delta_{t1} < t$) the body's motion continues without the presence of the active moment, using the energy accumulated by the moment of impulse.

The drag force which creates the moment of resistance M_w can be determined based on the drag coefficient, the rotational speed and the size of a surface the air resistance force acts on [17]. Based on the research results [17] it follows that for the speed interval $V < 3$ m/s the drag force can be neglected as a lower order quantity, *i.e.*:

$$M_a = 0, \quad F_w \to 0 \tag{2}$$

Having that in mind, differential equation (1) can be written in the following form:

$$I \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = M_t = F_t \cdot r \tag{3}$$

where: M_t – moment of friction; F_t – resulting friction force at contact; r – radius at which elementary friction forces act (Fig. 2).

The resulting moment of friction is the only unknown variable in the differential equation of motion and based on Fig. 2 it is determined by the expression:

$$M_t = \iint r \cdot \mathrm{d}F_t = F_t \cdot r \tag{4}$$

which defines the total friction force as an integral sum of the elementary friction forces over the contact surface.

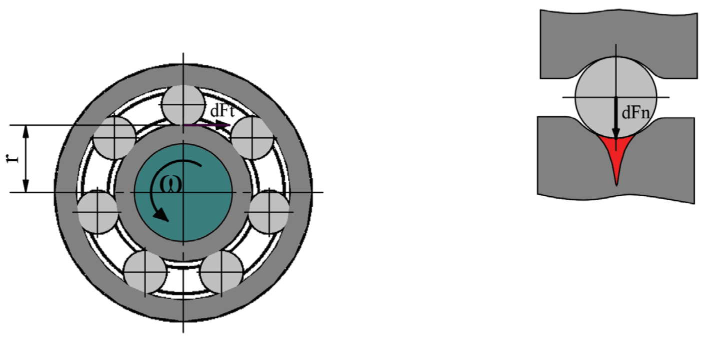


Fig. 2 – Schematic representation of the distribution of the sliding friction or rolling friction resistive forces when the body rotates around a fixed axis.

Based on the schematic representation given in Fig. 1, the friction force F_t can be defined as a function of the coefficient of friction, the component of the static contact load $F_s = (m + m_e) \cdot g$ and the component of the dynamic contact load $F_d = m_e \cdot R \cdot \omega^2 \cdot \cos\varphi$ by equation:

$$F_t = \mu \cdot F_N = \mu \cdot [(m + m_e) \cdot g - m_e \cdot R \cdot \omega^2 \cdot \cos\varphi] \tag{5}$$

Substituting the friction force value from equation (5) to equation (3) gives the equation:

$$I \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = \mu \cdot r \cdot [(m + m_e) \cdot g - m_e \cdot R \cdot \omega^2 \cdot \cos\varphi] \tag{6}$$

From (6) follows the final expression for the calculation of the kinetic friction coefficient value under dynamic loading conditions as a function of the angle of rotation and angular velocity.

$$\mu = \frac{I \cdot \dfrac{\mathrm{d}\omega}{\mathrm{d}t}}{r \cdot [(m + m_e) \cdot g - m_e \cdot R \cdot \omega^2 \cdot \cos\varphi]} \tag{7}$$

Based on the known theoretical expressions, angular velocity and angular acceleration can be determined by numerical differentiation using expressions:

$$\omega = \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{\varphi(t + \Delta t) - \varphi(t)}{\Delta t} \tag{8}$$

$$\varepsilon = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{\omega(t + \Delta t) - \omega(t)}{\Delta t} \tag{9}$$

From the previously stated theoretical considerations, it can be concluded that experimental determination of change in angle of rotation, as a function of time, enables the determination of kinetic friction coefficient value under dynamic loading conditions, for the entire time period, from motion initiation to the moment when body stops to rotate.

3. EXPERIMENTAL VERIFICATION OF THE THEORETICAL MODEL

Experimental verification of the previously stated theoretical model was realised at a preliminary device shown in Fig. 3.

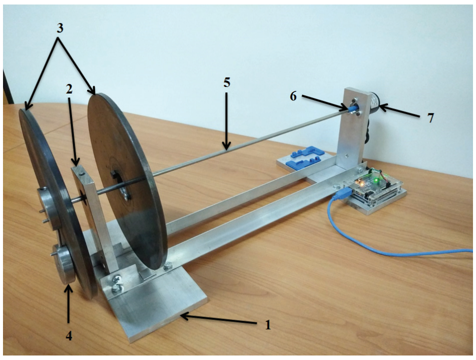


Fig. 3 – Device for determining the coefficient of friction of dynamically loaded bearings.

The device consists of a mount (position 1) to which the bearing housing is rigidly attached (position 2) in which the test specimen is situated (a rolling-element bearing). Two discs of the same shape and mass (position 3) are placed at the same distance on both sides of the rolling bearing, representing a symmetrical loading of the bearing m. Smaller weights (position 4) are attached to the discs (position 3) at a distance R from the bearing axis, representing an eccentric mass m_e. The m_e mass provides the dynamic bearing load during the testing. Thin shaft (position 5) transmits motion from the rolling bearing inner ring, *via* an elastic coupling (position 6) to a rotary encoder (position 7) which registers the change in angle of rotation as a function of time and which is the only measuring component necessary to determine the kinetic friction coefficient.

Customized software has been developed, based on the previous equations, which enables the display of current kinetic friction coefficient values, for the given loading conditions, throughout the entire time period of rotation, by using Arduino.

This software provides numerical values and diagrams of the friction coefficient as a function of time, as well as change in sliding speed, change in the normal contact load F_n, and changes in the angular velocity ω and angular acceleration ε with time, for the known input values (total mass $m = 4.3$ kg, eccentric mass $m_e = 0.3$ kg, mass moment of inertia $I = 0.02441$ kgm^2, eccentricity $R = 0.075$ m and bearing radius $r = 0.005$ m). Figures 4 to 8 show the corresponding diagrams.

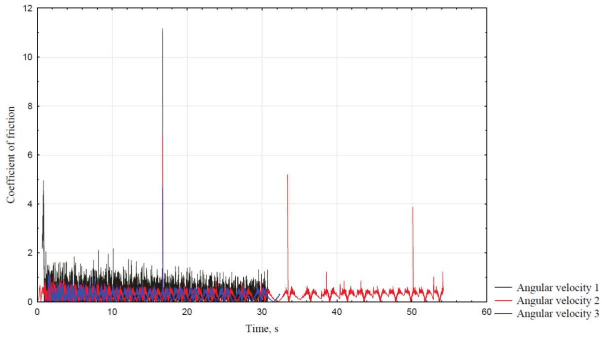
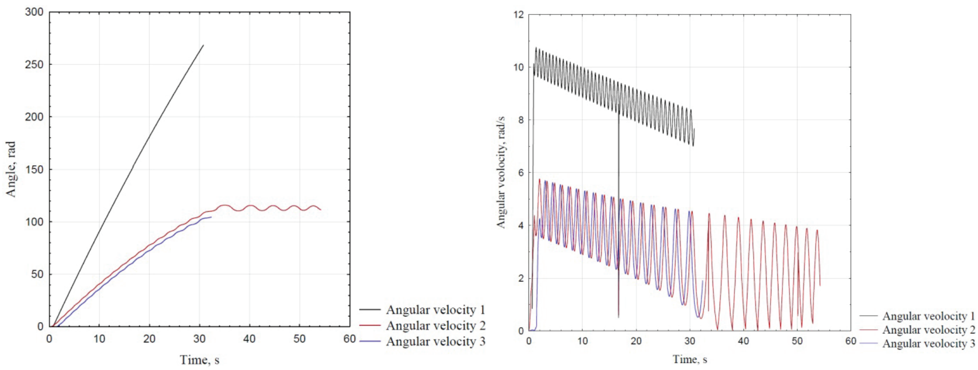


Fig. 4 – Friction coefficient change during time, for three different dynamic contact load levels.

The change in angle with time is shown in Fig. 5a, the change in angular velocity with time is shown in Fig. 5b, and the change dynamic contact load with time is shown in Fig. 6, and they all in correspondence with the change in the coefficient of friction diagram shown in Fig. 4.



(a) (b)

Fig. 5 – (a) Change in the angle of rotation as a function of time, (b) Change in the angular velocity as a function of time.

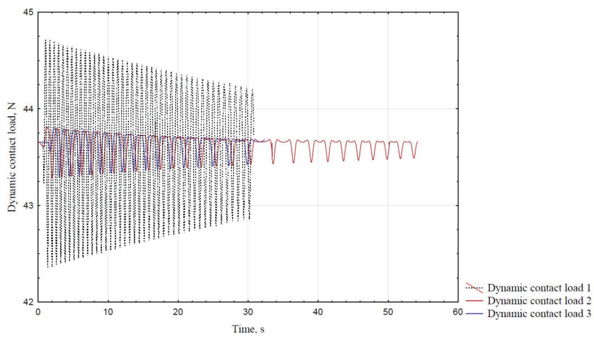
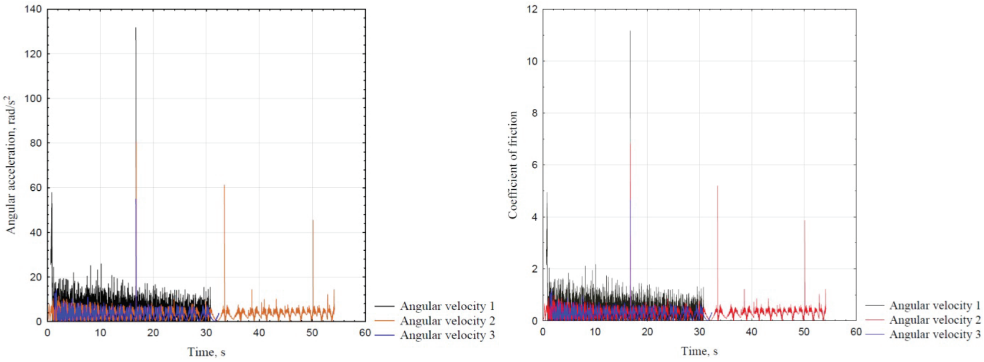


Fig. 6 – Change in the level of the dynamic contact load as a function of time.

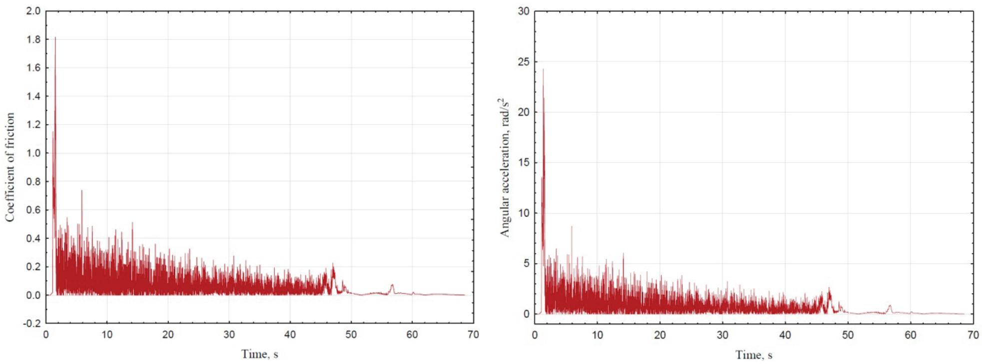
Comparative diagrams of change in angular acceleration and change in the coefficient of friction as functions of time are shown in Figs. 7a and 7b.



(a) (b)

Fig. 7 – Diagrams of change in angular acceleration (a) and change in the coefficient of friction (b) as functions of time.

A comparative diagram of change in the coefficient of friction with time, due to elimination − disassembly of the eccentrically positioned mass (shown in Fig. 3 at position 4) and a large reduction in the dynamic contact load component is shown in Fig. 8.



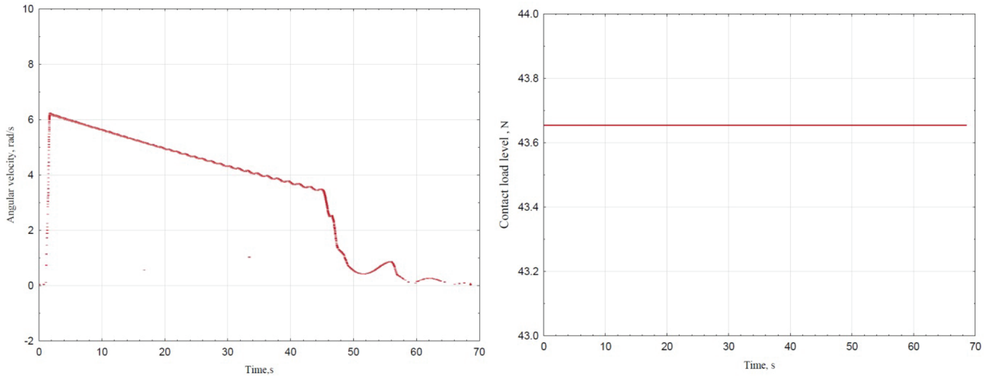
(a) (b)

Fig. 8 – Diagrams of change in the coefficient of friction (a) and angular acceleration (b) with time due to a reduction in the dynamic contact load component.

Diagrams of change in angular velocity and change in the normal contact load level shown in Fig. 9 (a, b) correspond to the diagrams shown in Fig. 8.

Levels of change in the speed of rolling at the friction zone in the rolling-element bearing are shown by the diagram in Fig. 10.

The influence of the dynamic contact load component for the stated test conditions is illustrated in Fig. 11 by a comparative diagram of the change in the coefficient of friction for a relatively short time interval.



(a) (b)

Fig. 9 – Diagram of change in angular velocity (a) and contact load level (b) with time.

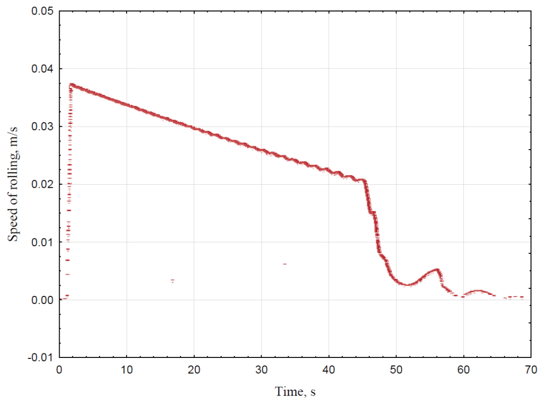


Fig. 10 – Levels of change in the speed of rolling at the friction zone in the rolling-element bearing.

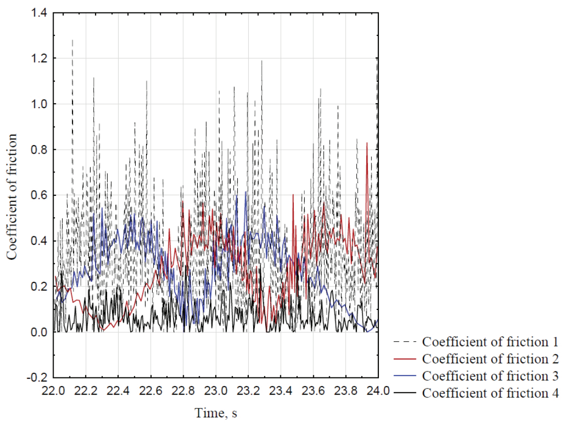


Fig. 11 – The influence of the dynamic contact load component illustrated by a diagram of the change in the coefficient of friction for a relatively short time interval.

4. DISCUSSION

From the literature analysis, it can be concluded that energy losses and the friction phenomenon are very interesting problems in the field of education, research, and development of contemporary tribodiagnostic equipment. Kinetic friction occurs during the relative motion of two bodies in contact, during the action of a force or a moment that provides the motion. Tribometry methods for experimental determination of the kinetic friction coefficient are based on the measurement of friction force at a certain level of normal contact load and a certain speed of relative motion of one contact element in regards to another, while the other contact conditions (microgeometry of contact pairs, temperature level in the contact zone, type of lubricant, etc) can be varied within wide limits. Research results related to the determination of the kinetic friction coefficient under dynamic loading conditions, using the conventional methods, are presented in papers [12, 18] and, in general, they indicate great compatibility of conventional methods with the method presented in our article. The stated conclusion of compatibility between the methods arises from the comparison of trends in the dynamics of changes in the coefficient of friction and the obtained values of coefficients of friction according to conventional methods and the method proposed here.

Based on analysis of differential equation (1) describing the motion of a body rotating in the presence of the resistive force of friction, that equation should be valid regardless of the tribometry method used for determination and measurement of the friction force value.

In the case of conventional methods based on friction force measurement, it follows that differential equation (1) will be satisfied only when the angular acceleration is $\varepsilon = \mathrm{d}\omega / \mathrm{d}t \rightarrow 0$, which these methods already imply. In order for the angular acceleration value to be zero, the angular velocity must be constant. Different types of electric motors (synchronous, induction, and DC motors) are the most common driving elements in tribometers. It a fact that the actual number of revolutions of an electric motor in real operating conditions can significantly deviate from the nominally declared number of revolutions, especially for induction and DC motors. That depends on the installed engine power, engaged engine power, and other factors. Even the small short-term deviations in angular velocity can lead to significant changes in angular acceleration value ($\varepsilon = \mathrm{d}\omega / \mathrm{d}t$) that, when multiplied by the mass moment of inertia (I), leads to a certain measurement error value.

The proposed method for determination of the kinetic friction coefficient under dynamic loading conditions indicates a theoretical dependence between angular acceleration and the coefficient of friction. The diagrams in Figs. 7 and 8 clearly show that the trend of changes in the coefficient of friction follows the trend of changes in the angular acceleration.

From the equation (7), when $m_e = 0$, follows:

$$\mu = \frac{I \cdot \dfrac{\mathrm{d}\omega}{\mathrm{d}t}}{r \cdot m \cdot g} \tag{10}$$

This means that the coefficient of friction under 'static' contact loading conditions is equal to the product of a constant and the angular acceleration, *i.e.*:

$$\mu = \frac{I \cdot \dfrac{\mathrm{d}\omega}{\mathrm{d}t}}{r \cdot m \cdot g} = C \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} \tag{11}$$

where constant C depends on the contact conditions and equals:

$$C = \frac{I}{r \cdot m \cdot g} \tag{12}$$

From the diagrams in Figs. 7 and 8, it can be seen that the experimentally obtained functions of the change in the coefficient of friction and the change in the angular acceleration have an identical responses at all times. Experimental functions which are shown in Figs. 7 and 8 are graphical illustration of the software processing of more than 6000 raw data points related to the current values of measured angle and time. The values of angular velocity, acceleration and coefficient of friction were calculated according to the theoretical model presented in this paper. Almost identical shape of the curves exhibited for the angular acceleration and the friction coefficient strongly indicate that we can consider acceleration to be closely connected with friction coefficient and can be taken as its indicator.

Equation (7) for a general case, which defines the coefficient of friction, for specific experiment conditions, can be written in the form:

$$\mu = \frac{C_1 \cdot \varepsilon}{C_2 - C_3 \cdot \omega^2 \cdot \cos\varphi} \tag{13}$$

where constants C_i ($i = 1, 2, 3$) have defined values, *i.e.*:

$$C_1 = I; \quad C_2 = r \cdot [(m + m_e) \cdot g]; \quad C_3 = r \cdot m_e \cdot R$$

Coefficient of friction μ defined by the equation (13), in a mathematical sense, represents a complex function of angle φ and angular velocity squared ω^2.

If regression analysis with linear function:

$$\mu = k \cdot \varepsilon \tag{14}$$

is performed on experimentally obtained series of values of acceleration and coefficient of friction, which for one performed and shown experiment amounts to around 6000 data for each of them, very high correlation were obtained with correlation coefficients close to one, for all performed experiments. Obtained constant values K_i (i = 1, 2, 3) were almost the same and for the demonstrated experiments they are in the range of 0.08358 ÷ 0.0849, with a maximum deviation of as little as 1.3%.

Based on the equations (13) and (14) it follows that the experimental data processing provided very similar equation:

$$k \cdot \varepsilon = \frac{C_1 \cdot \varepsilon}{C_2 - C_3 \cdot \omega^2 \cdot \cos\varphi} \tag{15}$$

where the constant k is:

$$k = \frac{C_1}{C_2 - C_3 \cdot \omega^2 \cdot \cos\varphi} \tag{16}$$

From the equation (3) arises the theoretical relation between the friction force and angular acceleration, as:

$$I \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = M_t = F_t \cdot r \rightarrow F_t = \frac{I \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t}}{r} = \frac{I}{r} \cdot \varepsilon \tag{17}$$

It is obvious that the friction force is, in this case, the product of the ratio of two constants, I / r and angular acceleration ε, which, in a theoretical sense, shows the angular acceleration as a kind of indicator for the friction phenomenon.

5. CONCLUSIONS

Based on the results of theoretical and experimental research presented in this paper, it can be concluded that the kinetic friction coefficient, under complex dynamic contact loading conditions, can be quite reliably determined based on the differential equations of body in motion, in presence of the resistive friction force. The methodology for determining the coefficient of friction has been reduced to measuring the two basic physical quantities, angle of rotation and time, which has a significant advantage over conventional methods for determination of the friction coefficient, from both the theoretical and experimental approaches. Our experimental verification has indicated theoretical relation between angular acceleration and coefficient of friction. Changes in acceleration, at any amplitude and frequency, exist in every dynamic system and they can largely explain the enigma of the kinetic friction coefficient. The authors of this paper consider acceleration as a kind of physical and energy indicator of friction and of energy dissipation in tribomechanical systems.

This method is especially significant in the field of education and essential understanding of the friction phenomenon. The fact that this method is based on the measurement of basic physical quantities (time, distance traveled, angle of rotation) allows the formation of reliable measurement chains and their location outside the contact zone, which considerably simplifies tribological research in high-temperature environments, controlled vacuum levels, aggressive environments, and other extreme conditions.

Here presented method showed high reliability of the results, thus indicating great potential for the application in research, and development of tribodiagnostic equipment.

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