ANALYSIS OF INFLUENTIAL FACTORS ON THE TENSILE STRENGTH OF POLYETHYLENE USING THE FACTORIAL DESIGN

ANALIZA UTICAJNIH FAKTORA NA ZATEZNU ČVRSTOĆU POLIETILENA PRIMENOM FAKTORNOG DIZAJNA

Sandra Veličković, *Faculty of Engineering, University of Kragujevac, SERBIA,*
E-mail: sandrav@kg.ac.rs

Abstract

Experimental results obtained by tensile test on specimens made of high-density polyethylene are presented in this work. Specimens were tested at low temperatures at different tension speeds. Factorial design and regression analysis were used for statistical analysis of the experimental results, more specifically the two-factorial experiment with three levels. By applying factorial design it can be determined which of observed factors have the greatest influence on the tensile strength of the tested specimens. The analysis of the results shows that the greatest impact on R_m *has the temperature. There are also shown linear regression equations for the tensile strength parameters in the function of the effective parameters. Applying this method are reduced testing time and reduced costs required for carrying out of the experiment.*

Key words: *factorial design, regression analysis, ANOVA, polyethylene, tensile strength*

Rezime

U ovom radu su prikazani eksperimentalni rezultati dobijeni zatezanjem epruveti od polietilena visoke gustine. Epruvete su testirane na niskim temperaturama pri različitim brzinama zatezanja. Faktorni dizajn i regresiona analiza su korišćeni za statističku analizu eksperimentalnih rezultata, tačnije korišćen je dvofaktorni eksperiment sa tri nivoa. Primenom faktornog dizajna može se utvrditi koji od posmatranih faktora ima najveći uticaj na zateznu čvrstoću epruveta. Analiza rezultata pokazuje da najveći uticaj na R_m *ima temperatura. Takođe, prikazana je linearna regresiona jednačina zatezne čvrstoće u funkciji od posmatranih parametara. Primenom ove metode smanjeno je vreme izvođenja eksperimenta i smanjeni su troškovi potrebni za obavljanje eksperimenta.*

Ključne reči: *faktorni dizajn, regresiona analiza, ANOVA, polietilen, zatezna čvrstoća*

1. INTRODUCTION

The experiment is a method of performing research or professional work in engineering practice. Factorial design of experiment is a statistical method for analysing of the experimental results when the test subject is affected by several factors with multiple levels [1]. The usage of statistical methods of experiments planning can significantly increase the efficiency of the experimentation process itself and lead to better and more reliable conclusions. Factorial design is a quick and economical method by which the most influential factors as well as the optimal solution can be determined [2].

The statistical principles underlying design of experiments were largely developed by R. A. Fisher during his pioneering work at Rothamsted Experimental Station in the 1920s and 1930s. Most of the early development was stimulated by applications in agriculture. Fisher systematically introduced statistical thinking and principles into designing experimental investigations, including the factorial design concept and the analysis of variance. The increasing interest of Western industry in quality improvement began in the late 1970s. Experimental design techniques are also becoming popular in the area of computer-aided design and engineering using computer/simulation models, including applications in manufacturing (automobile and semiconductor industries), as well as in the nuclear industry (Conover and Iman, 1980). Factorial designs are most frequently employed in engineering and manufacturing experiments [3, 4].

Polymeric materials are today among the most important technical materials. They are no longer used as a substitute for traditional materials, but are also used to create objects that have previously been produced from traditional materials [5]. These materials are characterized by their mechanical properties, which are a combination of properties of solids and liquids. Manufacture of the plastic masses increases significantly every year. According to some data, the production of plastic masses exceeds the production of all metals together [6].

Polyethylene is a macro-molecule hydrocarbon, and represents a very important material today. Industrially is produced by polymerization of ethylene, and in a laboratory's conditions can be also obtained from diazomethane. Polyethylenes are the most commonly used among all the polymers, because of their mechanical properties, easy processability and wide possibilities for the use. They have good toughness and excellent sliding properties. Their application on an industrial scale has been intensified over the last 60 years, primarily because of good mechanical qualities [7].

In this work the mechanical properties of polyethylene are tested at low temperatures at different speeds of testing. The specimens are made of high density polyethylene i.e. Hostalen GC 7260. This testing is essential in order to conclude how the material behaves under different low temperatures and at different speeds of testing. Applying factorial experiment with appropriate design leads to reliable conclusions with a smaller number of experimental tests, that is which of the considered factors have more influence on the tensile strength of polyethylene. Design and methods such as factorial design, Taguchi design and response surface methodology are now widely used in place of one factor at a time experimental approach which is time consuming and exorbitant in cost [8].

Factorial experimental design technique was used to study the main effects and the interaction effects between observed parameters.

This statistical method is used for processing experimental results, by applying the MINITAB 16 [9]. The applied factor experiment is 3^2, which has the three factors on the three levels, that is, a total of 9 combinations of the levels of factors.

The objective of planning the experiment is to obtain as much reliable information at the minimum cost.

2. FACTORIAL EXPERIMENT DESIGN

Design of experiment refers to the process of planning the experiments so that appropriate data can be analyzed by statistical methods, resulting valid and objective conclusions [8].

An factorial experimental design is the presentation of a detailed experimental plan in advance of doing the experiment. Chosen experimental designs maximize the amount of information that can be obtained for a given amount of experimental effort. Experimental design economically maximizes information and begins with determining the objectives of an experiment and selecting the process factors for the research [3].

When researching the determined phenomenon, the greatest importance has one-factor experiment. In addition to the phenomena that depend only on one factor, one-factor experiment is also used as the basis in experiments with multiple factors in the classical approach. The difference between the conventional and one-factor experiment is only in a randomization of factor level, which is required at one-factor experiment [1]. This causes that the external factors' effects, as random variables, are encompassed by an experiment's error. Factorial experiment is denoted by the product of factor's level, which precisely shows how large is the total number of combinations of factor levels.

Factorial design 3^n has n factors, each with three levels. In this experiment, the levels are fixed, and may be qualitative or quantitative [1, 10]. This type of plan, because of its good features, is often a basis of complex plans.

There are two factors with three levels in the experiment 3^2, i.e. total of nine combinations of factor levels. The levels of factors are divided into three types: the bottom level (mark 0), the basic level (mark 1) and the upper level (mark 2).

Marking of combinations of factor's levels is shown in Figure 1. In the labels shown on the Figure 1, the first digit indicates the level of factor A, and the second digit the level of factor B.

The classical dispersion analysis, which gives two each degree of freedom for the effects of A and B, is possible here. If there is no repetition for each combination of levels, for the interaction mixed with error of experiment, remains 4 degrees of freedom.

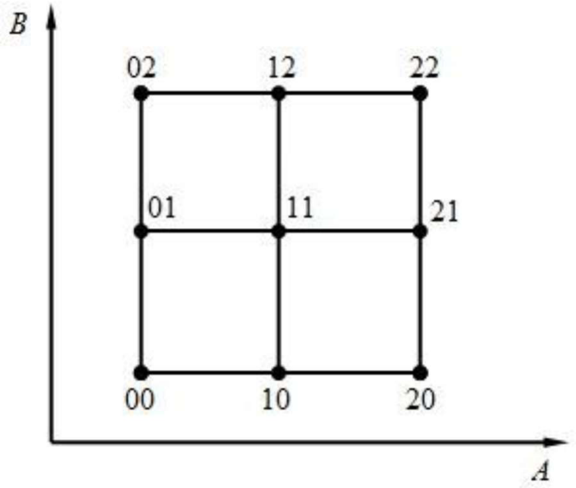

Figure 1. A combination of factor's levels

For further analysis, the coefficients for the results at the individual combinations of factors' levels are selected as follows:

- for linear effects -1 for the lower, 0 for the normal and +1 for the upper and
- for a square +1 for the lower, -2 for the normal and +1 for the upper.

The coefficients' scheme, selected as previously described, is given in the experimental part of paper in Table 4.

3. EXPERIMENTAL RESEARCH

The process of determining the tensile properties of polymeric materials is defined in ISO 527-1:1996 [11]. Tension testing of specimens was performed in the Laboratory for mechanical materials and deformation processing at Faculty of engineering, University of Kragujevac. Tests were performed on a universal testing machine ZWICK/ROELL Z 100.

The specimens are made from high-density polyethylene (Hostalen GC 7260). Injection of tensile test specimens was performed in company "21. Oktobar" in Kragujevac [5].

Specimens that are used for this type of testing were obtained by injection molding and they have a standard form of type 1A, shown in Figure 2 [12].

Figure 2. Specimens for testing of mechanical properties

High-density polyethylene has a high degree of crystallization, which leads to an increase in strength, elastic modulus and thermal stability [13].

Hostalen GC 7260 is high-density polyethylene with general purpose. Different parts such as covers, toy parts, bathroom's appliances and various types of packaging, are produced by injection [14]. The use of high-density polyethylene in applications requiring a long service lifetime has dramatically increased in the last years.

During testing the specimens, three temperature levels: 0, -20 and -40˚C and five different speeds of the tensioning: 10, 20, 50, 100 and 200 mm/min, were used. The tensile test specimens were cooled in a container with acetone, where it is added an appropriate amount of dry ice, in order to achieve the appropriate temperature (see Figure 3). The temperature is measured with a thermometer.

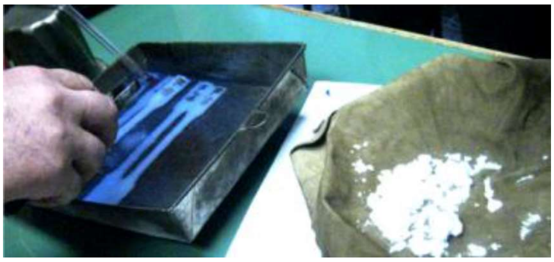

Figure 3. Cooling of test specimens

The obtained values of the elastic limit $R_{p0.2}$, tensile strength R_m, elongation corresponding to the tensile strength A_g and maximum elongation of A for various combinations of temperature and tensile speed values of specimens are given in Table 1 [12].

Table 1. The results obtained by testing of all series

Temperature [˚C]	Speed [mm/min]	Ordinal number of the series	$R_{p0.2}$ [MPa]	R_m [N/mm^2]
-40	200	1	25.63	39.11
-40	100	2	26.10	39.47
-40	50	3	25.72	37.88
-40	20	4	25.81	36.96
-40	10	5	25.96	36.48
-20	200	6	24.82	37.64
-20	100	7	24.98	37.84
-20	50	8	25.84	38.91
-20	20	9	25.47	36.51

-20	10	10	25.92	35.78
0	200	11	23.77	35.25
0	100	12	23.88	34.68
0	50	13	24.09	34.31
0	20	14	24.09	32.77
0	10	15	24.34	32.11

4. RESULTS AND DISCUSSION

In the course of this research are tested mechanical properties of high density polyethylene. Specimens of polyethylene have been tested at different speeds on tensioning and at different temperatures.

At the beginning of examination it was performed a testing of specimens at -40°C at different speeds tests defined by standards [8, 10]. The graph (Figure 4) shows the results of the examined test pieces at a temperature from -40°C at speeds of 10, 20, 50, 100 and 200 mm/min, while the results of testing at a temperature of -20˚C and 0°C, are shown in Figures 5 and 6, respectively.

Figures 4, 5 and 6 show the curves of tensile strength, where is with curve *a* marked stretching the tube at a speed of 10 mm/min, curve *b* marked stretching the tube at a speed of 20 mm/min, curve *c* marked stretching the tube at a speed of 50 mm/min, curve *d* marked stretching the tube at a speed of 100 mm/min and curve *e* marked stretching the tube at a speed of 200 mm/min.

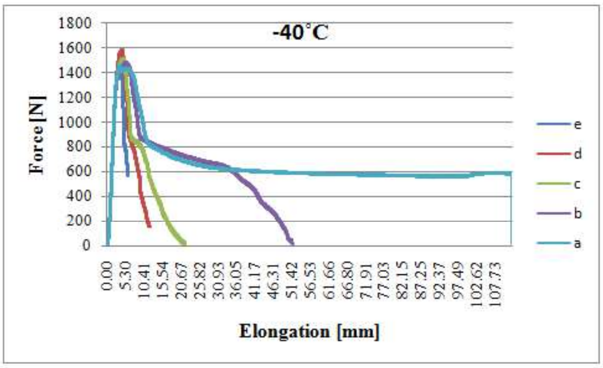

Figure 4. Force-elongation diagram for temperature at -40°C

On the basis of this examination it can be concluded that to the fracture of the test specimens comes before due to lower temperatures and due to the high speed testing. Results obtained by tension speeds of 50, 100 and 200 mm/min as well as results in the temperatures already mentioned in this paper were used for statistical analysis.

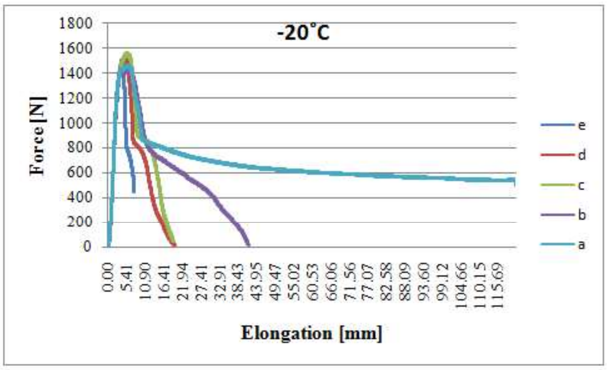

Figure 5. Force-elongation diagram for temperature at -20°C

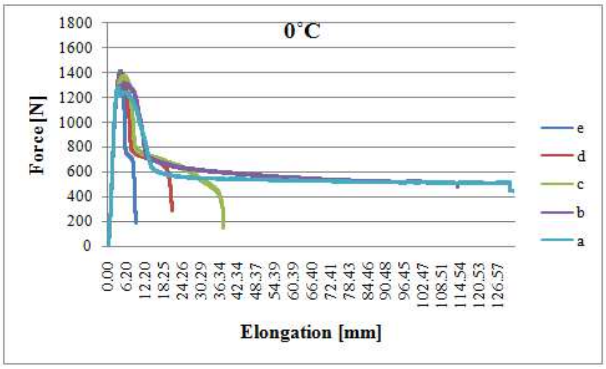

Figure 6. Force-elongation diagram for temperature at 0˚C

For the processing of previously obtained results, the Factorial design of experiment 3^n is used. Number 3 represents the level number, and n is the number of factors. In this case $n = 2$, which means that there are two factors (temperature and speed). It was investigated the effect of the mentioned factors on the tensile strength which is expressed in MPa.

Speed factor is denoted by *A*, and the temperature factor by *B*. The lower level of speed and temperature is denoted by 0, the basic level with 1 and the upper level with 2.

For a temperature factor indicated by the number 0, the temperature is 0˚C. Number 1 indicates the temperature of -20˚C and number 2 - the temperature of -40˚C. In addition, number 0 marks a speed of 50 mm/min, number 1 - speed of 100 mm/min and number 2 - speed of 200 mm/min.

Experimental tensile strength results are shown in Table 2.

Table 2. Experimental results and factor levels

Temperature (°C) *B*	Speed (mm/min) *A*		
	50	100	200
0	34.31	34.68	35.25
-20	38.91	37.84	37.64
-40	37.88	39.47	39.11

Analysis of the experimental values carried out using statistical software MINITAB 16, applying response surface [9, 15].

Experimental results were processed by the variable analysis (ANOVA), which is used for identification of significance of the factors that can have an impact on the response [9, 16]. The ANOVA results are shown in Table 4 and last column shows the percentage ratio (Pr) of each of the factors and their combination.

Based on the percentage of the factors in R_m (Table 3) it can be concluded that the greatest impact on the tensile strength has the temperature with 79.55% and quadratic effect of temperature with 11.59%.

Also, the factor influence can be determined by Fisher distribution for 99%, 95% or 90% of probability, based on the number of the degree of factor freedom and the number of the degree of error freedom. Values F gained by the analysis must have greater corresponding number of the degree of freedom than Fisher distribution values. According to Fisher distribution for 95 % of probability (F=10.1), it can be concluded that temperature has the highest impact on the tensile strength.

Table 3. Analysis of Variance for R_m

Source	DF	Seq SS	Adj SS	Adj MS	F	Pr(%)
Regression	5	28.6937	28.6937	5.7387	6.64	
Linear	2	24.9915	24.6193	12.3097	14.24	
A - Speed	1	0.1034	0.1350	0.1350	0.16	0.33
B - Temperature	1	24.8881	24.4843	24.4843	28.33	79.55
Square	2	3.7016	3.7016	1.8508	2.14	
A - Speed*A - Speed	1	0.0746	0.0746	0.0746	0.09	0.24
B - Temperature*B - Temperature	1	3.6270	3.6270	3.6270	4.20	11.59
Interaction	1	0.0006	0.0006	0.0006	0.00	
A - Speed*B - Temperature	1	0.0006	0.0006	0.0006	0.00	0.00
Residual Error	3	2.5926	2.5926	0.8642		8.29
Total	8	31.2864				100

During the analysis two graphs of tensile strength depending on speed and temperature are obtained (Figures 7 and 8).

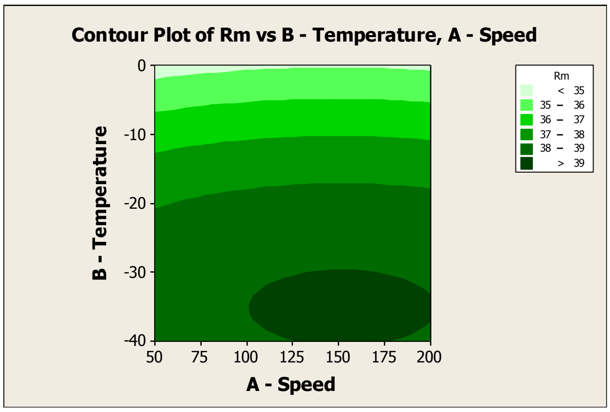

Figure 7. Contour plot for dependence between tensile strength of the speed and temperature

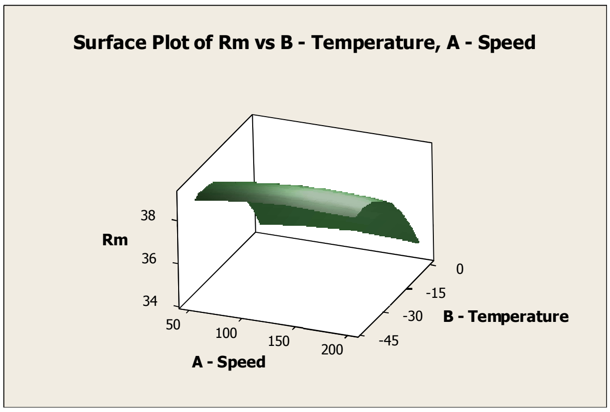

Figure 8. Surface plot for dependence between tensile strength of the speed and temperature

According to the Figures 7 and 8 it can be concluded that the highest value of tensile strength is for temperature varying from -30°C to -40°C and speed from 100 mm/min to 200 mm/min.

Also, the optimal solution is obtained through optimization, given in the Figure 9.

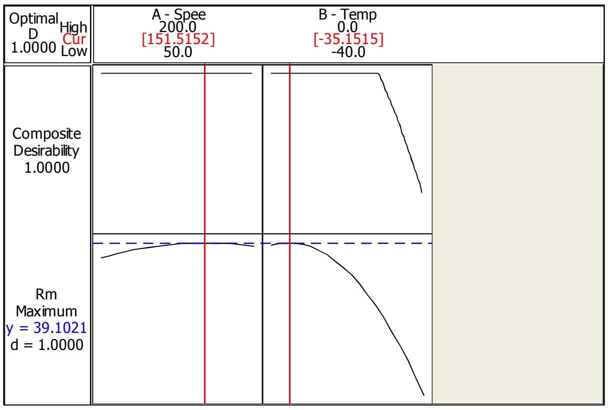

Figure 9. Optimal variant of the factors

In Figure 9 it is shown that the maximum value of tensile strength (39.1021 MPa) can be obtained with the speed at 150 mm/min and temperature at about -35°C. This diagram enables change of factor value in interval between given minimal and given maximal value, through which tensile strength for given values is obtained.

5. REGRESSION ANALYSIS

The mathematical model can be represented the effect of the speed and temperature on tensile strength using the general regression analysis.

This analysis give wider insight to understand any problem in general, and to optimize the factors influencing the response in particular [9, 16].

The coefficients of the equation of regression are shown in Table 4. The equation of regression for the tensile strength is as follows:

Table 4 Coefficients of regression analysis

Term	Coef	SE Coef	T	P
Constant	34.0736	1.98215	17.1902	0.000
A - Speed	0.0117	0.03531	0.3305	0.763
B - Temperature	-0.2355	0.07708	-3.0558	0.055
A - Speed*A - Speed	-0.0000	0.00013	-0.2938	0.788
B - Temperature*B - Temperature	-0.0034	0.00164	-2.0486	0.133
A - Speed*B - Temperature	-0.0000	0.00030	-0.0270	0.980

$$R_m = 34.0736 + 0.011669*A - 0.235542*B - 3.93333e\text{-}005*A*A - 8.21429e\text{-}006*A*B - 0.00336667*B*B \quad (1)$$

The model of linear regression was obtained through statistic software MINITAB 16.

By using regression model it can be estimated the course of change of one variable under the influence of the other variable. Regression model describes best the dependence between variations of the observed phenomena in reality by mathematical formulas with certain assumptions.

Linear regression line obtained on the base of experimental results of the tensile strength is given in Figure 10. Normal probability plot represents the comparison between the actual experimental results and the predicted values. The model given in the equation (1) corresponds to the diagram in Figure 10.

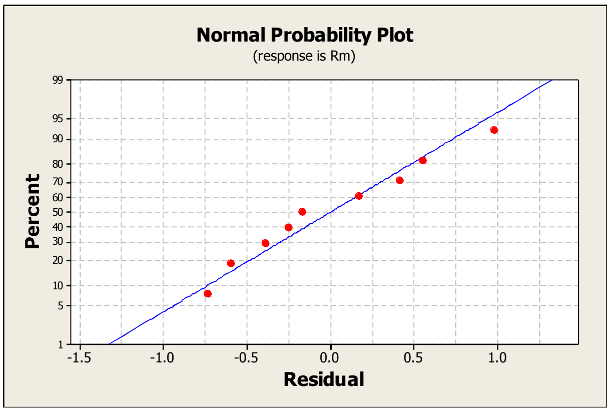

Figure 10 Comparison of the linear regression model with experimental results for the tensile strength

6. CONCLUSION

The main advantage of factorial experiment and its basic characteristic is that all levels of one factor combined with all levels of the other factors. Under the conditions of factorial experiments, choosing the favourable type of plan can significantly increase the efficiency of the experiment.

Factorial design experiment enables performing the experiment with a smaller number of experimental specimens from the classical experiments. By using factorial experiment with minimal expenditures and with shorter time of the experiment leads to reliable test results.

The ANOVA shows that the greatest impact on the tensile strength has the temperature.

The dependence between the tensile strength and the two factors, the temperature and the speed, is given using 2D and 3D graphs. The values of the influential parameters could be precisely defined on the base of these diagrams.

By the use of MINITAB 16 program the corresponding equation for the tensile strength.

The highest tensile strength is the result of the temperature at -35°C and the tensile speed at 150 mm/min.

Acknowledgements: Research presented in this paper was supported by Ministry of Science and Technological Development of Republic of Serbia, Grant TR 35041.

7. REFERENCES

[1] Pantelić, I. (1976). Introduction to the theory of engineering experiment, Workers' University Radivoj Ćipranov, Novi Sad.

[2] Qourzal1, S, Bakas, I, at al. (2014). Factorial Experimental Design for the Optimization of β-Naphthol Photocatalytic Degradation in TiO2 Aqueous Suspension, Canadian Chemical Transactions, 2(1), 1-11.

[3] Interim Report. (1995). Statistical Methods for Testing and Evaluating Defense Systems, Washington, DC, USA: National Academies Press, Web. 3 December 2014.

[4] Akçay Kasapoğlu, Ö. (2010). Process modelling for quality improvement in annealing process, Department of Production, Faculty of Business Administration, Istanbul University, Istanbul, Turkey, 39(2), 241-257.

[5] Velickovic, S. (2013). Investigation of the environmental factors' influence on the mechanical properties of plastics, Master thesis, Faculty of Engineering, University of Kragujevac, Serbia.

[6] Mikovic, J, Velickovic, S, Glisovic, J. & Catic, D. (2014). Analysis of the causes of the occurrence of an irregular process of obtaining polyamide 6, University of Kragujevac, Faculty of Engineering, Kragujevac, Serbia, 13(1), 23-30.

[7] Reis, J.M.L, Pacheco, L.J. & H.S. da Costa Mattos. (2014). Temperature and variable strain rate sensitivity in recycled HDPE, Polymer Testing, 39, 30-35.

[8] Das, S. R, Kumar, A. & Dhupal, D. (2013). Effect of Machining Parameters on Surface Roughness in Machining of Hardened AISI 4340 Steel Using Coated Carbide Inserts, International Journal of Innovation and Applied Studies, 2(4), 445-453.

[9] Stojanović, B, Babić, M, Veličković, S, & Blagojević, J. (2015). Optimization of wear behaviour in aluminium hybrid composites using Taguchi method, 14th International Conference on Tribology, Belgrade, Serbia, 81-86.

[10] Kabe, D, G, & Gupta, A, K. (2007). Experimental Designs: Exercises and Solutions, Springer, New York.

[11] ISO 527-1. Plastics - Determination of tensile properties, Part 1: General principles, 1996.

[12] ISO 527-2. Plastics - Determination of tensile properties, Part 2: Test conditions for moulding and extrusion plastics, 1996.

[13] Nedic, B, & Djukic, V. (2013). Plastics, Faculty of Mechanical Engineering, Kragujevac, Serbia.

[14] Hostalen GC 7260, http://www.ides.com/, downloaded in April 2013.

[15] Stojanović, B, Veličković, S, Blagojević, J. & Ćatić, D. (2015). Statistical analysis of roughness timing belt in operation using full factorial methods, Journal of the Balkan tribological association, 21(3), in print.

[16] Marwaha, R, Dev Gupta, R, Jain, V. & Kant Sharma, K. (2013). Experimental investigation & analysis of wear parameters on Al/SiC/Gr - metal matrix hybrid composite by Taguchi method, Global Journal of Researches in Engineering, 13(9), 15-22.