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THE TAGUCHI-GREY RELATIONAL ANALYSIS FOR OPTIMIZATION OF THE SAFETY COEFFICIENT FOR THE SURFACE DURABILITY OF A PLANETARY GEARBOX

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1. Introduction

The planetary gear transmissions are widely used in industrial applications for motion and power transmission such as automotive, aerospace, robotics, machinery etc., because of their significant advantages including the compact structure, self-centering capability, high power density, high transmission ratios, high degree of efficiency, durability in operation, low level of noise and vibration. Design of planetary gear transmissions depends on a large number of parameters, some of these are: material, module, addendum, profile shift, tooth thickness, dedendum, cutter tip radius, pressure angle, face width etc. [1, 2] The analysis and optimization of the planetary gearbox with the mark A_{ha}^{b} was performed in this paper. The optimization of a simple planetary gear train was performed by authors of [2]. They were observing the results obtained by changing the number of gear teeth, module, number of planets and face width. For development of mathematical model for multi objective optimization they used the Puerto and ELECTRE methods and noticed that the correlations between used methods are obvious. This kind of approach shows that the other planetary gear transmission types can be subjects of the multi objective optimization [3]. Multi objective approach in optimization is much faster with the use of the Taguchi-Grey method. For optimization of plastic gear production authors in [3] applied the hybrid integration of Taguchi parametric design, the Grey relational analysis and principal component analysis. For demonstration of efficiency and validity of the proposed hybrid optimization methods, in controlling all influential injection molding processing parameters during the plastic gear manufacturing, a plastic gear was used. The optimal combination of different process parameters is determined in order to minimize the shrinkage behavior in tooth thickness, addendum circle and dedendum circle of molded gear. They concluded that the proposed optimization method can produce plastic-molded gear with minimum shrinkage behavior of 1.8% in tooth thickness, 1.53% in addendum circle and 2.42% in dedendum circle and it should be noted that these values are less than the values in the main experiment. Therefore, defects related to shrinkage that lead to severe failure in plastic gears can be effectively minimized while satisfying the demand of the global plastic gear industry [4].

The Taguchi method is a well known optimization method, which is used in a variety of applications [4-5]. Application of this method ensures savings in labor and time costs, with simultaneously examining several parameters in several experimental conditions providing the quantitative information, [6-7]. The Taguchi method limitation is that it can only be applied to solve individual objective problems and cannot be used for problems with multiple objective optimization. However, this problem has been overcome and the optimization of multiple objectives can be performed by combining the Taguchi method and the Grey relational analysis to optimize multiple characteristics. In this paper, such a combination was applied for optimization of the safety coefficient for the surface durability of the entire planetary gearbox. The influence of parameters, such as the material, module and the gear width was considered to obtain the largest safety coefficients.

2. Design of experiments

The experiments were designed using the Taguchi's design of experimental methods, which are based on an orthogonal array of experiments, resulting in optimal setting of process control parameters. The orthogonal array provides a set of balanced experiments with less number of experimental runs, in order to evaluate the optimal parameters. For computing of the Taguchi data and data analysis the Minitab 16 was used.

In this paper, the orthogonal array L27 was chosen, which provides the least combination of parameters in a matrix. Those parameters were different and in this way the direct effect on the observed output was achieved. The chosen input parameters of the planetary gearbox are material, module and the gear width and they are all parameters of the third level. Levels of chosen parameters and their marks are listed in Tab. 1.

Tab. 1 *Control factors with their levels*

Control factors	Units	Level I	Level II	Level III
Material (A)	/	16MnCr5	28Cr4	C15E
Module (B)	[mm]	2.50	2.75	3.00
Gear width (C)	[mm]	30	33	36

3. Optimization using the Grey relational analysis

When there are multiple outputs, for optimization usually is used a combination of the Taguchi method and the Grey relational analysis. In this case, the Grey relational analysis is used to generate a single response for the two performance characteristics. Like in the Taguchi design, in the Grey relational analysis, there are also three quality characteristics: the higher the better, the nominal the better and the lower the better.

In Tab. 2 are given experimental results, the grade, the signal-to-noise ratio (S/N) for the Grey grade and results are ranked.

In this paper, for the desired higher values of the safety coefficients for the surface durability of a planetary gearbox at the output (the higher the better quality characteristics), normalizing in the range of 0 – 1 was done according to the following equation [8]:

$$x_i(k) = \frac{y_i(k) - \min y_i(k)}{\max y_i(k) - \min y_i(k)}, \quad (1)$$

where: k is the number of factors, in range 1-3, i is the experiment number, in range 1-27, $\max y_i(k)$ is the highest experimental value for the k-th output and $\min y_i(k)$ is the lowest experimental value for the k-th output, [9].

Tab. 2 Experimental design using L27 orthogonal array

No Exp.	A	B	C	Experimental results		The Grey grade	S/N for the Grey grade	Rank
				S_{H1-2}	S_{H2-3}			
1	16MnCr5	2.50	30	1.31	2.36	0.36716	-8.70297	25
2	16MnCr5	2.50	33	1.37	2.47	0.40345	-7.88425	21
3	16MnCr5	2.50	36	1.42	2.57	0.44152	-7.10098	19
4	16MnCr5	2.75	30	1.44	2.59	0.45408	-6.85741	17
5	16MnCr5	2.75	33	1.51	2.71	0.51924	-5.69263	12
6	16MnCr5	2.75	36	1.57	2.82	0.59492	-4.51088	8
7	16MnCr5	3.00	30	1.58	2.82	0.60218	-4.40554	7
8	16MnCr5	3.00	33	1.64	2.95	0.71780	-2.87995	4
9	16MnCr5	3.00	36	1.71	3.07	0.89541	-0.95958	2
10	28Cr4	2.50	30	1.33	2.39	0.37743	-8.46331	24
11	28Cr4	2.50	33	1.39	2.5	0.41588	-7.62063	20
12	28Cr4	2.50	36	1.45	2.61	0.46306	-6.68725	16
13	28Cr4	2.75	30	1.46	2.63	0.47241	-6.51365	15
14	28Cr4	2.75	33	1.53	2.75	0.54334	-5.29850	11
15	28Cr4	2.75	36	1.59	2.87	0.63134	-3.99473	6
16	28Cr4	3.00	30	1.51	2.86	0.57470	-4.81114	10
17	28Cr4	3.00	33	1.67	2.99	0.77659	-2.19617	3
18	28Cr4	3.00	36	1.74	3.12	1.00000	0.00000	1
19	C15E	2.50	30	1.24	2.24	0.33333	-9.54243	27
20	C15E	2.50	33	1.3	2.34	0.36149	-8.83814	26
21	C15E	2.50	36	1.35	2.44	0.39174	-8.14002	23
22	C15E	2.75	30	1.37	2.46	0.40161	-7.92385	22
23	C15E	2.75	33	1.43	2.57	0.44544	-7.02428	18
24	C15E	2.75	36	1.49	2.68	0.50000	-6.02060	13
25	C15E	3.00	30	1.49	2.68	0.50000	-6.02060	13
26	C15E	3.00	33	1.56	2.8	0.58017	-4.72887	9
27	C15E	3.00	36	1.62	2.92	0.68159	-3.32956	5

The absolute difference value is calculated according to expression, [9]:

$$\Delta_{0i} = \left\| x_0(k) - x_i(k) \right\|, \tag{2}$$

where: $x_0(k)$ referential normalized value ($x_0(k) = 1$).

The Grey relational coefficient is calculated according to [8, 9]:

$$\xi_i(k) = \frac{\Delta_{\min} + \varsigma\Delta_{\max}}{\Delta_{0i}(k) + \varsigma\Delta_{\max}}, \qquad (3)$$

where: $\Delta_{\min}$ and $\Delta_{\max}$ are the lowest and the highest values of absolute difference, respectively, ζ ς is the distinguishing coefficient in the interval 0 0 $\zeta \leq 1$, if the weighting of output is equal, then ζ = 0.5, [9].

Then the Grey relational grade (γ_i) can be calculated according to:

$$\gamma_i = \frac{1}{n}\sum_{k=1}^{n}\xi_i(k), \qquad (4)$$

where n is the number of outputs; γ_i has to be as high as possible.

The experimental data are analyzed by using the Taguchi signal-to-noise ratio to measure the performance of the process response. The S/N ratio is the objective function for optimization and with the use of a logarithmic function it helps in data analysis and in prediction of the optimal results. In this paper, the higher the better quality characteristics was used for calculating the S/N of the responses, [4, 5, 6]. In this way, the multiple response problems are converted into a single response problem.

4. Analysis of variance

For the purpose of finding the most influential parameter that contributes towards the outputs and how the variation in inputs affects the outputs, the ANOVA was performed. The effect of material, module and gear width on the safety coefficients for the surface durability of a planetary gearbox was analyzed with the 95% confidence level and 5% significance level, [7]. The probability values show the level of significance of each factor, the parameter is highly statistically significant if the corresponding P value is less than 0.05. In Tab. 3 are shown the ANOVA results.

***Tab. 3** Analysis of Variance for the S/N ratios for the Grey grade*

Source	DF	Seq SS	Adj SS	Adj MS	F	P	Pr %
A	2	15.748	15.748	7.8738	118.89	0.000	10.09
B	2	106.377	106.377	53.1883	803.11	0.000	68.17
C	2	28.120	28.120	14.0602	212.30	0.000	18.02
A*B	4	1.140	1.140	0.2849	4.30	0.038	0.73
A*C	4	0.820	0.820	0.2049	3.09	0.081	0.53
B*C	4	3.304	3.304	0.8260	12.47	0.002	2.12
Residual Error	8	0.530	0.530	0.0662			0.34
Total	26	156.038					100.00

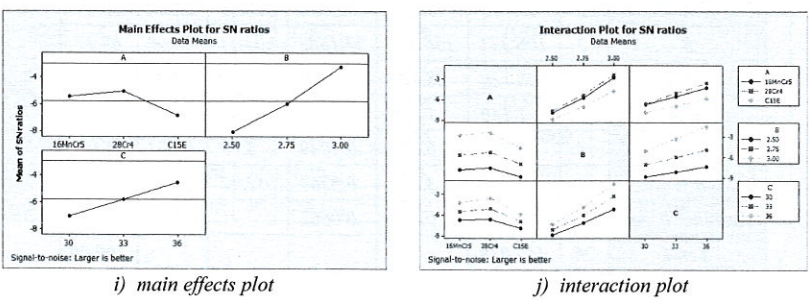
According to the ANOVA, the most significant control parameter is module with contribution of about 68.17%, the gear width with 18.02% and material with 10.09%. Interactions of observed parameters are insignificant because their effect on the safety coefficients was negligible. Analysis of parameters' influences was obtained from the response tables of the mean S/N ratio and the results are listed in Tab. 4. The larger values of the S/N ratio correspond to the better quality, thus the optimal combination of design parameters was obtained as A2B3C3, which is also shown in Fig 1.

5. Conclusions

Based on the Taguchi-Grey analysis, the following conclusions were drawn:
(i) according to results of the ANOVA analysis, the main contributing factors affecting the safety coefficients for the surface durability of a planetary gearbox were material, module and the gear width, with contributions of 10.09%, 68.17% and 18.02%, respectively. Interaction of factors B*C has influence of 2.12%, influences of other factors were negligible; (ii) the optimal combination of parameters, according to the Taguchi-Grey analysis is A2B3C3, which means that the highest values of the safety coefficients for the surface durability of a planetary gearbox are for material 28Cr4, module 3 mm and for the gear width 36 mm. The Grey grade for this optimal combination of parameters is 1 and the S/N ratio is 0; (iii) based on results from applied methods, one can conclude that the Taguchi-Grey method can be applied for optimization of the safety coefficients for the surface durability of a planetary gearbox.

***Tab. 4** Response for the Grey grade*

Level	*A*	*B*	*C*
1	*-5.444*	*-8.109*	*-7.027*
2	*-5.065*	*-5.982*	*-5.796*
3	*-6.841*	*-3.259*	*-4.527*
Delta	*1.776*	*4.850*	*2.500*
Rank	*3*	*1*	*2*



i) main effects plot *j) interaction plot*

***Fig. 1** Plots of the S/N ratio for the safety coefficients for the durability of a planetary gearbox*

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