EFFECT OF GEOMETRICAL PARAMETERS VARIATION OF THE CARDAN JOINT YOKE ON ITS LOAD CAPACITY

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Abstract

For the aims of mechanical parts and assemblies quality providing, it is necessary to meet the basic requirements of construction related to form, function, material and process development. Form of mechanical part resulted from the harmonization of those requirements and the task of designer is to identify the optimal construction form. This paper presents the analysis of effect of the geometric parameters variation on the stress level at joint yoke of Cardan shaft transmitter. The main function of the power transmitter with Cardan shaft is to transmit power and rotation between shafts with variable position, or certain angles, of the axis during exploitation. Joint yoke is one of the essential parts of the Cardan shaft. In most cases, the major influential factor to dimensions and reliability of Cardan shaft are the properties of its joint yoke. In the study presented in this paper, the stress limit at the joint yoke is calculated by analytical method, and then, obtained result is proven by numerical method. It is shown that small variations in geometry of the joint yoke lead to the significant reduction of stress level. Iterative procedure of the forms corrections and repeating of numerical calculations of stresses have been done to identify favourable relation between geometric parameters of the joint yoke.

1 Introduction

The main function of power transmitter with Cardan shafts is to transmit power and torque between the elements of car's power group which are on certain distance and that are not rigidly connected. They can be used between elements without coincide in axes or with variation in relative positions. The Cardan joint get its name after Italian mathematician Girolami Cardano, who invented this type of power transmitter. The Cardan shafts are used as part of power transmitter on vehicles which have engine in the front and rear wheel drive. They transmit the torque from the shaft to the main pinion gear and further to planetary transmitters. The Cardan shaft with two joints is usually used. This type of shaft provides flexibility of positions to the angle of 30° between of axes. This characteristic is important due to fact that driving wheels have continual up and down movement in order to reduce the influence of impacts from the road to the car body, so the need for flexible transmission of power and torque is essential. From this aspects, the usage of Cardan shafts in power transmitters of cars and trucks, building and agricultural machinery. The Cardan shafts are used to deliver power and torque to the additional equipment of tractors and other machines when position flexibility of power transmission is essential [1].

The identification of Cardan shafts and its elements are based on nominal power and torque from catalogs of producers. But, Cardan shaft, chosen by this method, do not always satisfied the requirements for reliability and duration of exploitation period. In most cases, the cause of early replacement of those elements are not production process, but design of whole power transmitter.

The causes of failures and design of power transmitters with Cardan shafts are analyzed by many researches. Hummel and Chassapis [2] researched on the design of the universal joints. They have given some suggestions on the configuration design and optimization of universal joints with manufacturing tolerances [3]. Bayrakceken et al. [4] performed the fracture analysis of a universal joint yoke and a drive shaft of an automobile power transmission system. Spectroscopic analyses, metallographic analyses and hardness measurements are carried out for each part. For the determination of stress conditions at the failed section, stress analyses are also carried out by the finite element method.

In most cases, the major influential factor to dimensions and reliability of Cardan shaft are the properties of its joint yoke. In the study presented in this paper, the stress limit at the joint yoke is calculated by analytical method, and then, obtained result is proven by numerical method. Finite elements method is used to analyze the stress distribution in critical zone. On the bases of the obtained results the optimization of joint yoke design was done.

2 Input yoke of Cardan joint

Cardan joint yoke, in dependence of type of connection with adjoining shaft can be equipped with bearing or flange. The flange couple the yoke of the Cardan joint with second shaft. The coupling can be obtained also by groove joint or by welding. The different design of Cardan joint yoke with different types of joining to adjoining shaft are presented at fig. 1. The difference between them are mainly in method of joining to shaft and in the form of bearings for the cross shaft.

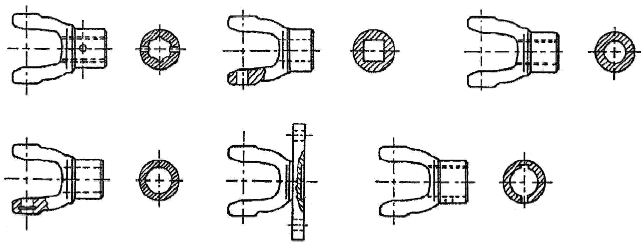


Fig. 1. Different designs of Cardan joint yoke [5]

The branches of yoke are loaded by different types of loads [6]:

- Flexion and torsion from the torques that acts in the plain of yoke and in the plain perpendicular to that plain. It is also loaded to flexion and torsion due to torque of friction forces in the same plains.
- Shear by forces reduced to centre of inertia in the plain of console,
- Tension in the zones of holes for groove of the needle bearing due to compression induced by compression from pressed joint.
- Pressure on surface on the cylindrical surface for the hole for groove of pressed joint.

Calculation of Cardan joint yoke is done on the basis of maximal torque and maximal force due to transmission of nominal power. Schematic drawing of the joint yoke is presented in Fig. 2.

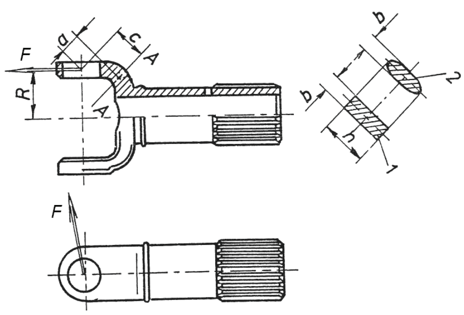


Fig. 2. A schematic drawing of the joint yoke [5]

2.1 Analytical calculation of Cardan joint yoke

Analytical and numerical calculation of stresses for Cardan joint yoke that was in exploitation was carried out with real dimensions. After that, the model parameters are varying and the stress distribution was calculated by numerical method in order to identify the best design solution with the lowest level of stresses [7]. The basic data for calculation are given in the Table 1.

Tab. 1. Basic data

Name	Value
Input power	P_U=10 *kW*
Input number of rotation	n_1=1500 min^{-1}
Angle of path	α_{12}=30°
Shear modulus	G=0,8·10^5 *MPa*
Dimensions of joint yoke	R=15.5 *mm*
	h= 27 mm
	b=8.5 *mm*
	c=18.5 *mm*
	a=5.5 *mm*

On the basis of the nominal input data and the presented load configuration, the calculation of maximal torque of $T_{u2\max}$ =77.775 *Nm* was done, so as the calculation of maximal load force at Cardan joint of F=2.509 *kN* [7, 8, 9].

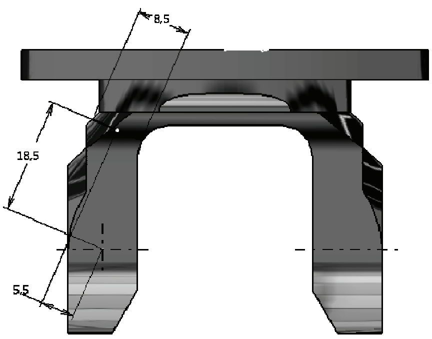


Fig. 3. Cardan joint yoke

Joint yoke is loading on bending and torsion torque. Bending stresses in the base of the Joint yoke sleeve (critical section A-A), can be determined as follows (Figure 2) [5]:

Rotary momentum of inertia W_b and W_s depend on shape of cross section of branch of Cardan shaft yoke. For the adopted ellipsoid shape of branch of Cardan shaft yoke, with certain assumptions following relation can be used:

- Resistant bending torque

$$W_b = \frac{b \cdot h^2}{10} = 6.179 \cdot 10^{-7} m^3 \tag{1}$$

- Resistant shear torque

$$W_s = \frac{1}{5} \cdot h \cdot b^2 = 3.902 \cdot 10^{-7} m^3 \tag{2}$$

- The bending stress

$$\sigma = \frac{F \cdot c}{W_b} = 74.9\ MPa \tag{3}$$

- The shear stress

$$\tau = \frac{F \cdot a}{W_s} = 33.57\ MPa \tag{4}$$

- The equivalent stress

$$\sigma_e = \sqrt{\sigma^2 + 3 \cdot \tau^2} = 96.76\ MPa \tag{5}$$

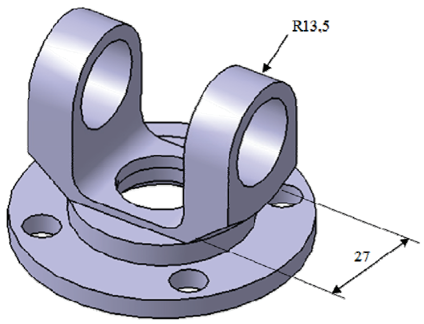
The stress is less than the allowable stress which is σ_d=150 *MPa* [5].

2.2 Structural finite element analysis of joint yoke

In this part of the paper, the structural analysis by finite element method, made in CATIA® software will be presented. It will be used in comparisons to stress values obtained analytically for the joint yoke. The comparisons are done to ensure the relevant of results. Upon the relevant results, the effect of changes in the shape to maximum stress was analyzed. It will be shown how the changes on the model affect the maximum stress.

The stress distributions for six different designs of input yoke of Cardan shaft were analyzed. Firstly, the tree dimensional tetrahedron discretization with variation in size of elements is used. The zone of shape transition between yoke and cylindrical part was discretized by finest elements (2 mm in dimension of edge) while the rest of the analyzed element was discretized by the elements with dimension of 2 x 4 mm. The boundary conditions were defined according to theoretical consideration of the input yoke of Cardan joint. The limits of movement were set on the surface which is overlapped on shaft. The load was set to force of 2509 N on the branch of the yoke for every case of design. The numerical model of Cardan joint yoke was completely defined and statically determined.

Case 1: Geometrical model was simplified in this case without profiling the branch of joint yoke (Fig. 4.) and the stress distribution for this case is presented in Fig. 5. At the zone of shape transition between the base of yoke and cylindrical part, the stress concentration is induced and the maximum stress reach the value of 106,42 MPa. The maximal stress obtained by analytical calculation is 96,76 MPa and the different to numerically obtained value is 10%.



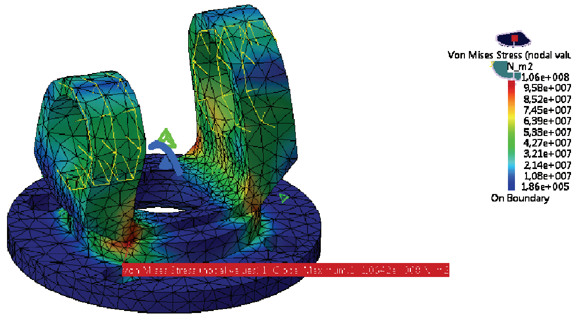
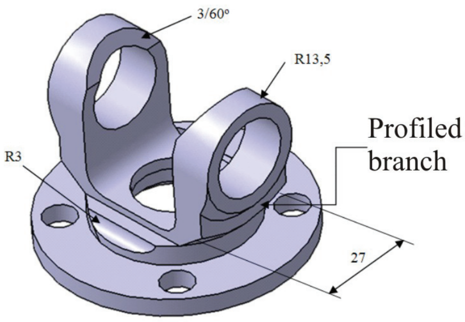


Figure 4. Simplified model joint yoke

Figure 5. Finite element stress analysis of the Cardan joint yoke (variant 1)

Case 2: Real model of yoke (Fig.6.) defer to simplified one in the profiling of yoke branches, while the rest remain the same. The stress distribution presented in Fig. 7. is similar to the stress distribution at simplified model from Case 1. Maximal stress act in the same position as it is in simplified model but it is 2% higher and reaches 108,532 MPa. The different in value to analytical calculation is 12% on side of numerically obtained value.



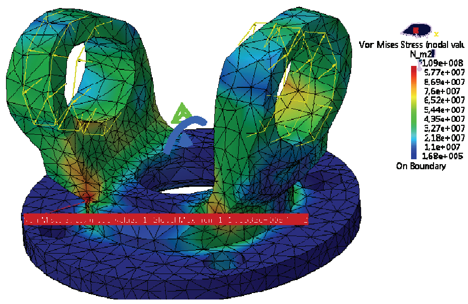
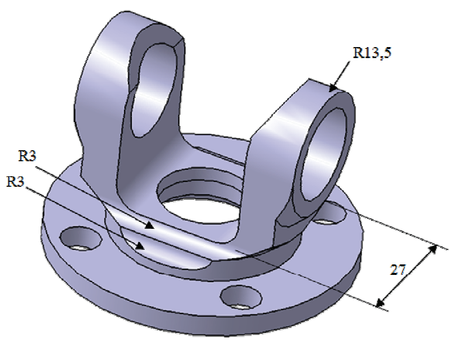


Figure 6. Model of the real joint yoke

Figure 7. Finite element stress analysis of the Cardan joint yoke (variant 2)

Case 3: The shape of the model analyzed in this case differs from shape of previous one by fillet of 3 mm at the zone of shape transition while the rest of the shape and dimension remain the same (Fig. 8.). The variation in stress distribution is minor (Fig. 9.). In relation to previous case the maximal stress is reduced by 0,5% and it reaches the value of 108,192 MPa. The position of zone within highest level of stress concentration was changed and transfer from the zone of shape transition between yoke and cylindrical part to inner edge side of the base of branch yoke.



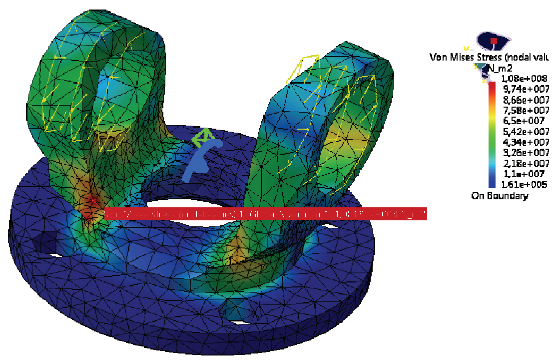


Figure 8. Model of the joint yoke (case 3)

Figure 9. Finite element stress analysis of the Cardan joint yoke (case 3)

Case 4: The fillet of 2 mm on defined edges was done on model (Fig. 10.) and resulted stress distribution is presented in Fig. 11. By filleting the maximal stress was increased by 19% in relation to former model, so the new maximal stress is 128,279 MPa. The zone of stress concentration is moved, in this model, to the central zone of the base of the branch yoke.

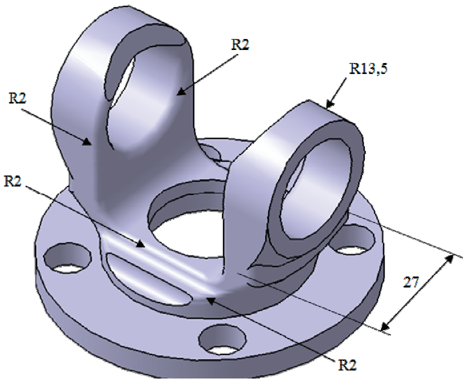


Figure 10. Model of the joint yoke (case 4)

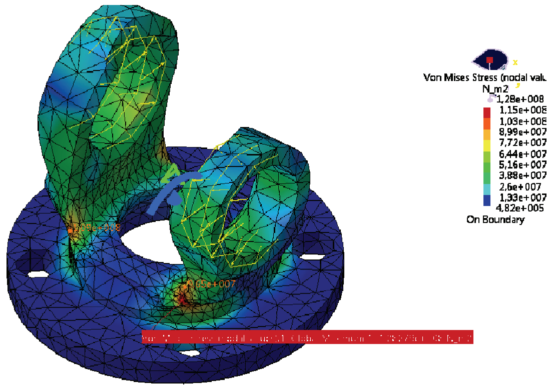


Figure 11. Finite element stress analysis of the Cardan joint yoke (case 4)

Case 5: In order to reduce the stress level the model with fillet of 10 mm was done. The other dimensions remain the same as in the previous case (Fig. 12.). The level of stress in this model was reduced (Fig.13.). In this case maximal stress is 119,101 MPa that is less to the maximal stress from previous case by 7%. The position of maximal stress concentration is moved and it is at the edge of zone of shape transition.

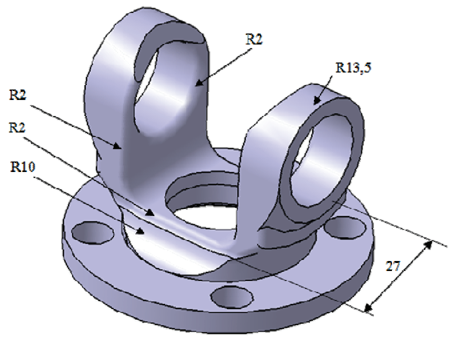


Figure 12. Model of the joint yoke (case 5)

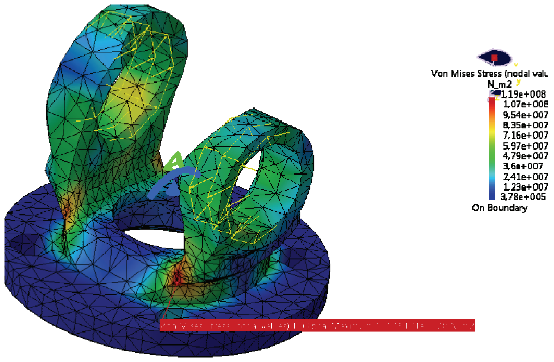
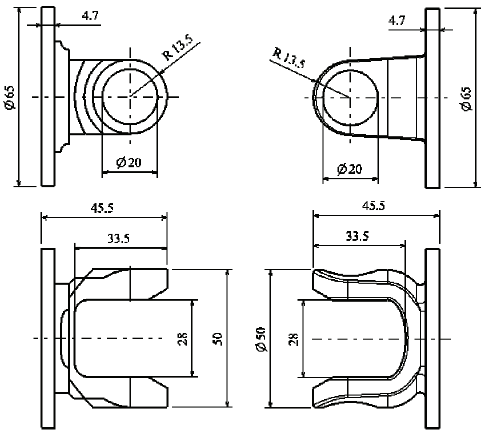


Figure 13. Finite element stress analysis of the Cardan joint yoke (case 5)

For the further analyses of variation of stress distribution the new model of input Cardan joint yoke was introduced. The functional and montage dimensions, so as load and exploitation conditions remain the same. The discretization mesh remains the same. The shape and dimensions of the optimized model in relation to real Cardan joint yoke is presented in Fig. 14.



a) b)

Fig. 14. Comparative shape and dimensions: a) Real Cardan joint yoke and b) Optimized model

Case 6: The model with optimized design of Cardan joint yoke (Fig.15.) do not have the sharp edges at the zones of shape transition between cylindrical part and the branches of yoke. The stress concentration at the base of the branches is reduced and stress distribution is presented in Fig. 16. The maximal stress in this model is 123,235 MPa that is higher to maximal stress in previous model by approximately 3%. The zone with highest stress concentration is moved to the zone of fillet in the body of yoke to the disc. The model in this case presents less adequate design due to higher value of maximal stress.

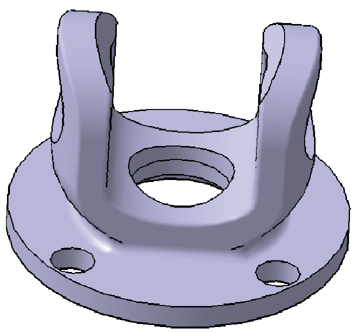


Figure 15. New model of the joint yoke (case 6)

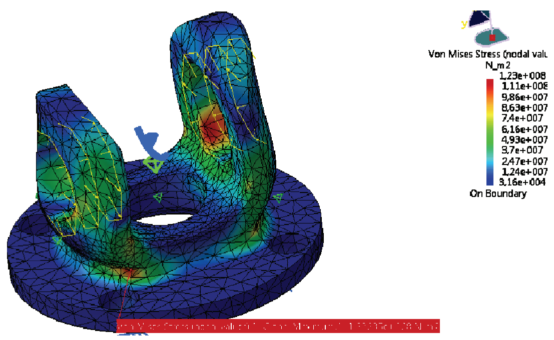


Figure 16. Finite element stress analysis of the new Cardan joint yoke (case 6)

3 Analysis of results

On the bases of the results obtained by analysis of Cardan joint yoke, presented in the paper, identification of the best design solution with lowest level of stresses was done. In Tab. 2. variation of maximal stresses at the Joint yoke is given. Case with lowest level of stresses is identified and chosen as most favourable design solution.

The sign minus represents that the variation of maximal stress in relation to maximal stress obtained in previous model are negative. So, the negative variation means that maximal stress is increased to previous maximal stress. The same manner of notation is used to indicate the variation of maximal stress obtained by numerical calculation to maximal stress calculated by analytical approach.

On the bases of the presented analysis the identification of the optimal design of Cardan joint yoke was done. The model with optimal design is the one with profiled branches of yoke and fillet of 3 mm at the zone of shape transition from cylindrical part to the branches of yoke, while the rest of dimensions remain the same as in the real Cardan joint yoke.

Table 2: Variation of maximal stresses at the joint yoke

The stress calculated analytically: 96.76 MPa			
No.	Maximal stress	Variation of maximal stress in relation to the previous model [%]	Difference to maximal stress calculated analytically [%]
1	106,42 MPa		-9,98
2	108,53 MPa	-1,98	-12,17
3	108,19 MPa	0,31	-11,81
4	128,28 MPa	-18,57	-32,57
5	119,10 MPa	7,15	-23,09
6	123,24 MPa	-3,47	-27,36

4 Conclusions

The results obtained by numerical analysis of stress distribution at input yoke of Cardan joint indicate that even small variation in shape can lead to significant variation in stress distribution. The stress concentration can be reduced by fillet. But increase of fillet can reduce stress concentration to some level. After that increase of fillet lead to increase of stresses.

The results obtained by analytical calculation can be used as relevant but those calculations considered general simplified mathematical model. The further complexity of shape with remain dimensions can induce the changing in stress values and it cannot be considered analytically. From that reason, the numerical calculation is favourable method because this method considered the influence of real shape, so variation in shape will be used for stress calculations.

At the input yoke with increase of fillet, the higher level of stress can be reached. The variation in stress level can be up to the 30% at certain design solutions, even if the dimensions remained the same as for analytical calculation. For the evaluation of the obtained results, it is recommended to use different software packages for the same models of design solutions and the same configuration of loads.

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