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APPLICATION OF GENETIC ALGORITHM AND LINEAR PROGRAMMING FOR DETERMINATION OF OPTIMAL PRODUCTION VOLUME

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Abstract. *The problem of determining the optimal production volume is one of the most important problems faced by the management of companies in modern industry. Determining the production volume primarily depends on the customer demands, i.e. demands from the market, but also from some other factors, such as production capacity, supplier capacity and flexibility, raw material costs, etc. In this paper, the way to determine the optimal production volume is defined by using one metaheuristic and one numerical method, namely Genetic Algorithm and Linear Programming, respectively. In addition, a comparative analysis of these methods was performed when it comes to the small-scale problem such as the problem discussed in this paper.*

Key words: *Production volume, Genetic Algorithm, Linear Programming*

1.INTRODUCTION

The problem of determining the optimal production volume is very complex and requires many relevant factors to be taken into account, both internal and external. Depending on the type of industry, the volume of production can be planned and adjusted for different time periods (eg daily, monthly, quarterly, etc.). In addition, it can be said that there is no universal way to determine the optimal volume of production, but the company's management is in charge of solving this problem. The main objective of this research is to test two methods for solving the considered problem. The first method belongs to the group of metaheuristics, and that is the Genetic Algorithm (GA) [4], while the second method belongs to the group of numerical methods, which is Linear Programming (LP).

In order to conduct a detailed analysis, GA and LP were tested under different conditions (different constraints). Accordingly, this paper considers the possibilities of applying these methods, as well as their advantages and disadvantages when considering this small-scale problem.

The basic idea for problem analysis and definition of the optimization model presented in this paper came from two scientific papers, i.e. Vidal and Goetschalckx [9] and Perron et al. [7]. In fact, Vidal and Goetschalckx [9] developed a new heuristic algorithm that successfully and according to certain rules applies linear programming to solve supply strategy problems. Referring to the idea of these authors, Perron et al. [7] solved a similar problem by applying Variable neighborhood search (VNS), for which starting point was theirresearch. So, the original idea for the analysis of this problem arose from these two papers but the considered problem is significantly modified and simplified.

In the mentioned papers, the problem is observed from the point of view of the company when looking for products (raw material) from suppliers, i.e. as a customer, relying on the costs of transport and procurement. On the other hand, in this paper, it is the case that the company should place products on the market (sell to customers). Therefore, the basic problem of this research can be presented as determining the optimal production volume in order to maximize profit.

GA is a metaheuristic that is used in the literature to solve various optimization problems in industry. Some of the problems solved by the application of GA are workflow scheduling [5], andselection of recycling center locations [8].Also, LP has found applications in solving problems in industry, such as determining the optimal production plan [6], and job sequencing and tool switching problem [2].

It is not uncommon for GA or LP to be used in combination with some other methods and approaches. Also, in some papers, a comparison of these two methods was performed [1], while some authors integrated this method into a hybrid approach [3].For these reasons, GA and LP were selected as methods to be tested in this paper to solve the considered optimization problem.The illustrated example and data presented in this paper come from a company that is part of the process manufacturing supply chain and, among other markets, operates in the Republic of Serbia.

The paper is organized as follows: The second section presents the problem statement. Section 3 presents an illustrative example through three different variants. In section 4, the discussion and conclusions are presented.

2. PROBLEM STATEMENT

Let us consider the company P_{ij}, which owns 5 manufacturing plants (factories) $i, i = 1, \dots, 5$ in different locations. Each of the manufacturing plants produces 5 different products $j, j = 1, \dots, 5$, which are the same for each plant, but with some variations. The production of each product requires 4 of the same types of raw materials $r, r = 1, \dots, 4$, and depending on the type of product, each raw material is represented in different quantities in the composition of the product. Also, it is considered that due to the distance of the plant locations, each plant has different suppliers of raw materials, and thus a different price of T_{rij}, which includes costs of transport, storage, production, labor, and other costs. In addition, companies place their products in different markets, so the unit price of C_{ij} products on the market is different for each product. The problem statement model is shown in Figure 1.

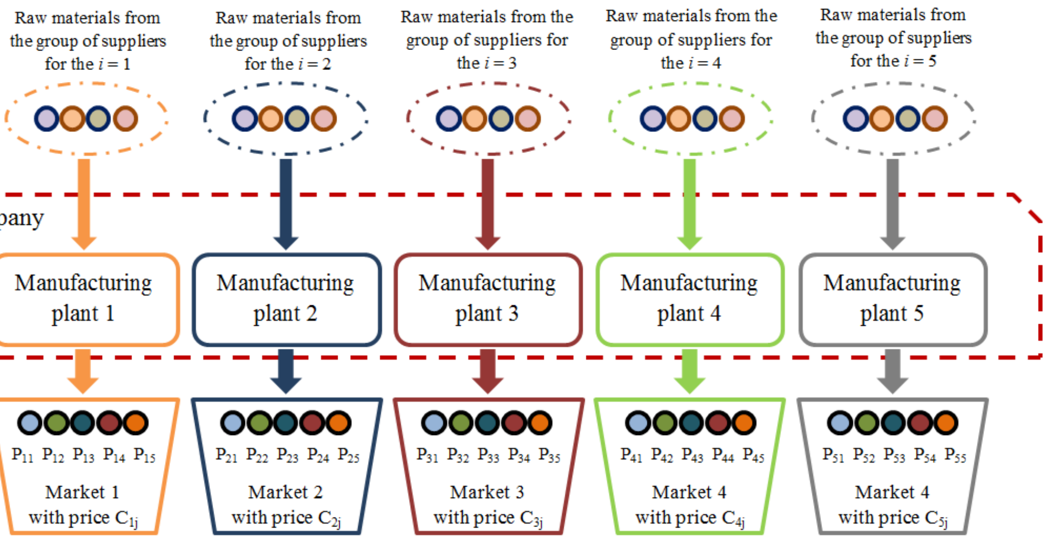

Figure 1. Graphical representation of the problem statement

Thus, the problem that this model needs to solve is to determine the optimal production volume $Q_{ij}$of each product type j in each plant i, in order to maximize the total sales profit D. In other words, it is necessary to determine how many each product type needs to be produced for each market while maximizing profits.

In this case, the objective function can be formally written as:

$$\max \left\{ \left(\sum_{i=1}^{5} \sum_{j=1}^{5} C_{ij} \cdot Q_{ij} \right) - \left(\sum_{r=1}^{4} T_{rij} \right) \right\}$$

With constraints:

(1) $C_{ij} = C'_{rij}$;

whereC'_{rij} is the unit price of the product j of the company P_{ij} on the market m_i in the considered time interval.

(2) $T_{rij} = T'_{rij}$;

where T_{rij} are costs of raw materials r needed to produce one product j in the manufacturing plant i, and T'_{rij}defined fixed costs of materials T_{rij} in the considered time interval.

(3) $Y_{ij\,min} \leq Q_{ij} \leq Y_{ij\,max}$;

where $Y_{ij\,min}$ is the minimum required quantity of product j from the manufacturing plant i to be delivered to the market m_i in the considered time interval, and $Y_{ij\,max}$ is the maximum required quantity of product j from the manufacturing plant i to be delivered to the market m_i in the considered time interval.

(4) $\min(Z_{ij}, W_{ij}) \geq Q_{ij}$;

where Z_{ij} is the total capacity of all suppliers of each manufacturing plant i to deliver the quantity of raw materials r in the considered time interval in order to produce the quantity of product Q_{ij}. W_{ij} denotes the capacity of the manufacturing plant i to produce a certain amount of product Q_{ij} in a defined time interval. The nature of these constraints is such that one value excludes another (a smaller value is taken into account).

After defining the problem statement, the following section provides an illustrative example of using GA and LP to solve a defined problem. The problem presented in this paper was solved in the software package Matlab R2018a when it comes to GA. In the case of LP, the problem was solved in the Lingo 17.0 software package.

When applying GA, the initial population was generated at random. For each of three variants in each generation, 50 individuals were selected, while the selection of individuals for crossover was performed using the rank method. The representation of individuals at the roulette wheel is determined by rank, not by proportion, in order to avoid the influence of the super-individual. Crossover and mutation were performed by a combined approach (the number and position of a crossover and the position of the mutation change through iterations, i.e. random selection of software (GA Solver)). The stop criterion is finding the optimal/suboptimal solution or performing a maximum of 2500 iterations.

3. ILLUSTRATED EXAMPLE

In order to determine the optimal production volume of each product j in each production plant i, it is necessary to define the input data. First, it is necessary to define the price of all products of the company P_{ij} in each of the markets m_i (Table 1). The price of the product on the market and raw material costs in this example are fixed and expressed in Serbian dinars (RSD). The quantity of products, i.e. the production volume, is expressed in $Q_{ij} \cdot 10^3$. Determining the optimal production volume, in this case, was done for a period of one month.

Table 1. The unit price of the product j of each manufacturing plant i in the market m_i

	Prod. 1	**Prod. 2**	**Prod. 3**	**Prod. 4**	**Prod. 5**
Mark. 1	75	89	66	56	62
Mark. 2	79	80	69	50	60
Mark. 3	80	78	63	42	72
Mark. 4	74	88	69	45	80
Mark. 5	73	85	74	51	67

After defining the selling unit price of all products, it is necessary to define the costs of procurement (price, transport, storage, production, etc.) of each type of raw material per unit of product (Table 2).

Table 2. Unit costs of procurement of raw materials needed to make each product j of each manufacturing plant i

	Product, $j = 1$				**Product, $j = 2$**				**Product, $j = 3$**				**Product, $j = 4$**				**Product, $j = 5$**			
	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Manuf. plant, $i = 1$	11	12	11	14	14	12	18	6	9	17	14	15	23	24	22	20	15	12	10	18
Manuf. plant, $i = 2$	11	11	13	11	12	15	15	14	11	13	12	22	10	14	20	20	10	15	21	27
Manuf. plant, $i = 3$	10	11	16	17	9	11	15	8	12	21	28	20	14	22	12	30	10	14	25	30
Manuf. plant, $i = 4$	10	15	28	12	12	18	16	15	11	21	23	24	19	22	22	24	11	10	21	25
Manuf. plant, $i = 5$	11	11	15	11	10	11	14	19	10	15	24	22	17	20	17	24	20	13	18	13

These data represent fixed values in the considered time interval and refer to constraints (1) and (2). This paper presents three variants of solving this problem: 1) the case when there is no limit on how much minimum product needs to be delivered to the market, 2) the case when all restrictions are taken into account and 3) when the supplier's capacity is neglected. It should be noted that the defined restrictions (1) and (2) apply to all three cases, and restrictions (3) and (4) change.

Table 3 shows the difference in a certain production volume when GA/LP (where difference exist) is used. This example is presented for variant 1, i.e. in the case when there is no limit on how much minimum product needs to be delivered to the market.

Table 3. Production volume ($\cdot\, 10^3$) for variant 1

	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
$i = 1$	840	900	980	0	849/900
$i = 2$	960	740	870	0	0
$i = 3$	850	1.060	0	0	88/0
$i = 4$	1.020	1.050	51/0	0	1.020
$i = 5$	1.100	930	950	0	990

The value of the objective function, i.e. the realized profit, in the case of GA is 314.730, while in the case of LP it is 316.210 (in both cases $\cdot\, 10^3$ RSD).The results of the other two variants will only be discussed in next section.

A discussion of the results obtained using GA and LP is given in section 4, where a review is also given of the advantages and disadvantages of applying these methods to this small-scale problem.

4. CONCLUSIONS

In this paper, GA and LP are applied to solve an identical problem, which is to determine the optimal production volume in the company of the process manufacturing.By solving each of the three mentioned variants of the optimization problem, different values of the objective function are obtained, which is a direct consequence of differently defined constraints. The values of the objective function for each variant differ, but they also differ when it comes to the same variant of the problem solved with the help of GA and LP.

The results for variant 1 have already been shown in the previous section. In variant 2, the value of the objective function for GA is 246.969, while for LP it is equal to 249.640. In the third variant, these values are 285.106 for GA and 287.550 for LP (in both cases $\cdot\, 10^3$ RSD).

From the above, it can be concluded that a better value of the objective function was obtained with the LP. The mentioned "errors" of the Genetic Algorithm caused the values of the objective function obtained by this method, in all three variants, to be slightly worse in relation to the values obtained by applying LP. Also, it can be said that the application of LP can give slightly better results in terms of the value of the objective function, but that it is a much more demanding method of application compared to GA. The number of iterations through which the problem is solved in this case cannot be taken as a measure, especially because these methods do not have the same logical basis, and LP is a solid mathematically based method, unlike GA. In addition, it is always important to emphasize that GA is extremely suitable when it comes to solving large-scale problems.

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