ANALYSIS AND SYNTHESIS OF EXTERNAL CYCLOIDAL SPUR GEARING

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Abstract : *This paper deals with a form of cycloidal gearing whose tooth profile consists of epicycloid and hipocycloid. Mathematical model, developed for determination of the gear tooth profile points coordinates, is used for the analysis of this gearing. The equations, given by the kinematic analysis, are used for construction of the diagrams of all the kinematic parameters: contact ratio, relative velocities, sliding velocities and specific sliding.*

Keywords : Cycloidal gearing, Contact ratio, Sliding velocity, specific sliding

Introduction

In this paper the kinematic parameters of the teeth pair with external cycloidal spur gearing is presented. The kinematic analysis was done for the teeth profiles contact points that perform the complex motion. Also is considered the detailed analysis of the sliding velocities in the contact points during the period, as well as the expressions for determinations of the specific sliding. As it depends on transition gear ratio of a pair, axial distance and cycloidal parameters, we constructed diagrams of the kinematic parameters which are significant for determination of the friction and wear parameters of the tooth surfaces in contact: contact ratio, relative velocities, sliding velocities and specific sliding.

Geometry of the cycloidal gearing

Active tooth profile of external cycloidal spur gearing consists of epicycloidal part, the addendum of the tooth and the dedendum is hypocycloid. Curves, which describe tooth profile, are obtained as result of the rolling, without slip, of the circles, radius r_{k1} and r_{k2} along the gear centroides, radius R_1 and R_2. To determine the geometrical and kinematic parameters of cycliodal gearing, we used the coordinate systems of the right hand orientation, for which the origin of coordinates is the same as the center of the gear. The ordinate axis of the fixed system is identical with the central axis of the gear pair, while the moving coordinate system has the ordinate which is passing through the center of the gear and the point of the tooth profile on the kinematic pitch circle. It is rotating also together with the gear to which it is fixed. All values are expressed as functions of the polar angle of rotation of gear φ which is measured from the central axis of the gear pair to the polar axis (ordinate of the moving

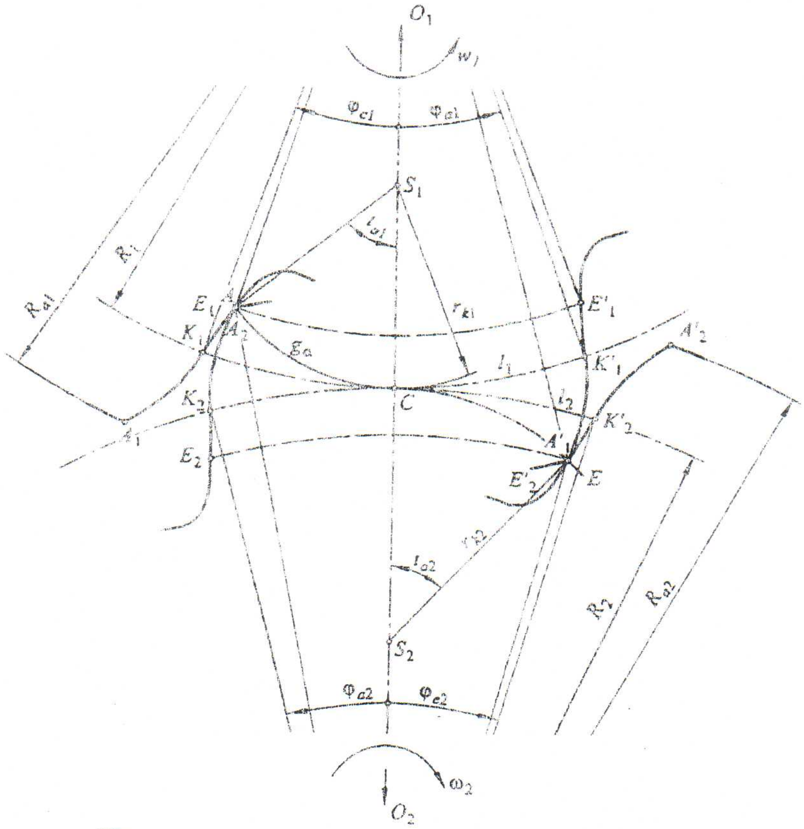

Figure 1 *Geometrical parameters of cycloidal gearing*

coordinate system) as it is shown in figure 1.

A characteristic parameter of the cycloidal curves is the module of the cycloide q, which is represented by the relation between radius rolling circle and fixed circle (radius of the centroide of the gear, in this case). If negative values of the rotation angle and the module of cycloid are used for hypocikloidal profile, than the tooth profile can be described with the uniform equations. In relation to moving coordinate system the equations of the tooth profile are of the following form:

$$x_i = R_i\left(1+q_{ji}\right)\sin\varphi_i - q_{ji}R_i\sin\left(\varphi_i + \frac{\varphi_i}{q_{ji}}\right),$$
$$y_i = R_i\left(1+q_{ji}\right)\cos\varphi_i - q_{ji}R_i\cos\left(\varphi_i + \frac{\varphi_i}{q_{ji}}\right), \qquad (1)$$

where: j - stands for the module of epi- or hipocycloid; i - subscript which defines if the angle φ_i is measured in relation to polar axis of the pinion (i=1) or of the wheel (i=2).

Further we shall determine some of the important geometrical parameters of the tooth profile in contact. Based on the geometrical relations (Fig. 3) and by application of the appropriate trigonometric transformations the formula is obtained for calculation of the distance from the contact point P to the pitch point C in the form:

$$\overline{CP} = 2\left|q_{ji}\right|R_i\sin\frac{\varphi_i}{2q_{ji}}. \qquad (2)$$

Radius of the circle, which is passing trough the contact point and is concentric with the pitch circle of the gear, is given by:

$$R_{Pi} = R_i\sqrt{1+4q_{ji}\left(1+q_{ji}\right)\sin^2\frac{\varphi_i}{2q_{ji}}}. \qquad (3)$$

Angle μ_i (see Fig. 3) between position vector of the specific profile point and tangent to the profile in this point and is calculated by the formula:

$$\operatorname{tg}\mu_i = \left(1+2q_{ji}\right)\operatorname{tg}\frac{\varphi_i}{2q_{ji}}. \qquad (4)$$

Angle χ between the tangent to the line of action and normal to profile in the pitch point P is defined by:

$$\chi = \frac{\varphi_i}{2q_{ji}}. \qquad (5)$$

Contact ratio

In the case when pinion is rotating in positive mathematical direction, then line of action is in position which is shown in figure 1. Active length of the line of action can be written as sum of the arcs, which are corresponding to contact of the addendum of the tooth profile of pinion and wheel (Fig. 1):

$$g_\alpha = l_{AC} + l_{CE}. \qquad (6)$$

In figure 1 is shown the pair of teeth in moment when they are coming in contact in point A and coming out from contact in point E. Contact of the part A_2K_2 with the part E_1K_1 can be defined as contact of the rolling circle of cycloide of the radius r_{k1}, with the pitch circles of the gears.

By this way the length of the contact arc of the pitch circles during relative motion, rolling without slip, is equal to the length of the arc of rolling circle, of the radius r_{k1}, along the pitch circle of the radius R_1 (analogly R_2), i.e., to the length of the arc l_{AC}, of the radius r_{k1}. Analogly is contact of the point A_1K_1 with E_2K_2 by the other cycloidal gear.

The number of teeth in contact, or contact ratio ε_α, is the ratio between the arc of action and the arc between successive teeth on the gear, which is defined by cycloidal tooth profile in following form:

$$\varepsilon_\alpha = \frac{l}{p} = \frac{\varphi_{ai} + \left|\varphi_{ei}\right|}{2\pi} z_i, \qquad (7)$$

where φ_{ai} and φ_{ei} are rotation angles of gear, which are corresponding to first and last contact point of active tooth profile. We shall write general equation to determination of these angles:

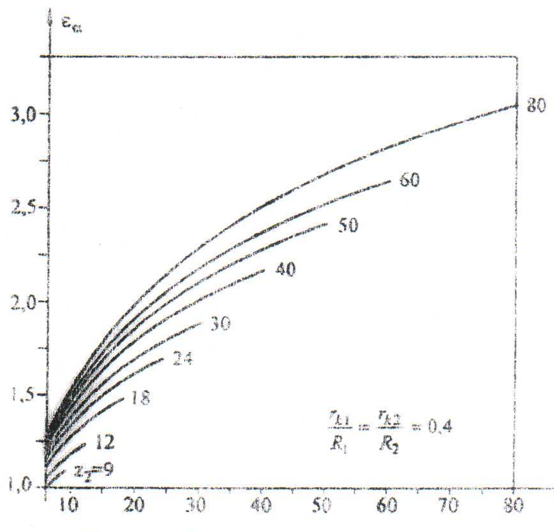

Figure 2 *Diagram of the contact ratio*

$$\varphi_i = q_{ji} \arccos \frac{q_{ji}(1+q_{ji}) + 0.5(1-Q_{Pi}^2)}{q_{ji}(1+q_{ji})}, \qquad (8)$$

where: j - stands for the module of the cycloide ($j=e,h$), i - stands for the pinion ($i=1$) or wheel ($i=2$), $Q_{Pi}=R_{Pi}/R_i$ - ratio of the radius of the circle of the observed point and radius of the corresponding pitch circle.

Based on defined and calculated relations the diagram of the contact ratio in function of the number of teeth z_1 (Fig.2) is constructed. Using the diagram, shown in figure 2, it is possible to determine the desired contact ratio for the given gear ratio (z_2/z_1) and number of teeth.

Kinematic analysis of the teeth profile in contact by cycloidal gearing

Contact point P referred to the fixed coordinate system is performing absolute motion and creating the line of the contact point - path of contact. Simultaneously, referred to the moving coordinate system the contact point is performing relative motion along the tooth profile. Motion of the moving coordinate system referred to the fixed one is transcendent, which is in fact rotation around the gear center. Absolute velocity of the point P can be written in the following form:

$$v = v_{pi} + v_{ri}, \qquad (9)$$

where v_{pi} is transcendent and v_{ri} relative velocity of the point P.

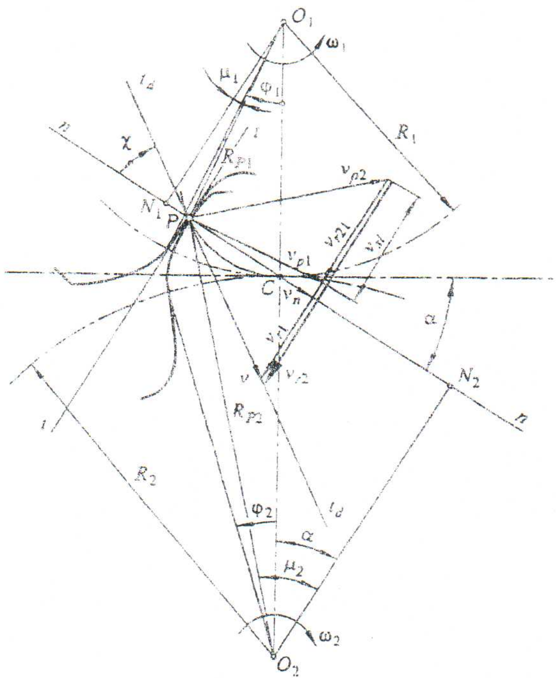

Figure 3 *Contact of the teeth profiles of cycloidal gearing*

Based on these conditions, which are defined with the low of conjugate gear tooth action, the teeth profiles in contact must have common tangent t-t and normal n-n in every point of contact, during the contact period.

Intensity of the absolute velocity of the point P, which is directed along the tangent to the line of contact in this point, is defined by kinematic and geometrical relations given in figure 3. In the procedure for determination of intensity of the absolute velocity the known relation for intensity of the transcendent velocity of the point P is used:

$$v_{pi} = \omega_i R_i \sqrt{1 + 4q_{ji}(1+q_{ji}) \sin^2 \frac{\varphi_i}{2q_{ji}}} \qquad (10)$$

By projecting of velocity vector from equation (9) on the common normal of profile and applying appropriate transformations we obtain:

$$v = R_i \omega_i = \text{const}. \qquad (11)$$

For constant angular velocities of the gears in contact, intensity of the absolute velocity of the contact point is also constant along the path of contact and equal to the intensity of the angular velocity of the pitch circles.

Vector of the relative velocities has the same direction as the common tangent of profile in the contact point. Intensities of the relative velocities of the contact points are determined trough the projection of the velocity vector from equation (9) on the common tangent of the profiles in contact:

$$v_{ri} = 2 R_i (1+q_{ji}) \omega_i \sin \frac{\varphi_i}{2q_{ji}}, \qquad (12)$$

where: i - signs the pinion or wheel ($i=1,2$), j - signs the module of epy- or hypocycloide ($j=e,h$), ω_i - absolute value of the angular velocity of the gear.

Sliding velocity of two profiles in contact is the velocity of the contact point during relative motion of the profiles. Sliding velocity is equal to the difference of the relative velocities of the contact point of the teeth profiles in contact and it is zero only when the profiles come in contact in pitch point.

By equations (2) end (12) we can obtain the general formula for determination of the sliding velocity of teeth profiles in contact:

$$v_{sl} = \overline{CP}(\omega_1 + |\omega_2|) = 2|q_{ji}|R_i(\omega_1 + |\omega_2|) \sin \frac{\varphi_i}{2q_{ji}} \qquad (13)$$

For wearing analysis, particularly for the lubrication, the value of the sliding velocity, as well as the value of the relative velocities sum in the contact point of the teeth profiles are important:

$$v_\Sigma = v_{r1} + v_{r2}. \qquad (14)$$

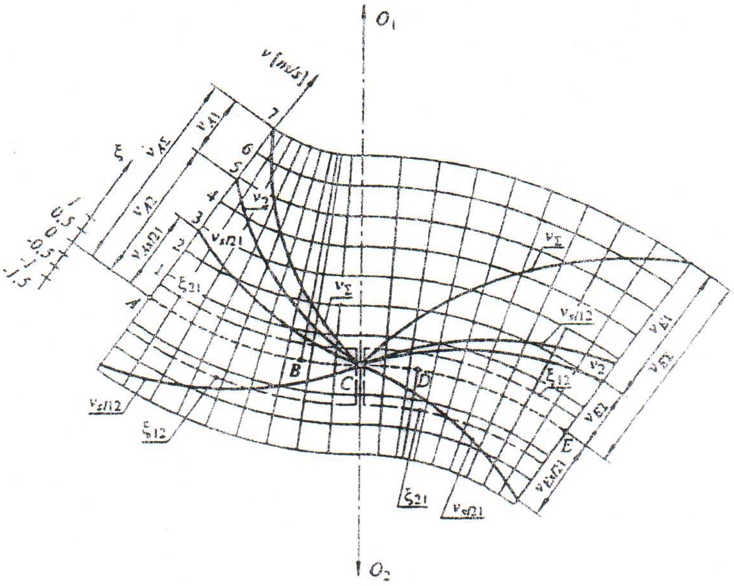

Figure 4 *Diagrams of the velocities and specific sliding*

For one pair of the gears with cycloidal gearing given $z_1=z_2=20$, $m=30$mm, $q_{h1}=q_{h2}=0.4$ and $\omega=10^{s-1}$ the values v_{r1}, v_{r2}, v_{kl} and v_Σ for the some points on the line of action are calculated and given in figure 4.

Specific sliding of the tooth profiles

For the analysis of sliding of the teeth profiles in contact and estimation of the efficiency of gears the sliding velocity and its change in relation to the corresponding relative velocity of the contact point are required. Specific sliding is relation between the sliding velocity and relative velocity of the contact point and can be written for a pinion as:

$$\xi_1 = \frac{v_{sl12}}{v_{r1}} = \frac{q_{j1}}{1+q_{j1}}\left(1+\frac{1}{|i|}\right) \tag{15}$$

and for wheel:

$$\xi_2 = \frac{v_{sl21}}{v_{r2}} = \frac{q_{j2}}{1+q_{j2}}\left(1+|i|\right) . \tag{16}$$

Diagram of specific sliding for the pair of gears with cycloidal gearing and data given above is shown in figure 4.

It can be concluded that specific sliding is influenced by the geometrical and kinematic parameters of the profiles in contact. It is clear, that in parameters in formula for specific sliding have constant values in the specific part of the tooth profile. Thus, specific sliding will be constant in these parts when gear ratio has constant value. Specific sliding will change its value and sign by the change of the module of cyclodal tooth profile. On the epycycloidal tooth profile it will be positive and on the hypocycloidal negative.

For cycloidal gearing when the contact of two teeth profiles is in the pitch point, the values of the sliding and relative velocities are equal to zero. Thus, point C is singular point for the distribution of specific sliding of the teeth profiles in contact.

Conclusion

Based on the given analysis of the external cycloidal spur gearing it can be concluded that contact ratio condition ($\varepsilon_\alpha>1$) is also fulfilled for small number of teeth.

For the higher values of module of hipocycloide higher values of contact ratio are obtained for the same number of teeth.

Analysis of velocities of the contact point motion of the teeth profiles shows that the absolute velocity of the contact point along the path of contact is constant and equal to the pitch line velocity.

Specific sliding is constant along the cycloidal profile when the geometrical parameters are constant.

References

[1] **Savelov A. A.** : Ploskie krive, Moskva, Fizmatgiz, 1960.

[2] **Leviitski K.V.** i dr. : Teorija mehanizmov i masin, Moskva, Nauka, 1979.

[3] **Hlebanja J.** : Izgube moci pri zobnikih s posebnim ozobjem, Strojniski vestnik, Ljubljana, (4-6) 1983.

[4] **Niemann G., Winter H.** : Maschinenelemente, Band II, Springer - Verlag, Berlin, 1989

[5] **Luck K., Modler K. H.** : Getriebetechnik, Analyse-Synthese-Optimierung, Akademie-Verlag, Berilin, 1990.