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STABILITY ANALYSIS OF CONCRETE ARCH DAM USING FINITE ELEMENT METHOD

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Abstract: This paper presents the procedure of numerical stability analysis of a concrete arch dam using the finite element method. Based on the geometry of the dam and the associated rock mass, an optimal finite element mesh of the model was created. Boundary conditions and loads were set so that they correspond to the real conditions of exploitation of the dam. The rock mass is divided into five quasi-homogeneous zones in accordance with the geological maps obtained from the field survey. The mechanical behavior of the dam model and the surrounding rock mass is described using the Hoek-Brown material model. The parameters of the material model were obtained through the identification process using experimental tests of soil samples at the site and in the vicinity of the structure. Numerical simulations of filtration, thermal and stress-deformation processes at the dam are performed using PAK software. In the presented analysis, the control of the permissible compressive and tensile stresses is performed according to the USBR recommendations, where the distance between the actual stress and the permissible stress in the concrete is shown. The global safety factor of the dam is determined using the shear strength reduction method.

Key words: Concrete arch dam, Finite element method, Hoek-Brown material model, PAK, Stability analysis

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1 INTRODUCTION

This paper presents the procedure for the stability analysis of the concrete arch dam using the finite element method [1, 2]. The analysis procedure is shown on the „Komarnica“ dam model. Komarnica is a river located in Montenegro on which the construction of a concrete arch dam is planned. Based on the geometry of the dam and the associated rock mass, a three-dimensional finite element model was created. Based on the conducted experimental tests, geological maps of the terrain were obtained. Within the FE model, the associated rock mass is divided into 5 quasi-homogeneous environments using obtained geological maps of the terrain. The Hoek-Brown material model [3, 4, 5] is used to describe the mechanical behaviour of the dam and the surrounding rock mass. The material model parameters are obtained by the identification procedure. For the purpose of parameter identification experimental tests of soil samples at dam site and surrounding area are used. Boundary conditions and loads are specified on the FE model so that they correspond to the real conditions of dam exploitation. Numerical analysis is performed using PAK software [6, 7], while FEMAP [8] software is used for pre and post-processing of the model. The results of the stability analysis of the dam are given in the paper. The global factor of safety of the dam is calculated using the shear strength reduction method. The value of the factor of safety for the specified load case is given in the paper. Permissible compressive and tensile stress according to USBR recomendations is used as strength criterion. Results of the permissible compressive and tensile stresses control according to the USBR recommendations is given [9].

In the second chapter, the theoretical basis of the Hoek-Brown material model, shear strength reduction method and control of permissible compressive and tensile stresses are given.

In the third chapter, the 3D FE model of the dam is described. In this chapter boundary conditions and loads assigned to the model, as well as the material parameters of the Hoek-Brown model are given.

In the fourth chapter, the results of stability analysis, control of permissible compressive and tensile stresses, as well as the obtained factor of safety are given.

2 THEORETICAL BASIS

2.1 Hoek-Brown material model

The mechanical behaviour of the rock mass is most often described using Hoek-Brown material model. This material model combines the results of research into the brittle failure of intact rock by Hoek [10] and on model studies of jointed rock mass behaviour by Brown [11]. Initially, this model started from the properties of intact rock and then added factors to reduce these properties based on the characteristics of joints within a rock mass [5].

The failure surface of Hoek-Brown material model is function of stress state and can be defined by applying the stress invariants in following form [13]:

$$f = \frac{I_1}{3} m_b \sigma_{ci} + m_b \sigma_{ci} \sqrt{J_{2D}} \left(\cos\theta - \frac{1}{\sqrt{3}} \sin\theta \right) - s\sigma_{ci}^{\ 2} + 4 J_{2D} \cos^2\theta = 0 \qquad (2)$$

where σ_{ci}, m_b, s, a represent material model parameters, while I_1, J_{2D} and θ are stress invariants.

The failure surface of this material model represents an irregular six-sided pyramid of hyperboloid sides whose axis, in main stresses space, coincides with the hydrostatic axis, as shown in Fig. 1.

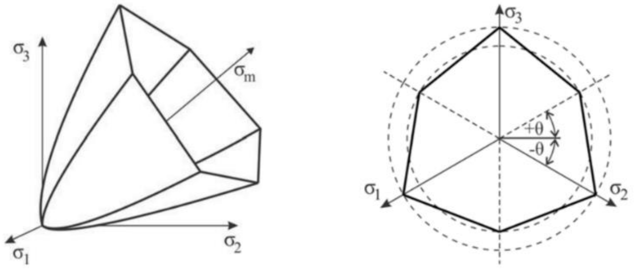

Figure 1. *Failure surface of Hoek-Brown material model*

2.2 Global factor of safety

The global factor of safety of the dam is determined using the shear strength reduction method. It represents the ratio of the material shear strength and the shear stresses in the material. This means that the global stability loss of the structure will occur in case when shear stress value in the material τ becomes equal to the value of the shear strength of the material τ_f [14]. The shear strength reduction can be described by the equation:

$$\tau^{red} = \frac{\tau_f}{F} \tag{3}$$

When it comes to the numerical stability analysis of dam, the initial stress state is first determined, where the reduction factor has a value of 1. Then the reduction factor is gradually increased, which reduces the material shear strength. The reduction factor is increased until there is a loss of dam stability i.e. until it is impossible to achieve the convergence or a increase the displacement increment. The maximum value of the shear strength reduction factor for which the dam is stable is the value of the global factor of safety *F* [15].

2.3 Strength criteria

Permissible compressive and tensile stress in accordance with the recommendations is used as strength criterion. Control of permissible compressive and tensile stresses, according to USBR (United States Bureau of Reclamation) recommendations, implies that the permissible compressive stress value, for the defined load case, is 10.3 MPa and the value of the permissible tensile stress is 1.03 MPa. Due to the triaxial stress state, an envelope of permissible stress values is formed, which is obtained by taking in to account the permissible compressive stress and the permissible tensile stress. For each integration point in the FEM model, the triaxial stress components are calculated, which is compared with the allowable stress value obtained on the basis of the allowable compressive and tensile stress values.

From the failure function of Hoek-Brown material model (2) material constants m_b and s are determined, while σ_{ci} is adopted to be the same value as permissible compressive stress. By solving the system of equations, the values of the material parameters s and m_b are obtained. By substituting the calculated values of the parameters m_b and s into the failure function of the Hoek Brown material model, the maximum value of the second stress invariant deviator for the envelope of permissible stresses $q^* = \sqrt{J_{2D}}$ can be determined. On the other hand, using the obtained values for the principal stresses σ_1, σ_2 and σ_3, stress deviator q can be determined. Finally, the distance from the surface representing the permissible stress envelope calculated by:

$$u = \left(1 - \frac{q}{q^*}\right) \cdot 100 \tag{5}$$

In this way, field of distance between the actual and permissible stress is formed, where, based on graphical display, more can be concluded about the load of structure.

3 3D MODEL OF CONCRETE ARCH DAM

3.1 Model geometry

The model geometry includes the dam, associated rock mass with geotechnical environments and injection curtain (Fig. 4). The model dimensions are 800x480 m. The lowest elevation of the model is 300 m asl, while the highest is 1000 m asl.

Figure 4. *Model geometry*

The concrete part of the model represents dam. The rock mass is modelled to include geological structures (faults and other characteristic zones) relevant to the model. In order to more realistically model the complex rock structure in the dam area, the rock mass is divided into 5 rock environments. The injection curtain, is also included in the model. Depth of the injection hols is 106 m at the lowest part of the dam, while the curtain thickness in the model is 5 m.

3.2 Finite element model

The finite element mesh of the arch dam was created using parabolic tetrahedral finite elements. The model consists of 1.3 million nodes and about 1.0 milion elements.

In order to achieve the optimal number of finite elements, different element sizes per volume were defined in different model zones. Elements with an average size of 3.5 m are generated in the concrete dam.

The terrain geometry is divided into five quasi-homogeneous zones based on experimental research. Zones are labelled A to B, where A and B represent fault structures. In order to preserve mesh elements quality, minimum size of the elements in the rock mass is dictated by the dimensions of the quasi-homogeneous zones. In the zones of narrow faults (width 1.5 m), the average size of the elements is 5 m with a gradual transition to a size of 10 m in faults of a larger width. The elements gradually increase in size up to the model boundary, where the average size of the elements is 40 m. By controlling the element sizes in different zones, the number of elements is minimized while preserving the optimal mesh quality. Fig. 5 shows finite element mesh of model in isometric view.

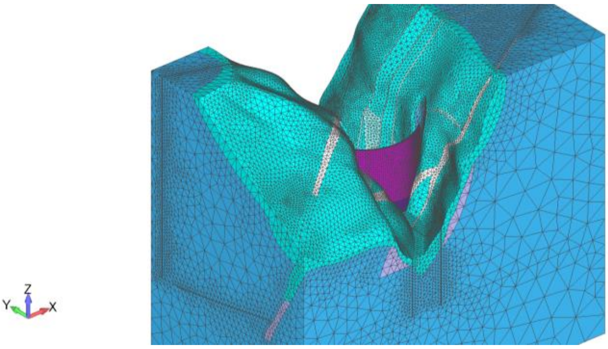

Figure 5. *Finite element mesh of model – 3D view*

3.3 Loads and boundary conditions of the model

Boundary conditions and loads are set so that they correspond to the real conditions of exploitation of the dam. The model dimensions are defined in such a way as to eliminate the influence of boundary conditions on the analysis.

The boundary conditions that were applied on the model are: fixed bottom of the model and constrained horizontal displacement of nodes in the direction perpendicular to the vertical sides that represent the model boundaries.

The loads used in the stability analysis of the dam are dead (self) weight, hydrostatic pressure, thermal loads and filtration forces.

In the model, the thermal effect is applied as follows: on the upstream and downstream surfaces below water level, the water temperature is set (temperature of reservoir water and downstream water). The water temperature is set as a boundary condition on the wetted surfaces of the model according to Bofang's distribution by accumulation depth [16]. The air temperature is set on all other surfaces of the terrain and the dam. At the bottom of the model, the rock temperature is applied, which can be considered constant after certain depth. Based on these boundary conditions, a steady state thermal analysis is performed. The temperature field obtained by the thermal analysis is then used in the structural analysis in order to take into account the influence of temperature on the stress-strain state of the dam.

The hydrostatic pressure is set on all model surfaces below water level. Therefore, there are pressures on the dam and terrain from the reservoir water, as well as pressures from the downstream water.

The filtration forces are determined in the analysis of filtration and are used as loads in the stability analysis. On the upstream surfaces of the dam and the terrain, a hydraulic potential corresponding to the water level in the reservoir is set. On the downstream surfaces of the dam and the terrain, a hydraulic potential corresponding to the water level on the downstream side is applied.

The model is loaded with 4 independent loads, which ultimately act simultaneously. In the first phase, only self-weight acts on model. After the self-weight reaches a maximum constant value, the displacements are reset, and obtained stress values represent the initial stress state for the next phase. In the second phase, hydrostatic pressure is applied on the model. After that, the third phase defines the effect of filtration forces. In the fourth, final phase, temperature is applied to the model.

The load case considered in this paper includes the following load combinations:

- water level in the reservoir at an altitude of 811.0 m asl and
- winter temperature conditions

3.4 Material parameters

The material model used for the numerical stability analysis of the dam is the Hoek-Brown material model. The material parameters for different zones in the model are given in Table 1.

Table 1. *Material parameters*

Model zones	σ_{ci} [MPa]	m_b [-]	s [-]	a [-]	Elasticity modulus (MPa)
Concrete	40.00	9.4860	0.4475	0.5000	34000
GT env. - A	61.48	0.8794	0.0008	0.5000	3400
GT env. - B	62.59	1.0746	0.0014	0.5001	5100
GT env. - C	49.25	1.7820	0.0065	0.5040	15250
GT env. - D	71.75	2.1300	0.0113	0.5030	24629
GT env. - E	92.50	2.5790	0.0205	0.5020	36051

Experimental tests were performed on the samples collected from the dam construction site. Based on the results of the experimental tests, using identification procedure, parameter values for different model zones are obtained.

4 ANALYSIS RESULTS

4.1 Stress-strain analysis

The analysis results are shown in 3D view and a vertical section along the dam axis. Fig. 7 shows the calculation results in form of the failure distance:

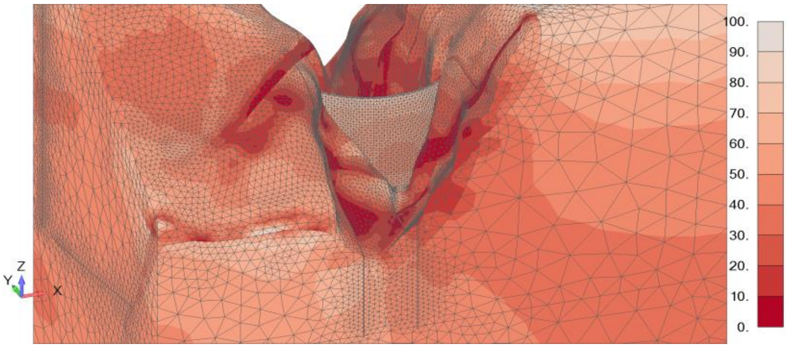

Figure 7. *Failure distance – 3D view*

Based on the Fig. 7, it can be concluded that the rock mass zones around the dam have lower values of failure distance. Therefore, those zones are more critical and eventual failure of the structure may occur within them. Zones further away from the structure have higher values of failure distance.

4.2 Strength criteria

Permissible compressive and tensile stress in accordance with the recomendations is used as strength criterion. The control of permissible compressive and tensile stress is performed for arch dam according to the USBR recommendations, as described in chapter 2.3. Fig. 9 shows the fields in which the permissible stresses for defined load case are exceeded (red colour) or satisfied (blue colour). Results are shown in upstream and downstream dam face.

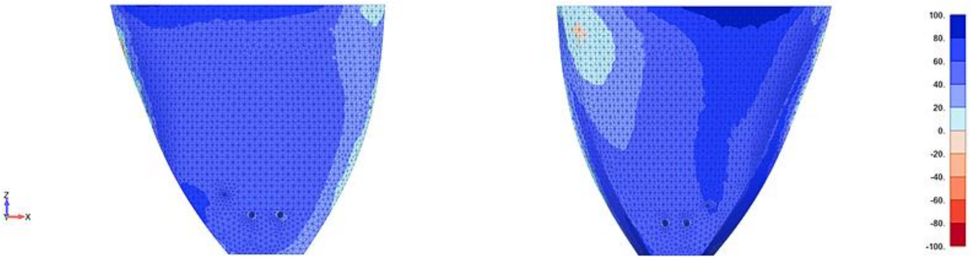

Figure 9. *Permissible compressive and tensile stress distance – upstream view*

Based on the Fig. 9 it can be concluded that strength criteria is satisfied in the greater part of the dam. In the dam flanks, smaller zones in which permissible stresses are exceeded can be observed.

4.3 Dam factor of safety

After the numerical analysis of the model for the defined load case is performed, the dam factor of safety is determined. The factor of safety is determined by the shear strength reduction method, as described in chapter 2.2. The obtained value of dam safety factor is 2.79.

5 CONCLUSIONS

In this paper, a numerical stability analysis of concrete arch dam is performed using the finite element method. An optimal finite element mesh is created based on the geometry of the dam and the surrounding rock mass. Boundary conditions and loads corresponding to real dam exploitation conditions are set on the model. The Hoek-Brown material model is used to describe the mechanical behaviour of the model. The calculation was performed using PAK software. The global factor of safety is determined using the shear strength reduction method. In addition to the calculation results and the global factor of safety, control of permissible compressive and tensile stresses according to USBR recommendations is given. The results obtained by FEM analysis show that this procedure can be used for effective stability analysis of concrete arch dams. It can be concluded that the finite element method has become almost indispensable in the stability analysis of geotechnical structures because it takes into account all the specificities of the structure with much greater reliability, unlike classical, analytical methods.

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