8th International Congress of Serbian Society of Mechanics
Kragujevac, Serbia, June 28-30, 2021

MATERIAL PARAMETERS IDENTIFICATION OF CONCRETE DAMAGE PLASTICITY MATERIAL MODEL

Dragan M. Rakić[1], Aleksandar S. Bodić[1], Nikola J. Milivojević[2], Vladimir Lj. Dunić[1], Miroslav M. Živković[1]

[1] University of Kragujevac,
Faculty of Engineering, Sestre Janjić 6, 34000 Kragujevac
e-mail: drakic@kg.ac.rs, aleksandarbodic.997@gmail.com, dunic@kg.ac.rs, zile@kg.ac.rs
[2] Jaroslav Černi Water Institute, Jaroslava Černog 80, 11226 Pinosava, Belgrade
e-mail: nikola.milivojevic@jcerni.rs

Abstract:

This paper presents a procedure for identifying concrete damaged plasticity material model parameters. A concrete damaged plasticity model is used to describe concrete behavior under quasi-static cyclic or dynamic loads. The theoretical basis of this material model and review of all parameters that define it are presented in the paper. The concrete damaged plasticity model parameters were determined by using experimental results of cyclic loading-unloading uniaxial compression and tension tests. A stress-strain dependence diagram is created based on the experimental data. That dependence is used to determine stress-plastic strain, stress-degradation diagrams and to identify material model parameters. Verification of identified parameters is performed using PAK software package. Finite Element Method (FEM) model was created for cyclic loading-unloading uniaxial tests simulations and numerical simulations are performed using the concrete damaged plasticity model. Results obtained by numerical simulations are compared with experimental results and numerical results from the literature. By comparing the results, it is concluded that the concrete damaged plasticity model parameters can be efficiently identified by applying proposed procedure.

Key words: concrete damaged plasticity model, PAK software, finite element method, material parameters identification

1. Introduction

Analysis of concrete structures based on the Finite Element Method (FEM) has become almost unavoidable in technical practice. As concrete consists of several materials that show different compressive and tensile properties, defining damage in concrete is very complex. Material models are used for this purpose, including concrete damaged plasticity material model. This material model is often used in technical practice due to its practicality and precision in describing the mechanical behavior of concrete structures.

In this paper, the material model parameter identification is performed, followed by numerical verification of cyclic loading-unloading uniaxial compression and tension tests of concrete specimens in PAK software package [1]. The paper contains the theoretical basis of the concrete damaged plasticity model for which the material parameters were identified and used to simulate the mechanical behavior of concrete in uniaxial tests. The procedure for identifying the parameters of the concrete damaged plasticity model based on experimental data from the literature [2, 3] is presented in the third chapter. At the end of the paper, numerical simulations of cyclic loading-unloading uniaxial compression and tension tests are presented and compared with experimental results and numerical results from literature [4].

2. Theoretical basis of concrete damage plasticity material model

The concrete damaged plasticity material model is used to describe mechanical behavior of concrete structures under quasi-static cyclic or dynamic loads. This material model defines the behavior of concrete using the yield function originally developed by Lubliner et al [5] and improved by Lee and Fenves [6].

According to the incremental theory of plasticity, the tensor of the total strain can be decomposed into elastic and plastic strain [7]:

$$\mathbf{e} = \mathbf{e}^E + \mathbf{e}^P . \tag{1}$$

The elastic part is defined by following equation:

$$\mathbf{e}^E = \mathbf{C}^{-1}\boldsymbol{\sigma} , \tag{2}$$

where $\mathbf{C}$ represents the elastic stiffness tensor.

The plastic part is an irreversible part of the total strain, which by applying equations (1) and (2), can be expressed by the relation:

$$\mathbf{e}^P = \mathbf{e} - \mathbf{C}^{-1}\boldsymbol{\sigma} . \tag{3}$$

Damage that occurs in the material is most often defined using stiffness degradation. The following equation defines elastic stiffness via scalar stiffness degradation parameter [7]:

$$\mathbf{C} = \left(1 - d\right)\mathbf{C}_0 , \tag{4}$$

where d represents the degradation, and $\mathbf{C}_0$ is the initial stiffness matrix.

Based on equation (4), current value of Young's elasticity modulus can be defined as:

$$E = (1-d)E_0 , \tag{5}$$

where E_0 denotes the initial value of Young's elasticity modulus.

Yield function of concrete damaged plasticity material model is defined using following equation [7, 8]:

$$F(\bar{\boldsymbol{\sigma}},\boldsymbol{\kappa}) = \frac{1}{1-\alpha}\left(\alpha \bar{I}_1 + \sqrt{\frac{3}{2}}\left\|\bar{\mathbf{S}}\right\| + \beta\left(\boldsymbol{\kappa}\right)\left\langle \bar{\sigma}_{\max} \right\rangle\right) - c_c\left(\boldsymbol{\kappa}\right) \le 0 , \tag{6}$$

where:

- $c_c\left(\boldsymbol{\kappa}\right) = \bar{\sigma}_c\left(\boldsymbol{\kappa}\right)$ - material cohesion
- $\bar{I}_1 = \mathrm{tr}\bar{\boldsymbol{\sigma}}$ - first stress tensor invariant
- $\left\|\bar{\mathbf{S}}\right\| = \sqrt{\bar{\mathbf{S}} : \bar{\mathbf{S}}}$ - stress tensor deviator norm
- $\bar{\mathbf{S}} = \bar{\boldsymbol{\sigma}} - \bar{\sigma}_m \mathbf{I}$ - deviator of effective stress: $\bar{\boldsymbol{\sigma}} = \mathbf{C}_0 : \left(\mathbf{e} - \mathbf{e}^P\right)$
- $\bar{\sigma}_m = \frac{1}{3}\mathrm{tr}\bar{\boldsymbol{\sigma}}$ - mean effective stress
- $\bar{\sigma}_{\max}$ - algebraic maximum of eigenvalues of effective stress tensor $\bar{\boldsymbol{\sigma}}$

The dependence of the effective stress on the strain for tension and compression was shown in Fig. 1.

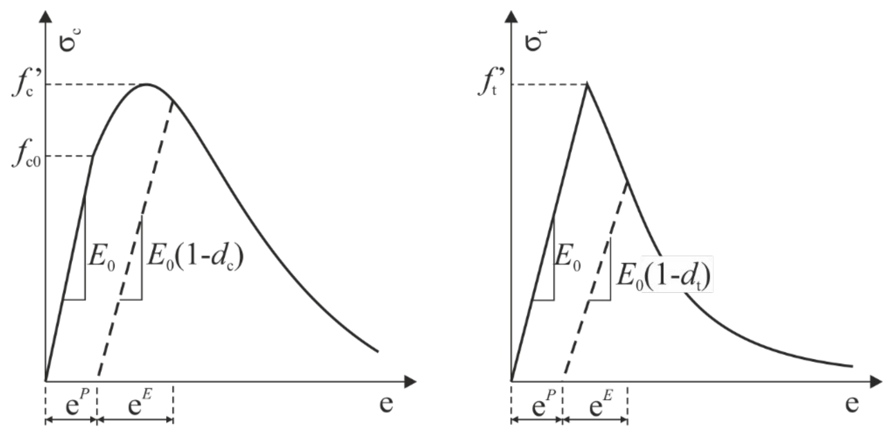

Fig. 1. Dependence of effective stress on strain for: a) compression and b) tension

Maximum stress (f_c', f_t') and yield stresses (f_{c0}, f_{t0}) for compression and tension are defined with following equations [7], respectively:

$$f_c' = f_{cm} = f_{c0}\frac{\left(1+a_c\right)^2}{4a_c}, \tag{7}$$

$$f_t' = f_{t0}, \tag{8}$$

where a_c represents compression curve parameter.

A review of the concrete damaged plasticity model parameters and their identified values for experiments from the literature [2, 3] are given in the next chapter.

3. Material model parameter identification

Identification of concrete damaged plasticity model parameters is performed by using experimental results of the cyclic loading-unloading uniaxial compression and tension tests.

Diagrams of stress-strain dependencies for uniaxial compression and uniaxial tension tests from literature [2, 3] are shown in Fig. 2.

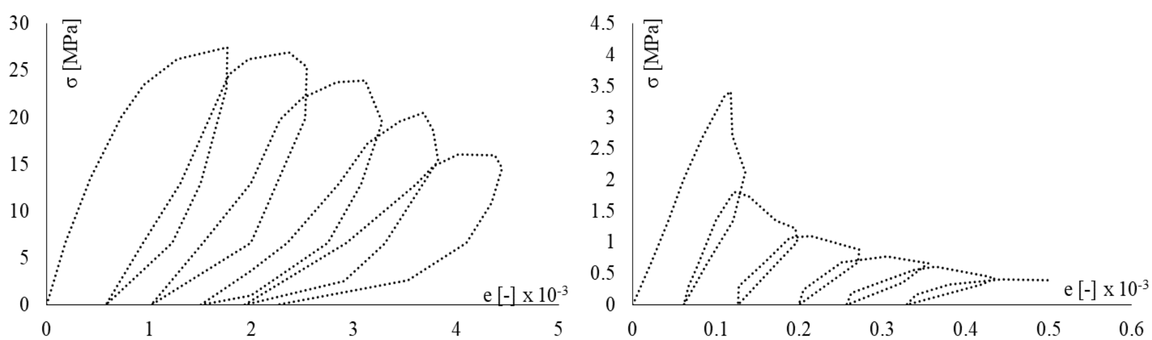

Fig. 2. Diagrams of stress-strain for a) uniaxial compression, b) uniaxial tension

Based on the stress-strain dependence, using basic relations from the theory of plasticity and damage theory, stress-plastic strain and stress-degradation dependences can be determined. According to equation (1), the total strain can be decomposed into elastic and plastic strain. Young's elasticity modulus represents the dependence between stress and strain in the elastic behavior region of a material, which is defined by Hooke's generalized law [9]:

$$\boldsymbol{\sigma} = \mathbf{C}^E \mathbf{e}^E, \tag{9}$$

from which it is possible to determine the value of the Young's elasticity modulus, by applying equivalent expression for axial load:

$$\sigma_z = E e_z^E \, . \tag{10}$$

Young's elasticity modulus represents the initial slope of the curve (linear behavior), so it can be easily determined using the stress-strain diagram.

Value of degradation can be determined at each unloading cycle of the specimen based on the current and initial elasticity modulus of the material by using equation (5). Based on the plastic strain and degradation values, stress-plastic strain and stress-degradation diagrams for uniaxial compression and uniaxial tension tests can be formed for each load-unloading cycle.

The stress-plastic strain and stress-degradation dependence, for a uniaxial compression and tension tests, obtained using diagrams shown in Fig. 2 and relations given in the previous section, are shown in Fig. 3 and Fig. 4, respectively.

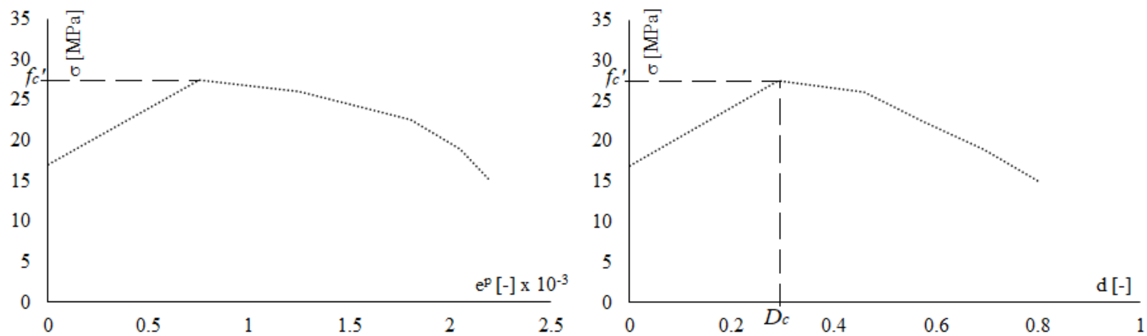

Fig. 3. Diagrams a) stress-plastic strain and b) stress-degradation for uniaxial compression

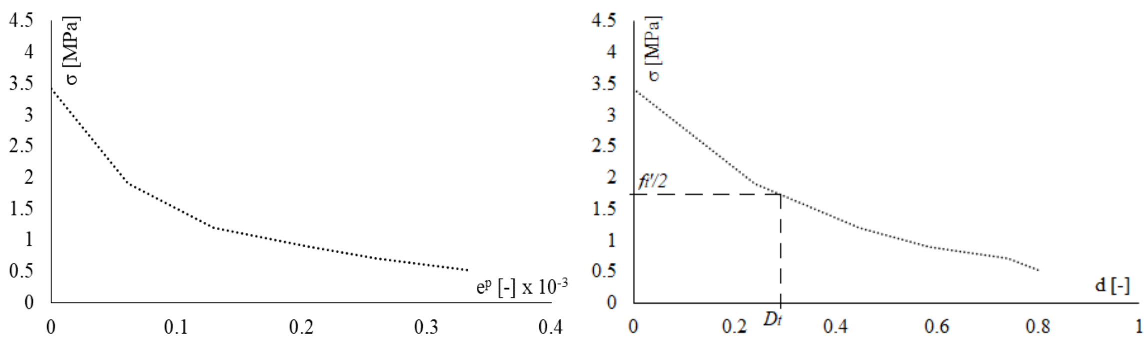

Fig. 4. Diagram a) stress-plastic strain and b) stress-degradation for uniaxial tension

The concrete damaged plasticity model parameters can be determined from stress-strain, stress-plastic strain and stress-degradation dependencies for uniaxial compression and tension tests in the manner shown below. The concrete damaged plasticity model parameters, their interpretation and description of method used for their determination based on the experiment results are given below.

The compressive strength (f_c') and the tensile strength (f_t') represent maximum values of stress that are reached during compression and tension tests (Fig. 1) and can be determined directly from the stress-strain diagram, shown on Fig. 2.

The compression curve shape parameter (a_c) defines relation between the material compressive strength and the value of yield stress in uniaxial compression test and can be calculated by equation (7).

The parameter D_c defines degradation value which corresponds to the compressive strength and can be determined from the stress-degradation diagram shown on Fig. 3b.

The degradation value which corresponds to half the value of tensile strength $f_t'/2$ in uniaxial tension test is denoted as D_t and can be determined from diagram on Fig. 4b.

The compressive fracture energy (G_c) represents the area under the stress-plastic strain curve in Fig. 3a. Analogous to the compressive fracture energy, the tensile fracture energy (G_t) represents the area under the stress-plastic strain curve in Fig. 4a.

The dilatation parameter is denoted as α_p and can be determined as the tangent of dilatation angle ψ. This parameter cannot be determined from uniaxial tests, so predefined value is used in this paper.

The triaxial compression parameter (γ) defines the ratio of uniaxial and biaxial yield stress for compression (α), and dilatation cannot be determined from uniaxial compression and tension tests and are used as predefined values in this paper. Parameter D_{cr} represents the critical value of degradation, i.e. the maximum value of degradation that can be achieved under load.

Table 1 shows a review of the concrete damaged plasticity model parameters obtained using identification procedure described previously and used in the numerical simulation of uniaxial compression and uniaxial tension tests, shown in the next chapter.

Parameter	Value
E	3.1·107
ν	0.2
f_c'	2.75·104
f_t'	3.42·103
ac	4.234
Dc	0.385
at	0.5
Dt	0.28
Gc	61
Gt	4.9·10-1
αp	0.26
α	0.12
γ	3
Dcr	0.95

Table 1. Identified parameters of concrete damage plasticity material model

4. Verification

Numerical simulations of cyclic load-unload uniaxial compression and tension tests are performed in FEM software package PAK [1] using the concrete damaged plasticity material model. Material parameters whose identification procedure is presented in the previous section are used. The results obtained by numerical simulations are compared with the experimental results [2, 3] and numerical results from literature [4].

Schematic representation of the model for numerical simulation of uniaxial compression and uniaxial tension tests, with defined boundary conditions and loads is shown in Fig. 5a. Dimensions of specimen are 0.5 m x 0.5 m x 1 m.

For numerical simulation, three-dimensional tetrahedral finite elements with midside nodes were used. The FEM model consists of 24 elements and 65 nodes (Fig. 5b).

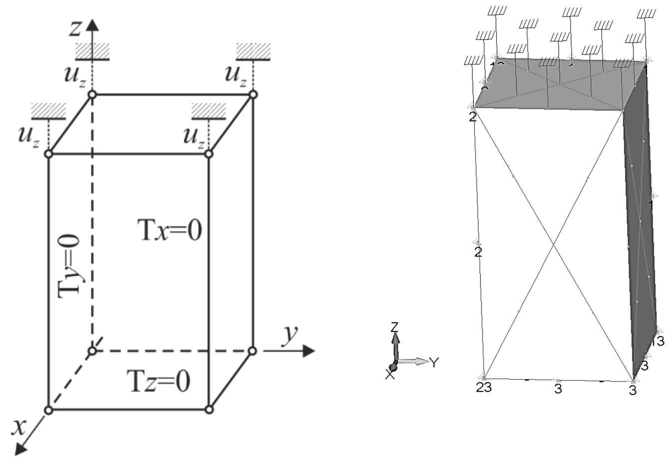

Fig. 5. Specimen model for uniaxial tests a) schematic representation, b) finite element model

Boundary conditions of the model are defined to correspond to the specimen experimental test conditions: nodes located in symmetry planes have boundary conditions of symmetry in that plane, as shown in Fig. 5b. The load of the model is set as prescribed displacement in the direction of the *z*-axis at the nodes on the upper surface of the model.

4.1 Uniaxial compression test

For numerical simulation of cyclic loading-unloading uniaxial compression test, direction of u_z displacement shown in Fig. 5 is opposite. Therefore, the value of the displacement u_z is negative and multiplied by the load function used in the experimental compression test. The numerical simulation results are compared with experimental results and numerical results from literature and are presented in Fig. 6 in the form of stress-strain dependence.

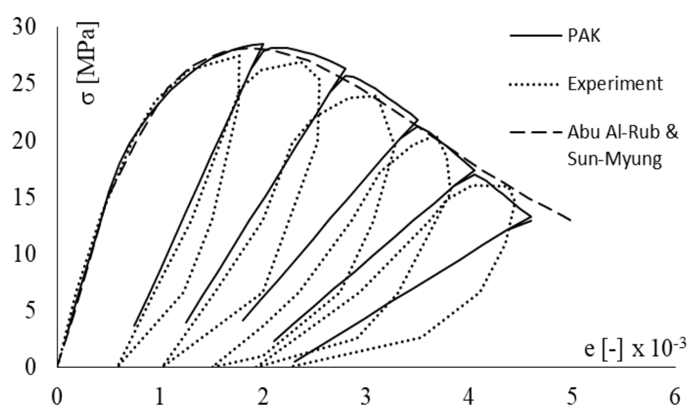

Fig. 6. Diagram stress-strain

In addition to the stress-strain diagram, Fig. 7 shows compared stress-plastic strain and stress-degradation dependence for numerical simulation in program PAK and experimental results.

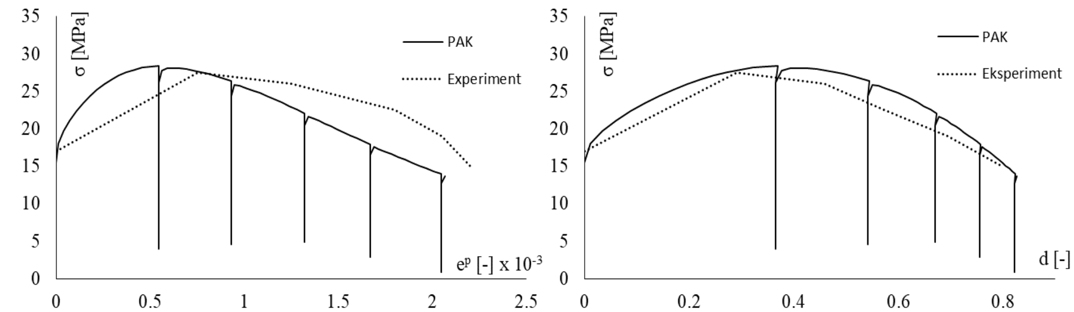

Fig. 7. a) Diagram stress-plastic strain, b) diagram stress-degradation

The numerical simulation results by character correspond to the experimental results, while values of the results deviate slightly. The deviation of numerical simulation results values from the experimental results values is not significant, so the match of previously compared diagrams is accurate.

4.2 Uniaxial tension test

For numerical simulation of cyclic loading-unloading uniaxial tension test u_z displacement shown in Fig.5a, which acts in the *z*-axis direction, is used. The prescribed displacement values are defined by tensile load function used in the experimental test. The numerical simulation results are compared with experimental results and numerical results from literature and are presented in Fig. 8 in the form of stress-strain dependence.

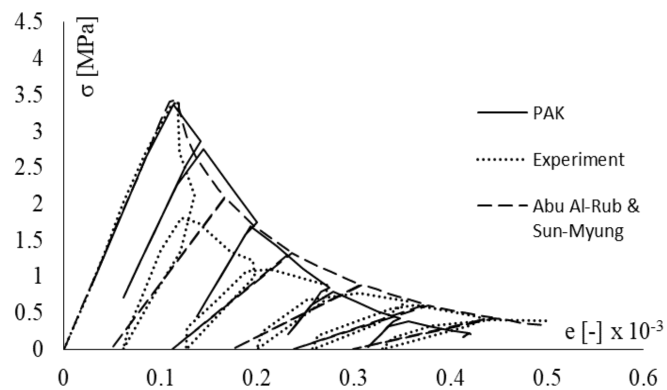

Fig. 8. Diagram stress-strain

In addition to the stress-strain diagram, the dependences stress-plastic strain and stress-degradation obtained by numerical simulation in program PAK are compared with experimental results. Comparative view of these quantities is shown in Fig. 9.

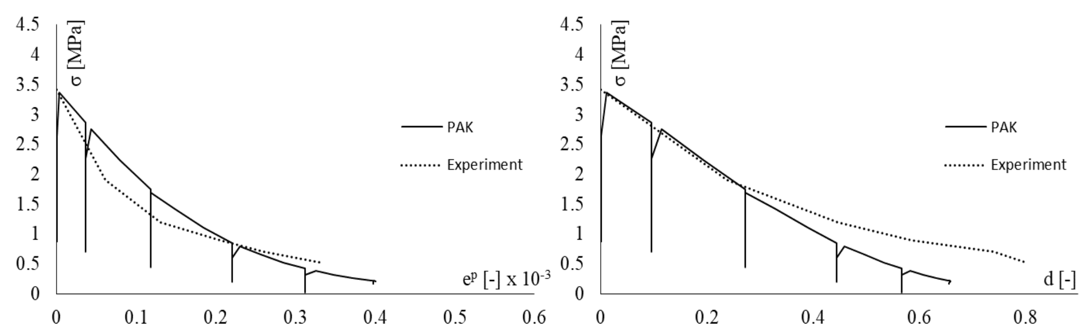

Fig. 9. a) Diagram stress-plastic strain, b) diagram stress-degradation

Analogous to results of compression test simulation, the numerical tensile test simulation results by character correspond to the experimental results. There are deviations in the results values that are not significant because match of curves shown in Fig. 8 and Fig. 9 is accurate.

5. Conclusions

This paper presents the procedure for identifying concrete damaged plasticity model parameters based on experimental data taken from the literature. Based on the experimental data of cyclic loading-unloading uniaxial compression and tension tests, stress-strain dependence diagram was created. This experimental data was used for determination stress-plastic strain and stress-degradation dependence. These dependencies are necessary to determine the parameters of the material model, using the proposed procedure. For the purpose of verification of the identified material model parameters, appropriate finite element models were created for numerical simulation of cyclic loading-unloading uniaxial compression and tension tests. Numerical simulation is performed using PAK software package, which has implemented concrete damaged plasticity material model, whose theoretical basis are presented in the paper. Numerical results are compared with the experimental data and the results of numerical simulation from literature. By comparing the obtained results, it can be concluded that, the concrete damaged plasticity material model parameters can be efficiently identified using proposed procedure, on the basis of cyclic loading-unloading uniaxial compression and tension tests.

Acknowledgements

This research is partly supported by the Ministry of Education and Science, Republic of Serbia, Grant TR32036 and Grant TR37013.

References

[1] Kojić M., Slavković R., Živković M. and Grujović N., *PAK-S: Program for FE Structural Analysis*, Kragujevac: University of Kragujevac, Faculty of Engineering, 2011.

[2] Karsan D. and Jirsa J. O., *Behavior of Concrete Under Compressive Loadings*, Journal of the Structural Division, Vol. 95, 1969.

[3] Taylor R., *FEAP: a finite element analysis program for engineering workstation. Rep. No. UCB/SEMM-92 (Draft version)*, University of California: Berkeley: Department of Civil Engineering, 1992.

[4] Abu Al-Rub R. K. and Kim S.-M., *Computational applications of a coupled plasticity-damage constitutive model for simulating plain concrete fracture*, Engineering Fracture Mechanics, Vol 77, 1577–1603, 2010.

[5] Lubliner J., Oliver J. and Onate E., *A plastic-damage model for concrete*, International Journal of Solids and Structures, Vol. 25, no. 3, 299-326, 1989.

[6] Lee J. and Fenves G., *Plastic-Damage Model for Cyclic Loading of Concrete Structures*, Journal of Engineering Mechanics, Vol. 124, no. 8, 1998.

[7] Lee J., *Theory and implementation of plastic-damage model for concrete structures under cyclic and dynamic loading*, Berkeley, California: University of California, 1996.

[8] Rakić D., Dunić V., Živković M., Grujović N. and Divac D., *Modeling of Damaged Concrete using Initial Degradation Parameter*, Journal of the Serbian Society for Computational Mechanics, Vol. 13, no. 2, 8-18, 2019.

[9] Kojić M., Slavković R., Živković M. and Grujović N., *Metod konačnih elemenata I*, Kragujevac: Mašinski fakultet Univerziteta u Kragujevcu, 1998.