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DESIGN OF THE GEARING SPECIAL FORMS

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Andreja Ilić[3]

Abstract

In this paper is analyzed the special forms of gearing generated by the different combination of the rolling and base circles of the kinematic pair in contact. Characteristic form of the gear tooth based profiles is trochoid. The advance of these profiles in relation to the standard involute profile is the convex-concave contact of the meshing profiles. To the aim of obtaining better characteristics of the special gearing is done the equidistant modification of the based profiles. It is also defined the geometrical constrains to the equidistant modification. Based on obtained analyzing are generated different gear pairs and given their application by some mechanical construction: rotary piston motors (Wankel engine), gerotor pumps and transmission of power (cyclo reductor), and these profiles can be used also for the other special applications.

Key words: special form of gearing, trochoid, equidistant modification, gerotor pump, cyclo reductor

1. INTRODUCTION

The based components of the more contemporary machines are gears, by them are using most often the gears with involute gearing which in addition the graet technical and economical advances passess also more defects. Good base for solution of thy majority gear pairs problems gives the trochoidal gearing. Trochoidals profiles are using in the practice at the external and also the internal gearing, by them is represented more at the internal gearing. At the internal trochoidal gearing the profile of the one gear is described with trochoid, until the meshing profile is yhe corresponding internal or external envelope. Thanks to specific geometry all profile can be used for meshing. Trough the equidistant modification is obtained the profile with the better functional characteristics. These advances are used at the constructions of the gearing profile by the numerous rotated machines of the different application as: rotated pumps, rotated motors, compressors, but also by the gearing profile of cycloreductors, special group of the planetary gear transmitters.

2. MODIFICATION OF EPITROCHOID

In the paper [2] is shown that as the geometrical form of the based tooth profile can be choosen equidistant of the epitrochoid with the parameters which content the determinated conditions. One of the important parameter of the modification is equidistance radius, whose size is limited with minimal values of the curve radius of the based epitrochoidal [5]. From this reason is necessary to define the epytrochoid curve radius and determinate its stationary points [1].

In this paper is starting of the equation of the epitrochoid point *D* (Fig. 1):

$$\begin{aligned} x_t &= e(\cos z\phi + \lambda z \cos\phi) \\ y_t &= e(\sin z\phi + \lambda z \sin\phi) \end{aligned} \tag{1}$$

by them are :

- z is teeth number of external gear,

- e is eccentricity (center distance between the internal and external gear),
- r_a is radius of pitch circle of the external gear, $r_a = ze$,
- r_t is radius of pitch circle of the internal gear, $r_t = (z-1)e$,
- λ is coefficient of trochoid, $\lambda = d/ez$,
- ϕ is rotation angle of generating coordinate system of trochoid.

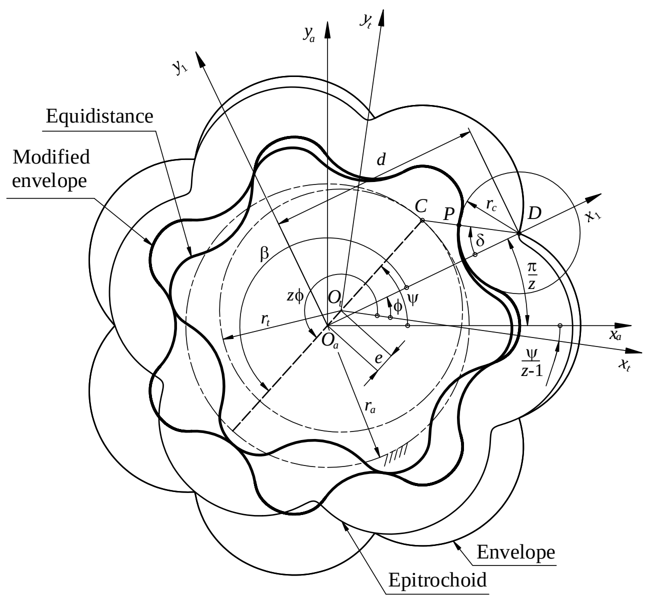


Figure 1: Generating of the gearing special forms

Basing on this can be written the formula to determinate the curvature radius in the point of epitrochoid, whose position is defined with referent angle $\beta = (z-1)\phi$ in following form:

$$\rho_t = \frac{ez\left[1+\lambda^2+2\lambda\cos\beta\right]^{3/2}}{z+\lambda^2+\lambda(z+1)\cos\beta} \quad . \tag{2}$$

Then are defined the conditions for existence of the stationary points of the epitrochoid curvature radius:

1) the first local extremum for the angle β=0º (the top of the epitrochoid lobe)

$$\rho_{t1} = \frac{ez(\lambda+1)^2}{\lambda+z}\,; \tag{3}$$

2) the second local extremum for the angle β=π (the bottom of the epitrochoid lobe)

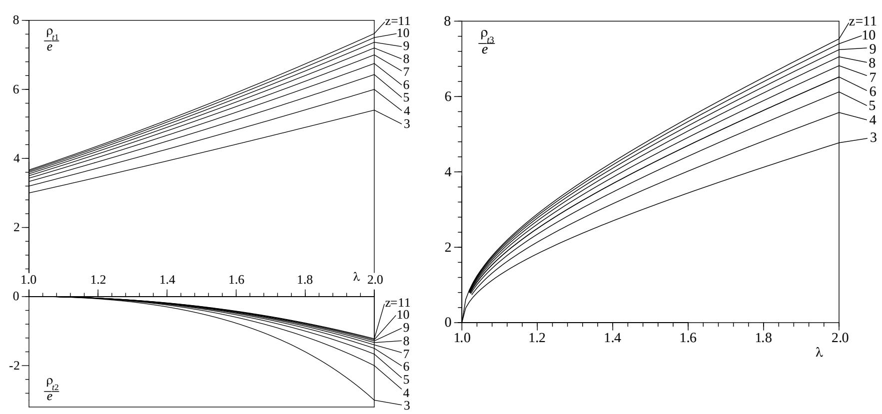
$$\rho_{t2} = \frac{ez(\lambda-1)^2}{\lambda-z}\,; \tag{4}$$

3) the third local extremum for the angle values determined to

$$\cos\beta = \frac{1-2z+\lambda^2(z-2)}{\lambda(z+1)}\,, \tag{5}$$

$$\rho_{t3} = ez\sqrt{\left(\frac{3}{z+1}\right)^3(\lambda^2-1)(z-1)}\,. \tag{6}$$

The graphical interpretation of the formulas (3), (4) and (6) is shown in Figure 2, a) and b).



(a) (b)

Figure 2: Diagram of the extrema of the radius of curvature of the epitrochoid: a) local extremum ρ_{t1} on the convex part of the profile, and local extremum ρ_{t2} on the concave part of the profile b) local extremum ρ_{t3} on the convex part of the profile

The real values for the epitrochoid coefficient λ are in the interval

$$1 < \lambda < \frac{2z-1}{z-2} \ . \tag{7}$$

If the epitrochoid coefficient is corresponding to the in equation (7) then exist all of the three extrema, defined to the equations (3), (4) and (6). Although, if is

$$\lambda \geq \frac{2z-1}{z-2} \tag{8}$$

then exist only the first and the second one of the extrema values.

To the testing of the radius of curvature epitrochoid function, except the stationary points, it is necessary to determine also the inflection point, in other words, the values of the angle β by them radius of curvature epitrochoid is not defined.

2.1. Inflection points in the epitrochoid

On the epitrochoidal form has the great influence epitrochid coefficient λ, which position of the generated point *D* is defined. The analysis of the influence of the coefficient λ on the epitrochid form is done based on the sign and size of the radius of curvature of the epitrochoid. If the radius of curvature has positive value the curve is convex, and for the negative value the curve is concave. In all points in them the radius of curvature is equal to zero, the curve has the cusps. The points, in them $\rho_t \to \infty$ are the inflection points, in them the curve has changing from the convex to the concave form and reverse. Starting from the equation (3) can be determined the condition which have filled that the epitrochoid could have the inflection point:

$$\cos\beta = -\frac{\lambda^2 + z}{\lambda(z+1)}. \qquad (9)$$

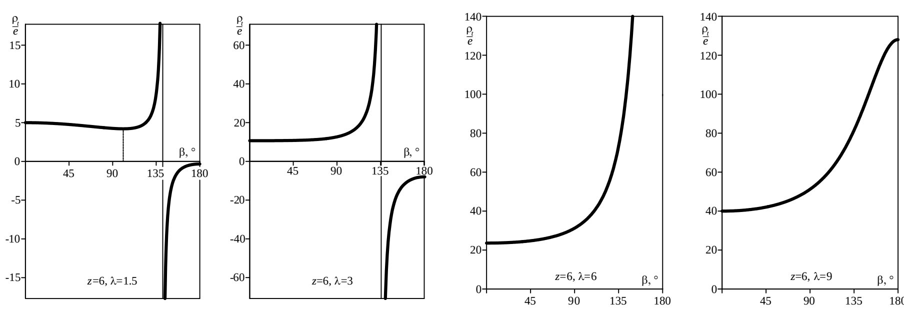
As λ and z are the positive and real values, so it is $\cos\beta < 0$, and $\frac{\pi}{2} < \beta \leq \pi$. According to, at the epitrochoid will be existence the inflection point is:

$$0 < \frac{\lambda^2 + z}{\lambda(z+1)} \leq 1. \qquad (10)$$

In the case when is:

$$\frac{\lambda^2 + z}{\lambda(z+1)} > 1 \qquad (11)$$

than is positive value of the radius of curvature for the all value of the angle β. By the solution of the inequation (11) is coming to the condition by them is the epitrochoid convex in all points, and it is for $\lambda > z$. On this way it can be conclused that for the existence of the third local extremum, except condition (7), is necessary to be filled also the condition $\lambda < z$, and in the case when is $\lambda = z$, the second local extremum is endless.



(a) $1 < \lambda < \frac{2z-1}{z-2}$ (b) $\frac{2z-1}{z-2} \leq \lambda < z$ (c) $\lambda = z$ (d) $\lambda > z$

Figure 3: Diagrams radius curves epitrochoid (z=6)

In accordance with forward conducted analysis in the Figure 3 is shown the functions of the epitrochoid curvature radius for z=6.

3. GENERATION OF SPECIAL PROFILE

The equidistant equations of the epitrochoid are derived in the coordinate system of the epitrochoid to Figure 1. In Figure 1 is shown that during the relative moving of pitch circles, until the point *D* is generating the epitrochoid, the point *P* generating the equidistant. The angle signed with δ is the angle between the normal *n-n* and the radius vector of the point *D*, and can be defined as the leaning angle. Coordinates of the contact point *P* in the coordinate system of the epitrochoid can be written in the following form:

$$\begin{aligned} x_t &= e(\cos z\phi + \lambda z \cos \phi) - r_c \cos(\phi + \delta) \\ y_t &= e(\sin z\phi + \lambda z \sin \phi) - r_c \sin(\phi + \delta) \end{aligned} , \qquad (12)$$

where is

$$\delta = \arctan \frac{\sin(z-1)\phi}{\lambda + \cos(z-1)\phi} \ . \qquad (13)$$

For this type of envelope is characteristic the notion of cusps. By curves with the cusps the equidistant modification in the cusp area is realized by the circle arc and its radius is corresponding to the selected radius r_c of the equidistant epitrochoid. By the equidistant modification is the new contact point *P* on distance r_c from the point *D* and always is on the common normal which connects the center of the circle profile with the instantaneous pitch point *C*. Besides, the advantage of the epitrochoidal gear pair consists in the same time realized contact all teeth and it is also illustrated in Figure 1.

To the correct engagement of the trochoidal gear pairs with modified profiles is necessary to be filled the numerous geometrical and kinematic conditions from that the most important is the absence of the profile undercutting. The tooth profile undercutting of the modified epitrochoidal profile is the geometrical section of curve that describes profile (Fig. 4). Profile testing on undercutting is doing trough the analysis of minimum value of the radius of curvature of the convex part of the real profile described with the internal equidistant of epitrochoid. In the chapter 2 of this paper are derived the equations with them is possible to determination by analytical method the minimum value of the curve radius on the convex part of the based profile.

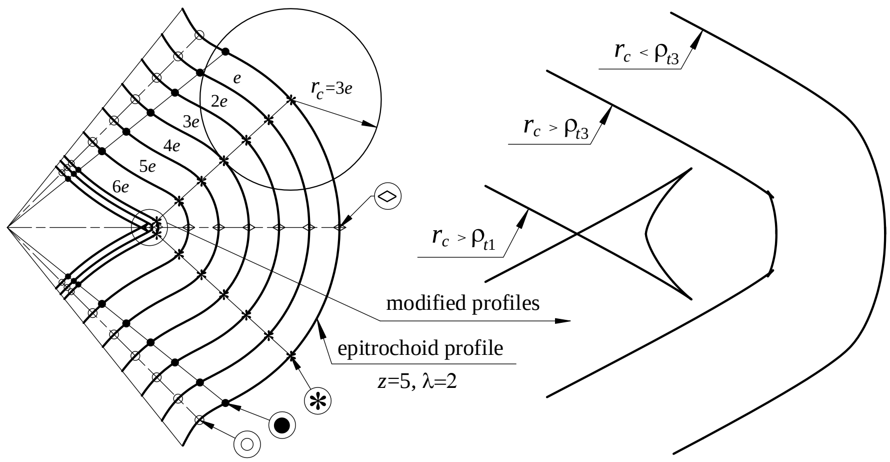


Figure 4: Influence of the size of the radius equidistant the form of modified profile epitrochoide and illustrations of stationary points on the profile ◇ - local extremes ρ_{t1}, ρ_{c1}, o - local extremes ρ_{t2}, ρ_{c2}, ∗ - - local extremes ρ_{t3}, ρ_{c3}, • - inflection point

The minimum values of the radius of curvature ρ_{t3} or ρ_{t3} on the convex part of profile determining the limited value for the negative modification, and ρ_{t2} on the concave part of profile determining the limited value for the positive modification. That values are the maximal geometrical constraints for modification of the based profile by them is coming to change of the curvature sign of modified profile.

4. APPLICATION SPECIAL PROFILES

The special forms are used by construction of the numerous rotated machines of the different applications. Some of them are given in the Figure 5. The adventages of the application of the trochoidal profile in these constructions are: compact construction, great speed ratio, small sliding, high coefficient of safety, small weight and dimensions and large working life.

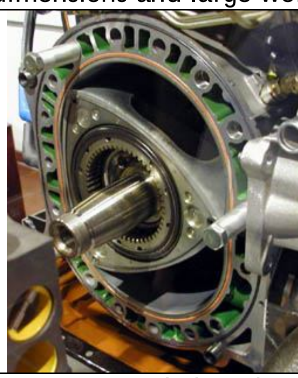
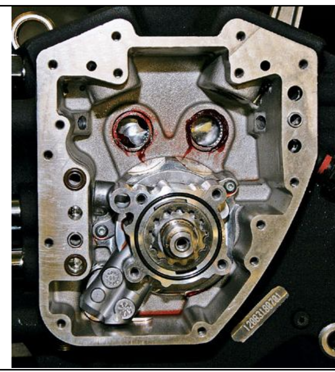
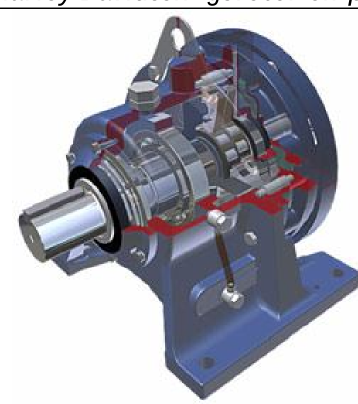
	
a) Wankel engine	*b) Harley Davidson gerotor oil pump*
	
b) Hydraulic motor	*d) Cyclo speed reducer*

Figure 5: Application special profiles

Wankel engine represents one of the most significant advances in automobile engineering (Fig.5a). This engine uses rotor instead of a pistons which allow it to deliver power without vibration in much higher RPMs [7]. A gerotor is a positive displacement pumping unit [8]. Gerotor pumps are generally designed using a trochoidal inner rotor and an outer rotor formed by a circle with intersecting circular arcs (Fig.5b). A gerotor can also function as a motor [9]. High pressure gas enters the intake area and pushes against the inner and outer rotors, causing both to rotate as the area between the inner and outer rotor increases (Fig.5c). Cyclo's unique epicycloidal design has advantages superior to speed reducers using common involute tooth gears [10]. Cyclo components operate in compression, not in shear (Fig.5d).

5. CONCLUSION

In this paper are defined the base principles of the generating special gearing. It is shown that with the varied geometrical parameters are obtained profiles with different sign of curvature. There are carried out the conditions for existence two or three local extrema of the radius curvature and on this way is defined the selection of the equidistance radius. Through the positive or negative equidistante modification are obtaining profiles with the better characteristics of meshing. It is making possible the different application newgenerated profiles. The results described in this paper are presented good base to investigation and development in the area of the special gearing and construction of the gear tools.

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