5th International Conference
″Research and Development in Mechanical Industry″
RaDMI 2005
04 - 07. September 2005, Vrnjačka Banja, Serbia and Montenegro

COMPUTER GENERATING OF MODIFIED TROCHOIDAL PROFILE OF GEROTOR

L. Ivanović, D. Josifović, G. Devedžić
Faculty of Mechanical Engineering, 34000 Kragujevac, SERBIA AND MONTENEGRO,
E-mail: lozica@kg.ac.yu

Summary: *In the paper is presented the theoretical model of the gearing generating with modified trochoidal profile which is applied by the gerotor machines. Gerotors belong to the category of rotated machines with the planetary moving of the working elements. In the paper is shown, how is possibly to describe in parameter form equidistant of the epitrochoidal profile. The profile in contact is the equidistant of the conjugate external envelope. The obtained equations to the calculation with the limiting factors have given the possibility to development of the algorithm and the program for automatic drawing of the trochoidal gearing. Based on the mathematical model is created the computer program in the standard programming language AutoLISP to the calculation of the given curves coordinates and to their automatic generating.*
Keywords: *gerotor, epitrochoid, envelope, generating*

1. INTRODUCTION

The trochoidal curves are modifying by the increase of the constant values that is drifting along the normal of the given curve. On that way is obtaining the curve which is parallel to the given curve and because their distance measured along the common normal is remaining constant, the obtained curve is also the equidistant. The parallel curves are characterized on the property that in the common points have the common normal and parallel tangents [1]. That means, by their choice for the gearing profile is satisfies the fundamental low of gearing. Equidistant can be defined too as the trajectory of the circle centre that is rolling to the obtained curve, or as the envelope of the circles of the constant radius r_c and their centers are located on this curve. According to this conclusion the constant increase r_c can be defined as the radius of equidistant.

The modification of the trochoidal curve can be positive or negative depended of the sign of the radius r_c so that are obtaining the external or internal equidistant. Chosen of the radius r_c value exists the geometrical constraint to the minimal radius $\rho_{\min}$ of the based curve. To can be obtained the smooth curve it is necessary to be filled the condition $r_c < \rho_{\min}$ [2]. If is $r_c = \rho_{\min}$ then is getting the curve whose characteristic is the cusps. If is $r_c > \rho_{\min}$ it is obtained equidistant whose characteristic is crossing over itself and it is practically impossibly to realize.

To the practically application are most suitable the prolate hypotrochoid and peritrochoid, because by corresponding choice of r_c complete profile can be used as the contact profile of the gearing. For the internal envelopes are recommending positive modification and for the external negative modification [3]. Keeping in mind the fact that by the hypotrochoidal profiles is more suitable the application of the internal envelope and with them also the external equidistant bringing to the dimension increase so that in this paper will be considered only the profiles with the basic peritrochoidal form (curtate epitrochoide).

2. MODIFICATION OF EPITROCHOID

In the first part of this paper is shown that as the geometrical form of the basic tooth profile can be chosen equidistant of epitrochoid with the parameters which content the determinated conditions. One of the important parameter of modification is the equidistant radius whose size is limited with the minimum radius of curvature of the basic epitrochoid. According to, it is necessary, before the modification, to determinate the stationary curvature of epitrochoid.

2.1. Stationary points of curvature of epitrochoid

The general notion of the stationary points is connected to the extrema values of the given function and in themselves are contained all points in them are attained the local extrema of the function, global extremum, as the inflection point too.
Based on the geometrical relations shown in Figure 1, the equations of the point D of the epitrochoid can be written in form:

$$\begin{aligned} x_t &= e(\cos z\phi + \lambda z \cos\phi) \\ y_t &= e(\sin z\phi + \lambda z \sin\phi) \end{aligned}, \qquad \textbf{(1)}$$

by them is $\lambda = d/ez$ - epitrochoid coefficient [4].

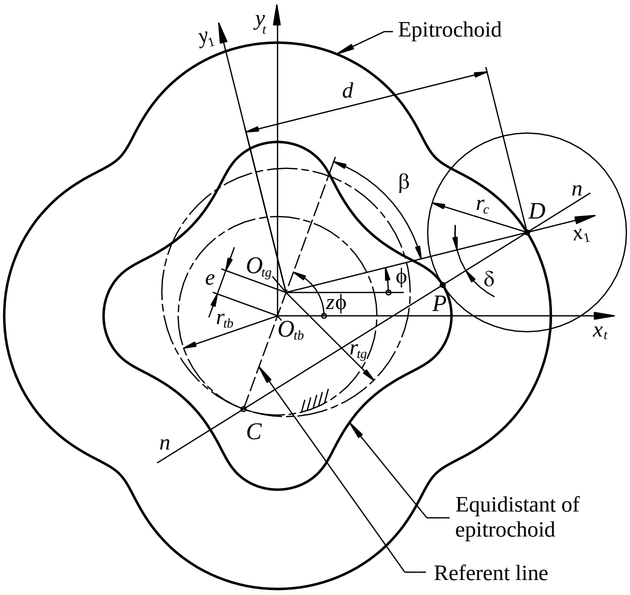


Figure 1: Generating of unmodified and modified epitrochoid

If the curve is defined by the equations in parameter form, the radius of curvature in anyone point is calculated to the formula:

$$\rho = \frac{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}{\dot{x}\ddot{y} - \ddot{x}\dot{y}} . \qquad \textbf{(2)}$$

By the finding first and second derivative of the equations (1) to the variable ϕ and after the simple trigonometric transformation is obtained the formula to determination of the radius of curvature in the point of the epitrochoid whose position is defined with the angle $\beta = (z-1)\phi$:

$$\rho_t = \frac{ez\left[1 + \lambda^2 + 2\lambda\cos\beta\right]^{3/2}}{z + \lambda^2 + \lambda(z+1)\cos\beta} . \qquad \textbf{(3)}$$

It is necessary to determinate the values of the angle β for them the radius of curvature has the minimum values. To this aim is done the differentiation of the equation (3) to the variable β and the make equal to zero. In that case it can be determinated the conditions to the existence minimum radius of curvature of the epitrochoid. The analysis of the conditions to exist of the local extrema of the radius of curvature epitrochoid is done by the consideration of the three cases by them is $\frac{d\rho_t}{d\beta} = 0$. On this way is obtained:

1) the first local extremum for the angle β=0° (the top of the epitrochoid lobe)

$$\rho_{t1} = \frac{ez(\lambda+1)^2}{\lambda + z} . \qquad \textbf{(4)}$$

2) the second local extremum for the angle $\beta=\pi$ (the bottom of the epitrochoid lobe)

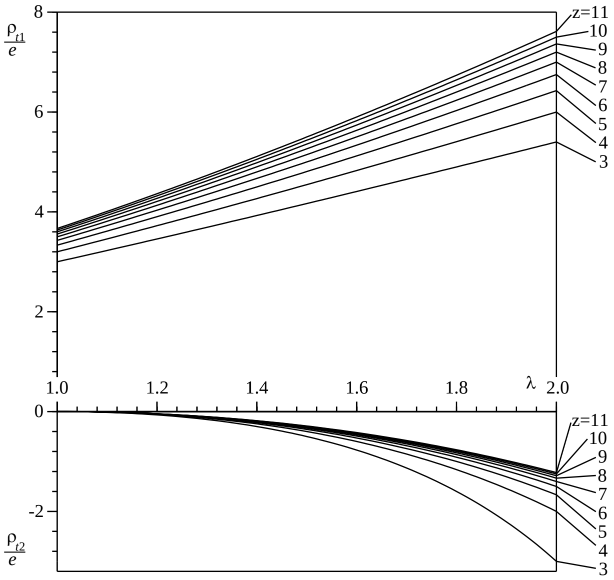
$$\rho_{t2} = \frac{ez(\lambda-1)^2}{\lambda-z} \tag{5}$$

3) the third local extremum for the angle values determined to

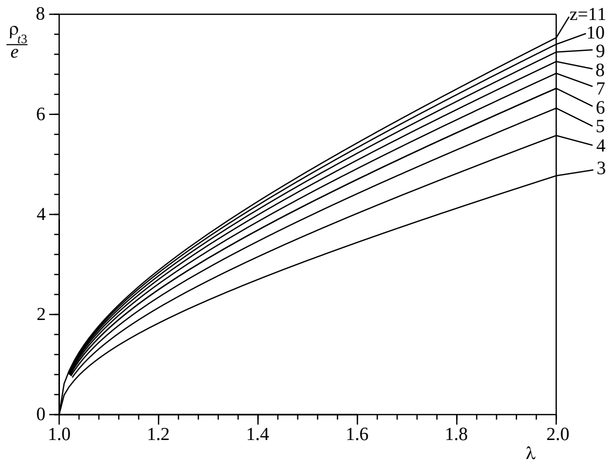
$$\cos\beta = \frac{1-2z+\lambda^2(z-2)}{\lambda(z+1)} \tag{6}$$

$$\rho_{t3} = ez\sqrt{\left(\frac{3}{z+1}\right)^3(\lambda^2-1)(z-1)}\,. \tag{7}$$

The graphical interpretation of the formulas (4), (5) and (7) for the extrema of the radius of curvature epitrochoid in function of the coefficient λ for the constant values of the gear teeth number is shown in figure 2, a) and b).



(a)



(b)

Figure 2: Diagram of the extrema of the radius of curvature of the epitrochoid:
a) local extremum ρ_{t1} on the convex part of the profile, and local extremum ρ_{t2} on the concave part of the profile b) local extremum ρ_{t3} on the convex part of the profile

The real values for the epitrochoid coefficient λ are in the interval

$$1 < \lambda < \frac{2z-1}{z-2}\,. \tag{8}$$

If the epitrochoid coefficient is corresponding to the inequation (8) then exist all of the three extrema, defined to the equations (4), (5) and (7). Although, if is

$$\lambda \geq \frac{2z-1}{z-2}\,, \tag{9}$$

then exist only the first and the second one of the extrema values. To the testing of the radius of curvature epitrochoid function, except the stationary points, it is necessary to determine also the inflection point, in other words, the values of the angle β by them radius of curvature epitrochoid is not defined.

2.2. Inflection points in the epitrochoid

On the epitrochoidal form has the great influence epitrochid coefficient λ, which position of the generated point *D* is defined. The analysis of the influence of the coefficient λ on the epitrochid form is done based on the sign and size of the radius of curvature of the epitrochoid. If the radius of curvature has positive value the curve is convex, and for the negative value the curve is concave. In all points in them the radius of curvature is equal to zero, the curve has the cusps. The points, in them $\rho_t \to \infty$ are the inflection points, in them the curve has changing from the convex to the concave form and reverse. Starting from the equation (3) can be determined the condition which have the filled that the epitrochoid could have the inflection point:

$$\cos\beta = -\frac{\lambda^2 + z}{\lambda(z+1)}. \qquad \textbf{(10)}$$

As λ and z are the positive and real values, so it is $\cos\beta < 0$, and $\frac{\pi}{2} < \beta \le \pi$. According to, at the epitrochoid will be exist the inflection point is:

$$0 < \frac{\lambda^2 + z}{\lambda(z+1)} \le 1. \qquad \textbf{(11)}$$

In the case when is:

$$\frac{\lambda^2 + z}{\lambda(z+1)} > 1 \qquad \textbf{(12)}$$

them is positive value of the radius of curvature for the all value of the angle β. By the solution of the inequation (12) is coming to the condition by them is the epitrochoid convex in all points, and it is for $\lambda > z$. On this way it can be conclused that for the existence of the third local extremum, except condition (8), is necessary to be filled also the condition $\lambda < z$, and in the case when is $\lambda = z$, the second local extremum is endless.

2.3. Stationary points of curvature of the modified epitrochoid

Starting of the fact that the modified epitrochoid is defined as the equidistant of the based epitrochoid curve and according to the distance along the normal on the curve is constant and equal to r_c, can be written the general form to determination of the radius of curvature ρ_c of the modified epitrochoid by the following equation:

$$\rho_c = \rho_t \pm r_c. \qquad \textbf{(13)}$$

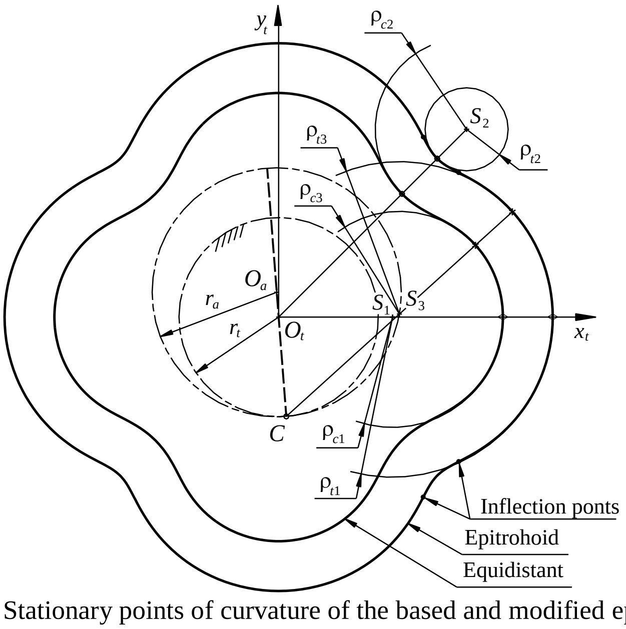


Figure 3: Stationary points of curvature of the based and modified epitrochoid

To the change of the sign is coming in the inflection point. In the equation (13) the sign "+" is relating to the concave, and the sign "-" to the convex part of the curve.

By the inflection points all conditions, which are derived for the epitrochoid, are valid for her equidistant too. In that case entered in the equation (13) the formulas to the determination local extrema of the radius of curvature of the epitrochoid it will be obtained the extremes values of the radius of curvature of the equidistant curve. The position of the stationary points (local extrema and inflection point) for the one lobe of the parallel curves is given in Figure 3.

3. THE EQUIDISTANT EQUATIONS

The equidistant equations of the epitrochoid are derived in the coordinate system of the epitrochoid to Figure 1. In Figure 1 is shown that during the relative moving of pitch circles, until the point *D* is generating the epitrochoid, the point *P* generating the equidistant. The angle signed with δ is the angle between the normal *n-n* and the radius vector of the point *D*, and can be defined as the leaning angle. The contact point *P* of the modified profile is on the normal of the based profile and during the gear moving will be in contact with the points of conjugate profile which are located from the left and the right side of the common normal. According to the contact is realized with the own part of the conjugate profile that is limited with the maximum angle δ_{max}.
Coordinates of the contact point P in the coordinate system of the epitrochoid can be written in the following form:

$$\begin{aligned} x_t &= e(\cos z\phi + \lambda z \cos\phi) - r_c \cos(\phi+\delta) \\ y_t &= e(\sin z\phi + \lambda z \sin\phi) - r_c \sin(\phi+\delta) \end{aligned} \quad . \tag{14}$$

Based on the geometrical relations, shown in Figure 1 and by the application of the known trigonometrically transformation can be obtained the formula to determination of the leaning angle δ in the form:

$$\delta = \arctan\frac{\sin(z-1)\phi}{\lambda + \cos(z-1)\phi} \ . \tag{15}$$

To the equation (15) is constructed the diagram of the angle change for the different values of the epitrochoid coefficient λ in relations to angle β and given in Figure 4.

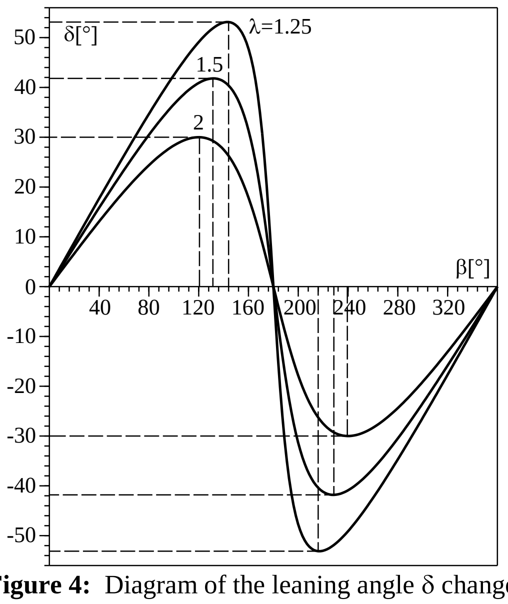


Figure 4: Diagram of the leaning angle δ change

During the moving the angle δ is change from zero to the maximum values of amplitude $\pm\delta_{max}$. To the determination of the extremum of the angle δ is necessary the differentiation of the formula (15) to the variable and make equal to zero:

$$\frac{d\delta}{d\phi} = \frac{\lambda(z-1)\cos(z-1)\phi + z - 1}{1+\lambda^2 + 2\lambda\cos(z-1)\phi} = 0 \ . \tag{16}$$

The equation will be contented in the point in that is filled the following condition:

$$\cos(z-1)\phi = -\frac{1}{\lambda} \ . \tag{17}$$

When the formula (17) enters in the equation (15) is getting the maximum value of the leaning angle δ obtained by equation:

$$\delta_{\max} = \arctan\left[\pm\left(\lambda^2 - 1\right)^{-1/2}\right]. \tag{18}$$

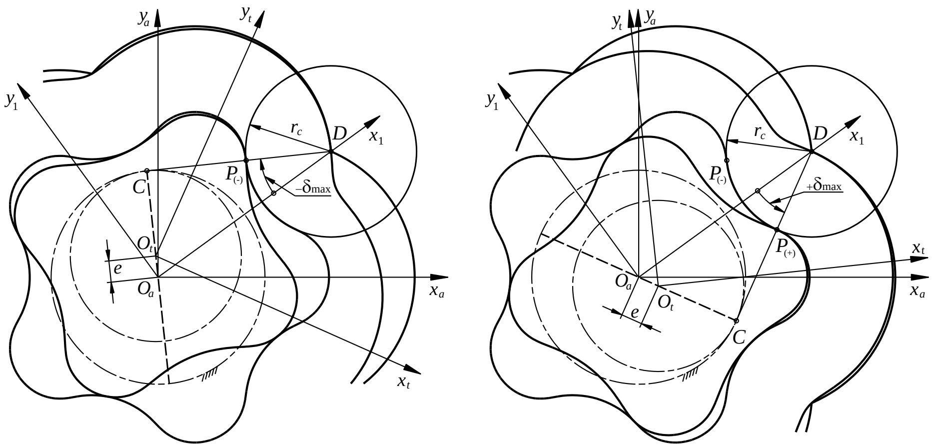


Figure 5: Geometrical relations corresponding to maximum values of the leaning angle (general case): (a) negative $-\delta_{max}$ and (b) positive $+\delta_{max}$

It is evident, that the value of the angle δ_{max} is depended only from the value of epitrochoid coefficient λ. In consideration of that fact that is of the small values of the angle δ also small the contact area and the consequence of that is the intensive wear of these places. To the normal working conditions of gerotor with these profiles is recommended the higher values of the angle δ, and to Figure 4, that means smaller values of the epitrochoid coefficient λ.

In Figure 5 are given, as the illustration, geometrical relations which are corresponding to the maximum values of the leaning angle. It can be considerable that in these cases, normal on the conjugate profiles in the contact point *D* is the common tangent on the pitch circles in the pitch point *C*. That means, by the maximum values of the leaning angle δ_{max}, the pressure angle, is defined as the angle between the profile normal and tangent on the pitch circles in the pitch point is equal zero.

4. GENERATING OF MODIFIED ENVELOPE

For the selected type of envelope is characteristic the notion of cusps. By curves with the cusps the equidistant modification in the cusp area is realized by the circle arc and its radius is corresponding to the selected radius r_c of the equidistant epitrochoid.

By the equidistant modification is the new contact point *P* on distance r_c from the point *D* and always is on the common normal which connects the center of the circle profile with the instantaneous pitch point *C*. According to the position of the point *P* is on the circle arc whose radius r_c surrounds angle $2\delta_{max}$ with the center in the point *D*. It is the active tooth profile of the modified envelope, and the remained part of the profile, which does not participate in contact, can be approximated with the circle arc or by combination of the circle arcs which is out or on the obtained envelope (Figure 6). Possibility to determination of the approximation is one of the main advantages of epitrochoidal profile because with is possible the easier production, dimension control, and the more simple form of the derived equations is obtained. Besides, the advantage of the epitrochoidal gear pair consists in the same time realized contact all teeth and it is also illustrated in Figure 6.

To the correct engagement of the trochoidal gear pairs with modified profiles is necessary to be filled the numerous geometrical and kinematic conditions from that the most important is the absence of the profile undercutting. The tooth profile undercutting of the modified epitrochoidal profile is the geometrical section of curve that describes profile. Profile testing on undercutting is doing trough the analysis of minimum value of the radius of curvature of the convex part pf the real profile described with the internal equidistante of epitrochoide. In the chapter 2.1 of this paper are derived the equations with them is possible to determination by analytical method the minimum value of the curve radius on the convex part of the based profile.

The minimum value of the radius of curvature ρ_{t3} on the convex part of profile (positive curvature) is given by the equation (7) determining the limited value for the negative modification.

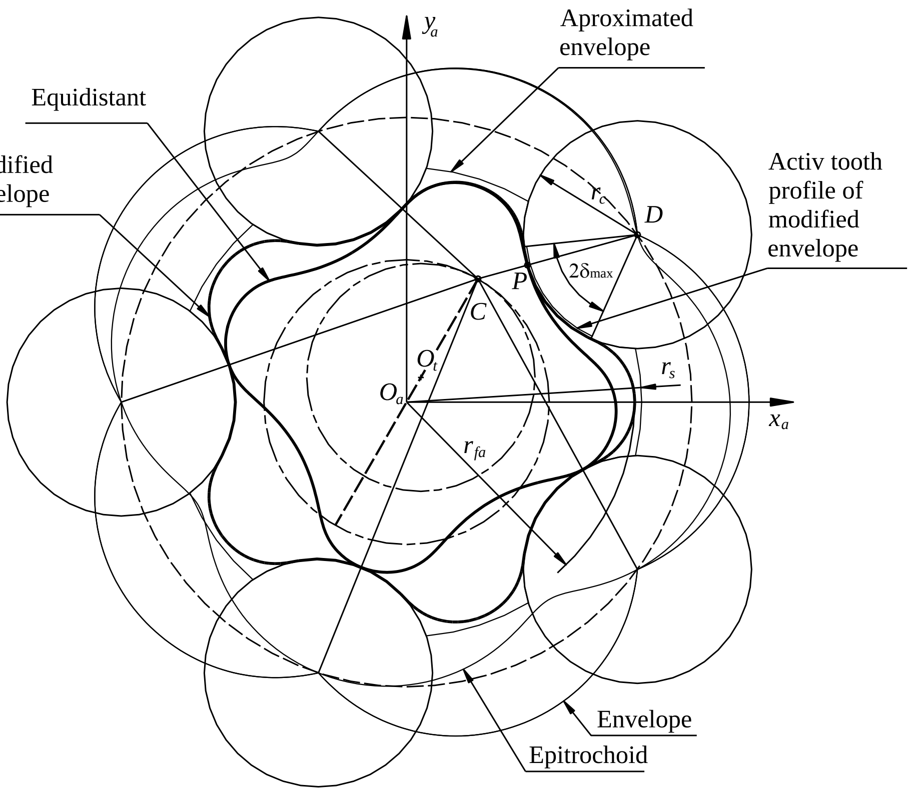


Figure 6: Definition of modified conjugate profiles of the epitrochoidal gear pair

The minimum value of the radius of curvature ρ_{t2} on the concave part of profile (negative curvature) is given by the equation (5) determining the limited value for the positive modification.
That two values are the maximal geometrical constraints for modification of the based profile by them is coming to change of the curvature sign of modified profile.
Based on Figure 2 can be conclused that the absolute minimum values of the radius of curvature are greater on the convex part of profile them on the concave that is one of the next proof to efficiency of application of the negative modification. On that way by internal modification the limited value of the equidistant radius $r_{c\max}$ is defined by equation (7). Although in practice are accepted values which are less then ρ_{t3} because is get out the notion of the cusps at the curve and also the obtained contact stresses and wear are more less.

5. COMPUTER GENERATING OF GEROTOR PROFILE

On the base of presented mathematical model is created computer program EKVIT in the standard programming language AutoLISP to calculation of coordinates of obtained curves and to their automatic generation. As result of program testing are obtained the conjugate gear pairs of gerotor mechanism. For obtained internal gear pair developed 3D geometrical models by the application of the Part Design module of the computing program CATIA V5R14.
Results of 3D models of the conjugate gear pairs are shown in Figure 8. Further, it is possible to make the simulation of conjugate for the presented models by application of the DMU Kinematic module of the given programming package ant then also the optimization by application of corresponding module.

6. CONCLUSION

From the aspect of engineering construction negative characteristic of trochoidal curves is phenomenon of cusps on the conjugate envelopes. In the paper is shown that is possible to avoid by application of the equidistant modification of the based profile, with that is obtained the smooth curve, and the conjugate is more suitable from the aspect of the increase of the radius of curvature of the profiles in contact.

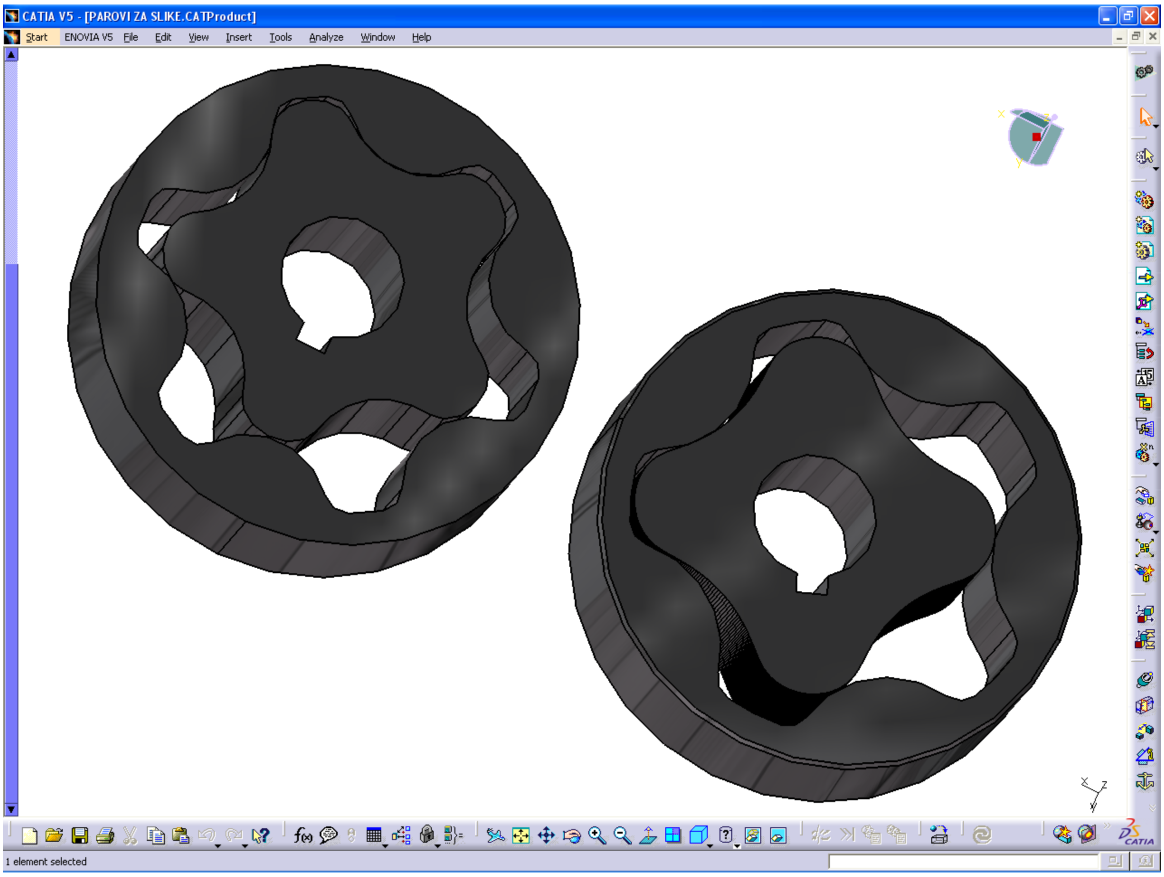


Figure 8: Examples of 3D geometrical models of gerotor gear pairs

Thanking to this fact is developed the simple mathematical model and corresponding program to automatic drawing of the modified epitrochoid and conjugate envelope, what further, lays the possibility for modeling and moving simulation of the engaged gear pairs. On that way can be followed and analyzed one to another position of the gear pairs elements in different phases of engagement.

ACKNOWLEDGEMENT

This paper is result of investigation on project (TR-6218A) that is financed by Ministry of Science and Technology of Republic Serbia.

REFERENCES

[1] Savelov A.A.: Ploskie krivie, Fizmatgiz, Moskva,
[2] Lehman M.: Sonderformen der Zykloidenverzahnung, Konstruktion, 1979, Vol. 31, 429-433
[3] Robinson F. J., Lyon J. R.: An Analysis of Epitrochoidal Profiles With Constant difference Modification Suitable for Rotary Expanders and Pumps, Journal of Engineering for Industry, Trans. ASME, 161-165, Feb. 1976.
[4] Ivanovic L., Josifovic D., ″Geometrical parameters of trochoidal profile and their application at the internal gearing″, Conference IRMES, Kragujevac, 2004.