International Symposium
"MACHINES AND MECHANISMS"
Belgrade,
September, 2- 5, 1997
IFToMM

GEOMETRICAL CONSTRAINTS IN CYCLOIDAL TEETH SYNTHESIS

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Abstract: In modern constructions the most frequently are used gears with evolvent gearing, that, besides great technical and economical advantages, has a series of disadvantages. The main deficiency is the small carrying capacity of the gear teeth, and that is related to the contact of the convex tooth sides surfaces with relatively small curvature radius. This deficiency can be overcome by application of the cycloid gearing, where the convex-concave contact is realized, as well as the greater carrying capacity. Also, the cycloid forms are suitable as a basis for creating the new kinds of gearing. Due to the mentioned reasons, in this paper are considered the geometric characteristics of the cycloid gearing from the aspect of the geometric constraints in its synthesis. The expressions were derived that served as a basis for the computer program that was written for drawing the diagrams for choosing the possible combinations of the tooth number of the cycloid gears, at given gearing parameters.

Key words: Cycloid gearing, geometric constraints, cycloid profiles path of contact.

1. INTRODUCTION

Cycloid gearing, compared to evolvent, has greater carrying capacity and greater wear resistance. The mentioned advantages are being realized thanks to conjugate action of the convex with concave tooth surfaces, with large values of the average curvature radii of the tooth surfaces in contact, greater contact surface and presence of bigger oil layer between the tooth surfaces in conjugate action. Also, this gearing is characterized by the possibility for realization of gears with very small number of teeth ($z < 10$), without the phenomenon of the tooth side undercutting.

Although it posses numerous advantages, the application of the cycloid gearing is, for the present, limited. The main reasons for this are the complex manufacturing of the rack and gearing, significant influence of the axial distance change on the regularity of the conjugate action, as well as insufficiently developed procedures for calculation of its geometry. Taking into account present theoretical considerations in that area, the basic objective of this paper is the contribution to better insight into the possibilities for realization of different teeth pairs by variation of geometrical parameters of cycloid gearing. In regard with that the check was done of certain geometrical conditions in gearing synthesis.

As the first is defined the basic profile form, as well as the geometrical and kinematical relations in gear tooth profiling. The equations were derived that describe the tooth profile, and also are determined the limiting points of the active tooth profile. Then the basic parameters were determined of the spur gears cycloid tooth profiles.

Geometrical conditions, that ought to be checked in cycloid gearing synthesis, are the following: possibility of the rack tooth profile forming, generating of the tooth profile without undercutting, absence of interference of cycloid teeth of the two gears in conjugate action, as well as of gear and rack in conjugate action, the area of definetness of the path of contact and limiting tooth thickness the addendum circle.

Based on the derived conditions the computer program was conceived for checking the possible combinations of the gear teeth number and drawing the diagram. From that diagram the maximum values can be red off of the contact ratios that are possible to be realized with the chosen number of teeth of the pinion and the coefficient of the hypocycloid portion of the gear. In the concluding remarks are given the limiting factors in cycloid gearing synthesis.

2. THE BASIC PROFILE FORM AND GEARING GENERATING

The gear tooth profile is obtained according to the relative rolling method. Relative motion, in gear teeth profiling, is rolling without sliding of the rack pitch line along the gear pitch circle. In Figure 1 is shown the rack tooth profile that generates the profile of the gear. The rack tooth profile consists of two arc of the orthocycloid l_{LO_0} and l_{O_0G}, that are profiling the active tooth profile, and the circular arc of the radius r_{a0} with the center in S, that is forming the tooth fillet. In the profiling procedure the point O_0 lies on the rack pitch line.

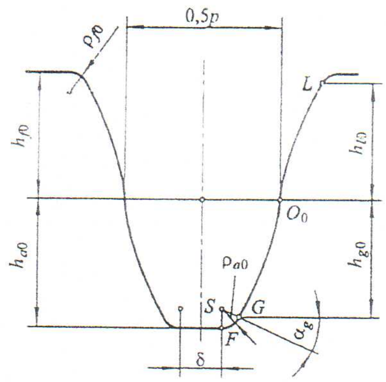


Figure 1. Rack tooth profile

In figure 2 is shown the relative position of the rack and the pinion during the profiling procedure of the active gear tooth profile. The rack tooth profile in the coordinate system fixed for the rack can be described by the following equations:

$$x_0 = r_{ki}(t - \sin t),$$
$$y_0 = r_{ki}(1 - \cos t), \qquad (1)$$

where r_{ki} is the constructive parameter (the radius of the circle that generates the cycloid and has the positive value for the profile of the dedendum profile and has the negative value for the addendum profile), i is the notation for the gear and pinion ($i = 1, 2$), and t is the angular parameter of the cycloid and always has the positive value.

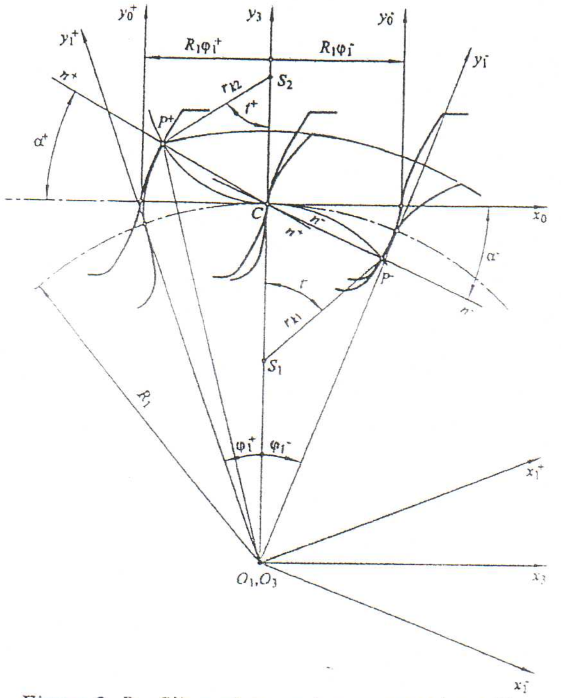


Figure 2. Profiling of the active gear tooth profile

The gear tooth cycloid profile is being generated during the straight line motion along the pitch line, with simultaneous rotation of the gear for the angle φ_i that is equal to:

$$\varphi_i = q_{ji} t \,, \tag{2}$$

where q_{ji} is the cycloid coefficient and it represents the ratio of the moving circle radius r_{ki} and the fixed circle radius R_{ki}, which for cycloid gears corresponds to pitch circle, and j is the notation for the epicycloid or hypocycloid coefficient ($j = e, h$). For the gear pair in the conjugate action the choice is recommended of the values of the dedendum hypocycloid profile coefficient values from the interval (0.35 - 0.4) with the negative sign, and values of addendum epicycloid profile coefficient are being calculated according to the following relations:

$$q_{e1} = i\, q_{h2} \,, \qquad q_{e2} = \frac{q_{h1}}{i} \,, \tag{3}$$

with the remark that in gears with the outer gearing the value of the contact ratio i is negative.

In the coordinate system fixed for the gear the useful portion of the tooth profile can be described by equations:

$$x_i = R_i\left(1+q_{ji}\right)\sin\varphi_i - q_{ji} R_i \sin\left(\varphi_i + \frac{\varphi_i}{q_{ji}}\right),$$
$$y_i = R_i\left(1+q_{ji}\right)\cos\varphi_i - q_{ji} R_i \cos\left(\varphi_i + \frac{\varphi_i}{q_{ji}}\right). \tag{4}$$

In further considerations, for the sake of the simpler presentation the subscripts are omitted, and by their proper choice the equations were obtained that are valid for both gears in the gear pair.

In generating the epicycloid profile of the addendum, the upper limiting point is determined by the crossing point of the path of contact and the gear addendum circle R_a. Its position is defined by the gear rotation angle

$$\varphi_a = q_e \arccos\frac{q_e\left(1+q_e\right)+0.5\left(1-Q_a^{\,2}\right)}{q_e\left(1+q_e\right)} \,, \tag{5}$$

where is $Q_a = \dfrac{R_a}{R}$.

In profiling the dedendum the limiting point is the last point of the hypocycloid profile, which is generated by the point G on the rack tooth. Its position on the gear tooth is defined by the rotation angle, that is determined by the relation:

$$\varphi_g = q_h \arccos\left(1+\frac{2h^*_{g0}}{q_h z}\right). \tag{6}$$

Another important geometrical parameter of the tooth cycloid profile is the pressure angle α, which is equal to the angle between the normal to the tooth profile and tangent to the centroid, and it amounts to:

$$\alpha = \frac{\varphi}{2q} \,. \tag{7}$$

3. GEOMETRICAL CONSTRAINTS

Limiting form of the rack tooth profile. In design the gear with the cycloid teeth it is necessary to perform the check of the possibility of the rack tooth forming. The limiting case is when the addendum (or dedendum) profile is formed by one part of the orthocycloid (Figure 3).

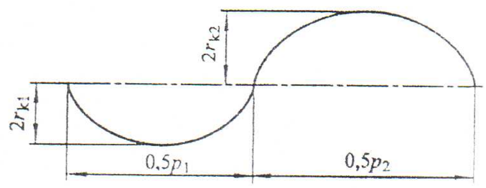


Figure 3. Limiting form of the rack tooth cycloid profile

Keeping in mind the properties of the orthocycloid, the inequality can be written:

$$0.5p \le 2 r_k \pi \,, \tag{8,a}$$

from which the condition is obtained:

$$z \ge \frac{1}{2|q|} \tag{8,b}$$

Tooth undercutting. Tooth undercutting, that characterizes the evolvent gearing and is the undesired phenomenon, is not present for cycloid gearing.

Interference. Tooth interference is the phenomenon of the partial teeth overlapping of teeth in conjugate action.

During the real contact of gears the interference is the phenomenon when the tip of one gears touches the fillet of the other. This leads to the irregular conjugate action and at higher torque the tooth fracture can occur.

In Figure 4,a are shown teeth profiles of gears in the moment of conjugate action of the tooth tip of one gear.

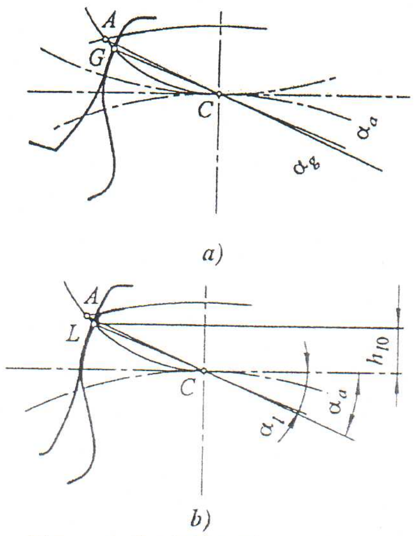


Figure 4. Cycloid teeth interference
a) two gears; b) gear and rack

From Figure 4,a the following statements are obvious:

- if $\alpha_a<\alpha_g$ interference does not occur,
- if $\alpha_a= \alpha_g$, points A and G coincide, and the transmitters is on the verge of interference.
- if $\alpha_a< \alpha_g$, interference occurs.

In this way, the appearance of interference can be realized according to the inequality:

$$\alpha_a \le \alpha_g$$

namely

$$\frac{q(1+q)+0.5-0.5\left(1+\frac{2h_a^*}{z}\right)^2}{q(1+q)} \ge 1-\frac{2h_{g0}^*}{qz}. \qquad (9)$$

From this inequality the condition is obtained for the limiting number of the gear teeth, in order to avoid interference:

$$z \ge \frac{h_a^*}{h_{g0}^*(1+|q|)-h_a^*}, \qquad (10)$$

which, at $h_a^* = h_{g0}^* = 1$, gets the form:

$$z \ge \frac{1}{|q|}. \qquad (11)$$

The interference can occur also during the conjugate action of the gear with the rack, when it leads to cutting off of the tooth tip (Figure 4,b). If the gear is being profiled by the straight rack, it is necessary the check for the absence of interference of the transition curve of the rack tooth profile (circular arc of the radius ρ_{f0}) with the gear tooth tip. In order to do that the pressure angle is determined α_l of the rack tooth profile at the limiting point L, that must be greater than the pressure angle α_a of the gear tooth tip, namely

$$\frac{q(1+q)+0.5-0.5\left(1+\frac{2h_a^*}{z}\right)^2}{q(1+q)} \ge 1-\frac{2h_{f0}^*}{qz}. \qquad (12)$$

If the equations (9) and (12) are compared to each other, it is obvious that form the latter one the condition analogous to (10) is obtained, while at $h_a^* = h_{f0}^* = 1$the condition is obtained identical to condition (11).

Area of definetness of the path of contact. The area of the possible path of contact is within the angular limits:

$$0 \le \frac{\varphi}{q} \le \frac{\pi}{2},$$

namely

$$1 \ge \cos\frac{\varphi}{q} \ge 0. \qquad (13)$$

The upper limiting point on the path of contact corresponds to the contact of the crossing point of the addendum epicycloid profile and the addendum arc (Figure 2), and its position is defined by the equation (5).

This equation can be written in the form

$$\cos\frac{\varphi_a}{q_e} = 1-\frac{2(1+z)}{z^2 q_e(1+q_e)} \qquad (14)$$

and when it is entered into (13) one obtains:

$$q_e(1+q_e)z^2-2z-1 \ge 0. \qquad (15)$$

The solution has the form:

$$z \ge \frac{1+\sqrt{1+q_e(1+q_e)}}{q_e(1+q_e)}. \qquad (16)$$

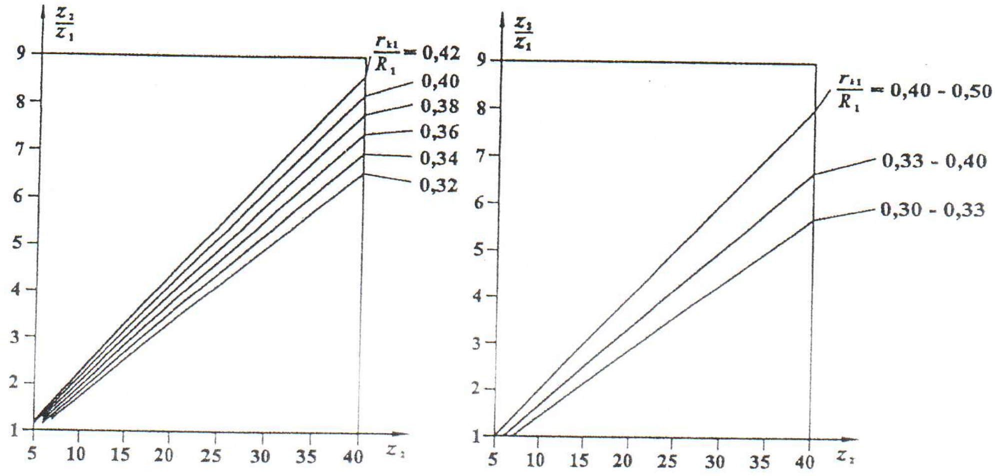
The lower limiting point on the path of contact corresponds to contact of the limiting point G on the rack tooth addendum profile and the corresponding point of the gear tooth dedendum profile. Its position is determine by the angle, that is defined by the equation (6). This equation can be written in the form:

$$\cos\frac{\varphi_g}{q_h} = 1+\frac{2h_{g0}^*}{q_h z} \qquad (17)$$

and when it is entered into (13) one obtains:

$$z \ge \frac{2h_{g0}^*}{|q_h|}. \qquad (18)$$

Based on the equation (16) the program was written for determination of the minimum number of teeth that is possible to realize with the chosen gear hypocycloid coefficient, as well as the maximum values of the contact ratio at given teeth number of the pinion, in order to realize normal conditions of the cycloid gears conjugate action. The program was written in the standard programming language AutoLISP, and it enables also drawing of the corresponding diagram that is presented in Figure 5,a. The



a) b)

Figure 5. Diagrams of the maximum contact ratios

similar procedure was conducted also for the condition (18), and with the chosen value $h^*_{g0} = 1$, and the obtained diagram is shown in Figure 5,b. By the analysis of the obtained diagrams, it can be noticed that, according to equation (18), the greater values of the minimum teeth numbers are obtained, thus this condition has the greater importance than the condition expressed by inequality (16).

The limiting thickness of the tooth on the addendum circle. The limiting value of the addendum thickness is the zero thickness, that leads to the phenomenon of the pointed teeth. In that case, the crossing point of the different named profiles of one tooth lies on the addendum circle. The position of that point on the profile is determined by the gear rotation angle for which the transcendent trigonometric equation is obtained /2/. That equation can be solved approximately, by application of the corresponding iterative method. However, in reference /2/ is shown that solving of that complex equation can be avoided since conditions (16) and (18) eliminate the phenomenon of the pointed teeth. From that reason, as well as due to its huge size, that equation is omitted in this paper.

Having in mind the aforementioned statement, as well as by mutual comparison of expressions obtained in this analysis, with the fact that $h^*_{g0} \geq 1$, one can come up with the conclusion that the inequality (18) is the most important limiting condition in cycloid gearing synthesis.

4. CONCLUSION

In the paper are first defined the conditions that have the role of the limiting factors in cycloid gearing synthesis. Then the check was performed of the following conditions: limiting form of the rack tooth profile, interference, pointed teeth phenomenon and area of definetness of the path of contact. It was shown that this last condition is the most important limiting factor in gearing parameters choice. If this condition is satisfied, the phenomenon of the teeth interference is eliminated, as well as the phenomenon of the pointed teeth. The obtained expressions, in combination with equations of the general gearing theory, are used for determination of the possible contact ratios for different values of the hypocycloid coefficient. Then the diagram was drawn for choosing the possible combinations of the gears teeth numbers. It was shown that by choosing the larger values of the hypocycloid coefficient the larger values of the contact ratios can be realized. It was noticed that the teeth pairs can be realized with very small numbers of teeth.

The presented theoretical model enables development of the corresponding systems for automatic design and construction of gears with cycloid gearing, what further, lays the possibility of manufacturing on the CNC machines. In that way the problem of the complexity of manufacturing would be overcome, with simultaneous lowering of the production costs.

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