International Congress Motor Vehicles & Motors 2008

Kragujevac, October 8th-10th, 2008

MVM20080036

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METHODOLOGY FOR SELECTION OF THE OPTIMAL TROCHOIDAL GEAR TOOTH PROFILE AT THE LUBRICATING PUMPS

ABSTRACT: Methodology for selection of the optimal trochoidal gear tooth profile at rotary (gerotor) pumps is considered in this work. The procedure of identification of the optimal geometrical parameters of the trochoidal gearing at the actual IC engines lubricating pump is also given in the paper. Described methodology is illustrated on the example of the actual pump and obtained solution is proposed as optimal for given conditions. The profile in contact by the theoretical model of the gearing is the equidistant of the conjugate external envelope. Based on theoretical considerations, physical models of gears were made and simulation experiment performed, wherewith the results of the analysis of the functional characteristics of the pump and described methodology were entirely verified.

KEYWORDS: lubricating pump, trochoidal gearing, optimal geometrical parameters

INTRODUCTION

The gerotor pumps, which are the subject of the study in this paper, is to provide a high level functional characteristics of the pump and to preserve competitiveness. Basic requirements set for the hydraulic system pumps are to provide needed fluid flow, pressure and endurance, with minimal pump weight and volume. In order to provide the best construction, it is necessary – in the process of design - to observe influence of various parameters on the functional characteristics of the pump. For these reasons, the methodology of optimal design of the shape of trochoidal teeth profile was formulated on the basis of a mathematical model given in the literature [2]. The developed mathematical model was used for optimal construction, with the purpose of correction of the existing construction solution on the lubrication pump gears (3120.216.89), produced in PPT, Trstenik. The main part of the gerotor pump is a gear pair which is assembled to the main shaft by a gear key. The pump body has fluid outlets which are connected to pump suction line and delivery line. The cover is on the front side. Both gears rotate counterclockwise.

The functional characteristics of the pump are significantly influenced by the gear pair parameters. This paper will analyze the influence of geometrical and kinematical parameters of the gear pair profile on the pump functional characteristics.

1 DESIGNING AND MATHEMATICAL MODELING OF THE TOOT PROFILE

For kinematical analysis of two profiles in contact are considered the basic geometrical relations by generating trochoidal gearing. The basic geometrical relations during unmodified and modified peritrochoid gearing generation

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are illustrated in the Fig.1. Peritrochoid is presented by the point *D,* fixed in a circle plane of radius r_a, which is rolling inward side on the outside of an immovable circle of the radius r_t. The equations of the peritrochoid are obtaind in the coordinate system of the trochoid $O_t x_t y_t$, according to Fig.1, [1], [2].

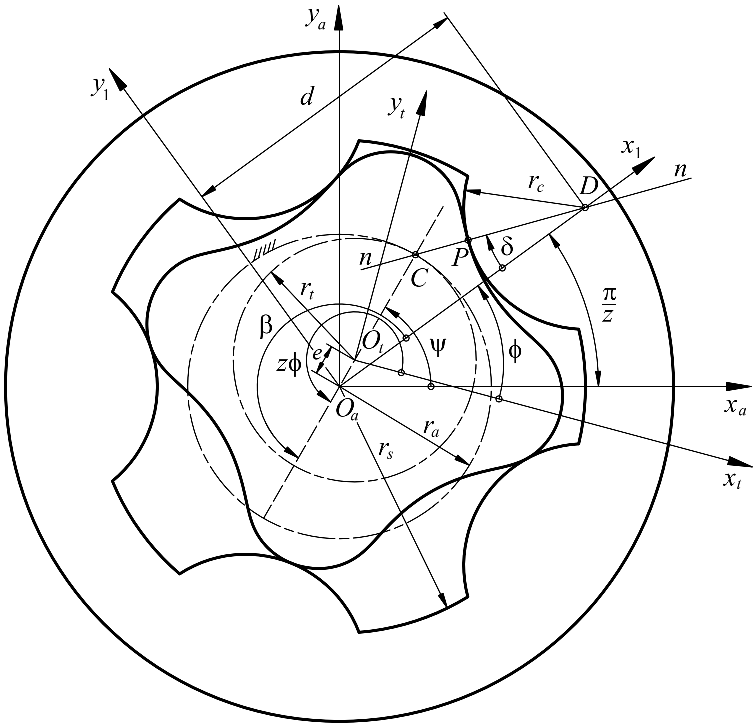


Figure 1 *Generating of trochoid gearing*

Figure 1 illustrates that during the relative moving of kinematical circles, point *D* generates peritrochoid, while point *P* generates equidistant [6], [10]. The angle specified as δ is an angle between the normal *n-n* and radius vector of the point *D*, and can be defined as leaning angle [7]. Coordinates of the contact point *P* in the coordinate system of trochoid can be written as:

$$\vec{r}_t^{(t)} = \begin{bmatrix} x_t^{(t)} \\ y_t^{(t)} \\ 1 \end{bmatrix} = \begin{bmatrix} e[(\cos z\phi + \lambda z \cos\phi) - c\cos(\phi + \delta)] \\ e[(\sin z\phi + \lambda z \sin\phi) - c\sin(\phi + \delta)] \\ 1 \end{bmatrix} \tag{1}$$

where are:

- z is teeth number of external gear,
- ϕ is rotation angle of generating coordinate system of trochoid,
- λ is coefficient of trochoid, which defines relation between the values of the trochoid radius and the radius of moving circle, $\lambda = d/ez$,
- *c* is coefficient of equidistant, which defines relation between values of trochoid radius and eccentricity, $c = r_c/e$

Based on geometrical relations from the Fig.1, the formula for determination of angle δ can be obtained:

$$\delta = \arctan\frac{\sin(z-1)\phi}{\lambda + \cos(z-1)\phi} \tag{2}$$

Establishing geometrical relations between angles of rotation in different coordinate systems and applying transformation of coordinates, equations can be derived which describe meshing gear profiles and their contact line.

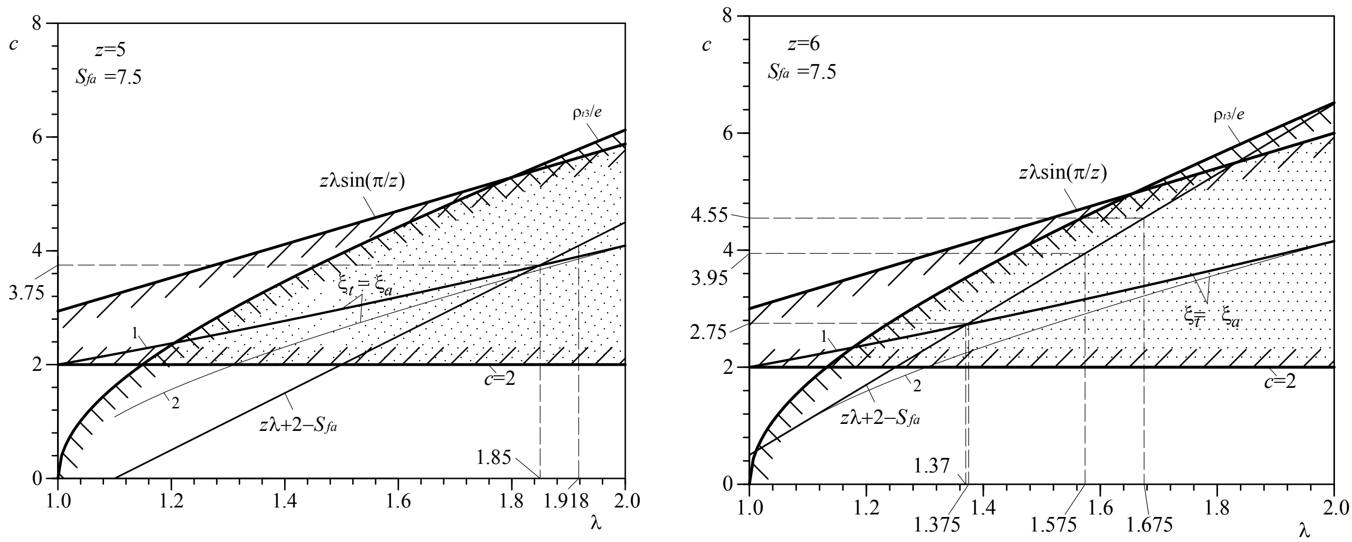
2 METHODOLOGY FOR IDENTIFICATION OF OPTIMAL GEAR TOOTH PROFILE

The structure of algorithm for identification of optimal gear tooth profile is based on theoretical principles of optimal design. To obtaining of the methodology, it is necessary to follow the next procedure:

1. Definition of the problem,
2. Definition of the parameter and functional limits,
3. Choice of alternative solutions,
4. Definition of the criteria for evaluation of the solutions,
5. Choice of best solution,
6. Experimental verification.

To definition of the problem it is necessary to define starting values of the projected gear pair. Gear width *b*, eccentricity *e* and radius of the outer gear root circle, denoted by the parameter S_{fa} of the conjugate envelope, are constant [2]. Values of coefficients λ and *c* must be determined to define optimal shape of gear tooth profile, which means establishing the best indicators, where the same or bigger volumetric capacity of the pump is provided. It is possible to vary number of gear teeth. The main aspect of the choice of the optimal gear profile is the higher volumetric efficiency.

By defining domain of practical use of geometrical parameters for the gear pair of trochoid pump are necessary some limits, which are given in [2]. Using the analyzing of these limits in the Figure 2 are shown the defining domain of practical use of geometrical parameters.



a) b)

Figure 2 *Defining domain of practical use of geometrical parameters*

In the Fig. 2, a is shown the defining domain of practical use of the geometrical parameters for the teeth number of external gear $z = 5$, and in the Fig. 2, b for $z = 6$.

On the base of this graphical interpretation are chosen of the geometrical parameters to define of the gear toot profiles of the alternative solutions.

After that, basing of the defined criteria for the modification of trochoid profile is done the control if the chosen value is compatible with the conditions of quality.
The quality of the designed pumps can be determined by a number of indicators, when certain criteria are accepted:

- sliding velocity of the trochoidal gear pair tooth profile,
- rolling velocity of the meshing gear tooth profile,
- specific sliding of profile at the point with highest value,
- flow rate irregularity.

In the Fig. 3 are given alternative solutions of the gear pairs.

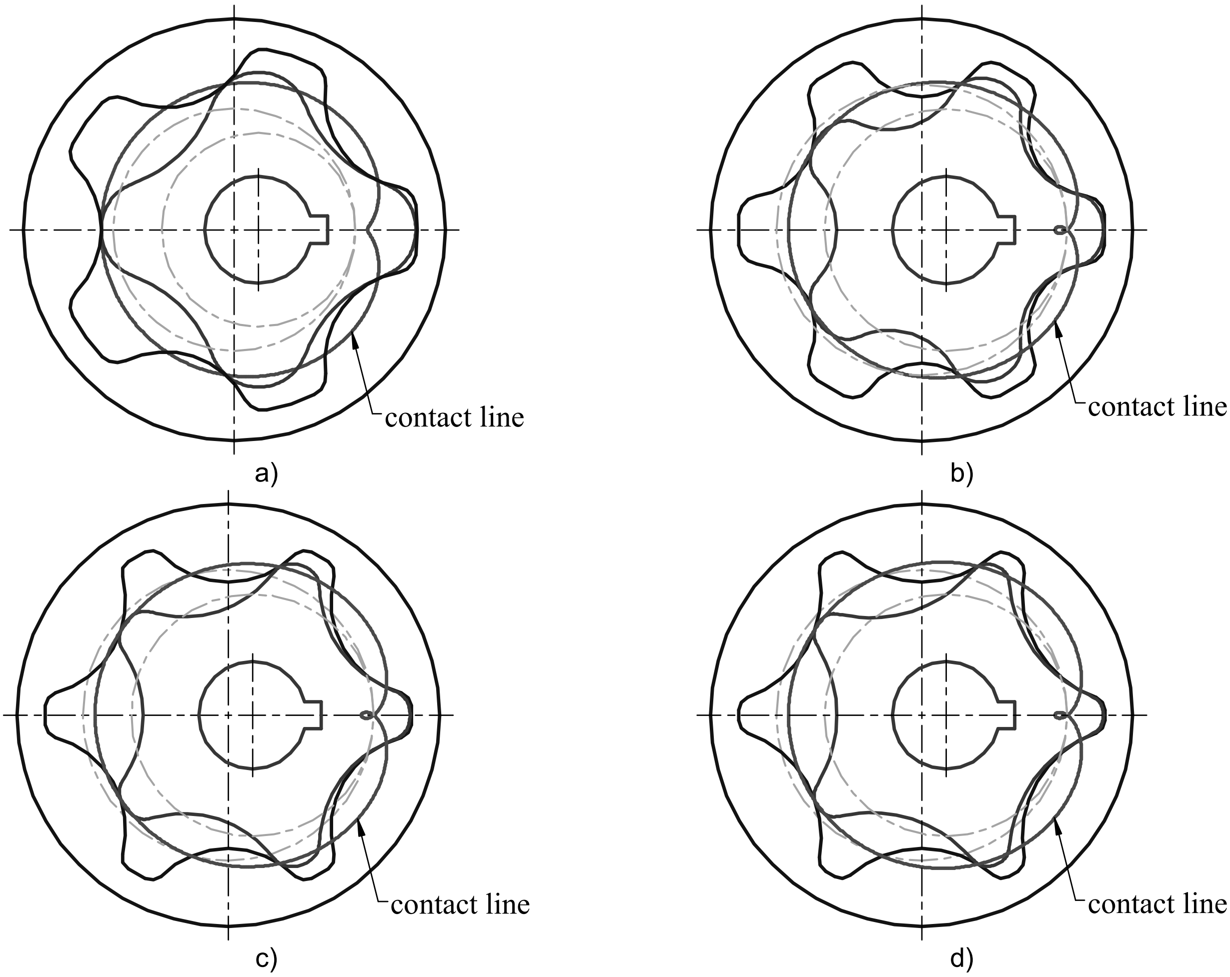


Figure 3 *Alternative solutions of the gear pairs theoretical profiles: a) GP-850, b) GP-375, c) GP-575 (commercial) and d) GP-675*

Starting with the chosen criteria for evaluation of solutions and considering the parameters which need to be calculated, the values of parameters needed for calculation are adopted. All initial data are given in Tab.1.

Table 1 *Initial data*

Parameter	Sign	Value
Needed volumetric capacity [m^3/rev]	q	$14 \cdot 10^{-6}$
Working pressure [bar]	p	6
Number of revolutions of the pump shaft [rpm]	n_t	1500
Angular velocity of the pump shaft [s^{-1}]	ω_t	50π
Coefficient of dynamic viscosity of hydraulic fluid [kg/ms]	η	0.02
Density of hydraulic fluid [kg/m^3]	ρ_f	900
Width of gear pair [mm]	b	16.46
Eccentricity of pump [mm]	e	3.56
Real radius of the root circle of external gear [mm]	r_s	26.94
Theoretical radius of the root circle of external gear [mm]	r_{fa}	26.702

In order to obtain functional dependency which would provide projecting of the pump gear pair, based on the given starting data, a mathematical model of trochoidal gearing pump capacity characteristics has been developed. First to be considered was distribution of working fluid and definition of characteristic phases in the pump operating cycle, followed by description of the methods for defining working capacity and actual rate of delivery of a pump [2], [8]. Based on the analysis of results, relevant values were identified which have influence on pulsation of the rate of delivery and uneven flow.

3 EXPERIMENTAL VERIFICATION

After the analytical check-up of the theoretical models and computer designed program for automatic generation of the gear tooth profile and also the other parameters of the pump, an experimental verification of results was conducted. Measuring was conducted on four different models of gear pairs in the laboratory PPT Hidraulika, with simulation of real conditions of pump exploitation [11]. Models for experimental investigation were cylindrical trochoidal gears. One of them was a gear pair made in PPT factory, shown in Fig. 4, a, and the other three gear pairs, whose profiles were derived from calculations, are made in Unior Components Ltd. according to appropriate technical documentation. Illustration of the gear pair with suggested optimal solution is given in Fig. 4, b, c and d.

a) GP-575 b) GP-375 c) GP-675 d) GP-850

Figure 4 *Models of tested gear pairs*

Investigation program for pump according to internal norms of factory:

- Working fluid: oil – kinematical viscosity ν=22 mm^2/s
- Temperature: T =40±2 °C
- Highest value of the working pressure: p_{max}=10 bar
- Measuring of the flow rate Q
- Variability of values: p and n=500 rpm, n=1000 rpm and n=2000 rpm

The Figures 5, 6 and 7 show diagrams of volumetric efficiency for the different values number of revolutions of the pump shaft. Volumetric efficiency is defined trough relation

$$\eta_V = \frac{Q_{p_r}}{Q_{p_0}} \tag{3}$$

where are:

- Q_{p_r} is measured value of the flow rate of the working pressure $p = p_r$
- Q_{p_0} is value of the flow rate measured at the minimal pressure at the outlet from pump $p = p_o$.

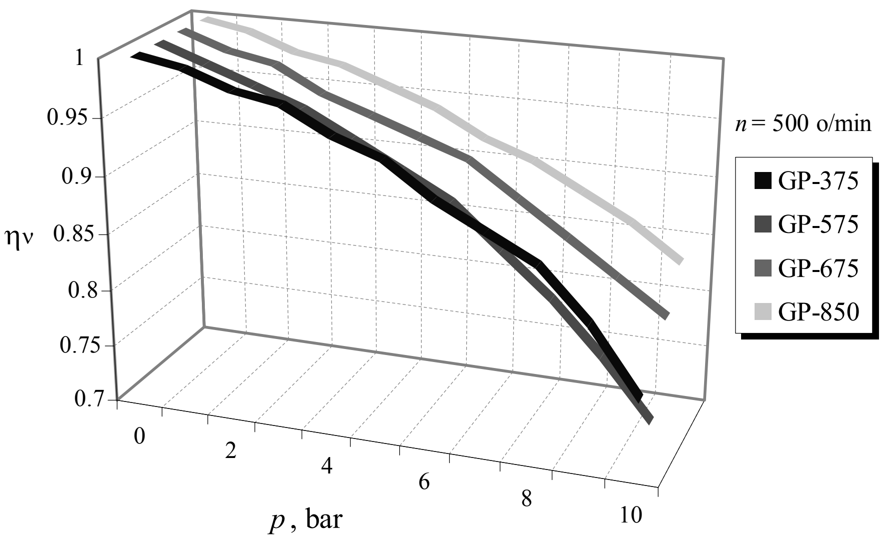


Figure 5 *Diagrams of volumetric efficiency of the gerotor pumps for the values n=500 rpm*

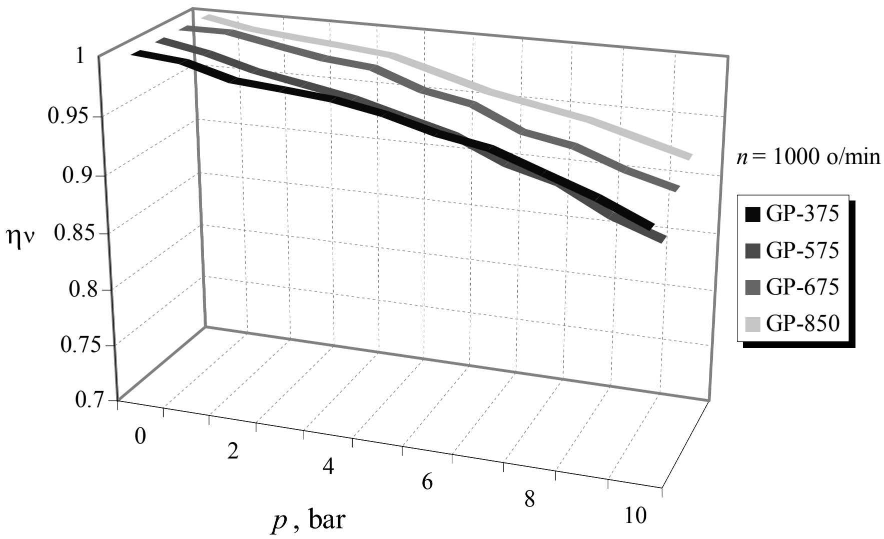


Figure 6 *Diagrams of volumetric efficiency of the gerotor pumps for the values n=1000 rpm*

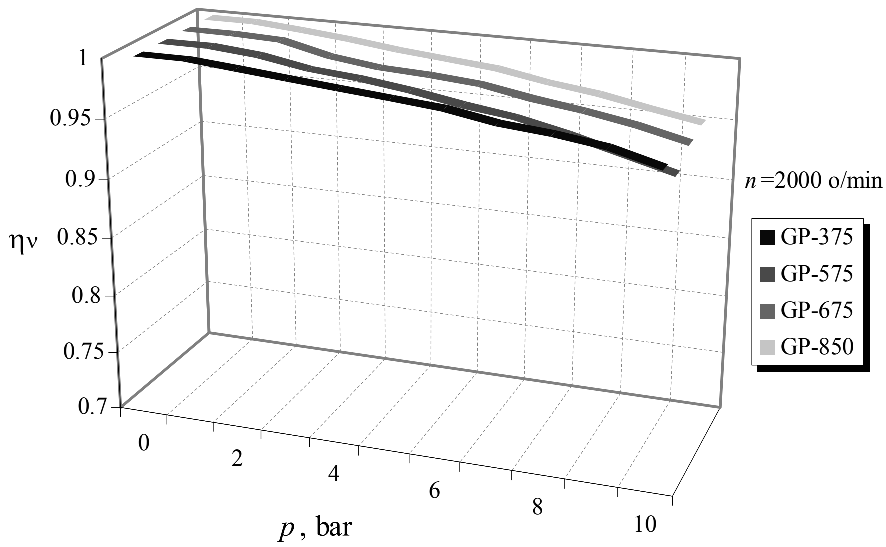


Figure 7 *Diagrams of volumetric efficiency of the gerotor pumps for the values n=2000 rpm*

On the base of given diagrams can be concluded: by *n*=500 rpm volumetric efficiency of pump is higher with model which has smaller number of chambers, than in the other cases highest volumetric efficiency can realize pump with the optimal gear pair.

CONCLUSIONS

Based on the results of analysis kinematical parameters and the optimal solution and also generating of the new model of the gear toot profile can be obtained the following conclusions:

- Functional limits obtained at the definition tooth profile gives elimination of the different kind interference and on the base of that is possible right function and montage of the gear pair;
- From the kinematical aspect, new designed tooth profile is giving the same wear of the meshing tooth profile;
- Through the application of internal equidistance modification comes to the small dimensions and at the same time to increase of the working volume of the gerotor pump;
- Maximal working capacity of the pump was realized, both theoretically and practically.

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