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IDENTIFICATION OF THE OPTIMAL GEOMETRICAL PARAMETERS OF THE TROCHOIDAL GEARING AT THE IC ENGINES LUBRICATING PUMPS

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Keywords: lubricating pump, trochoidal gearing, optimal geometrical parameters

Abstract

The paper presents the theoretical model of the gearing generating with modified trochoidal profile which is applied at the IC engines lubricating pumps. Methodology for selection of the optimal shape of trochoidal gear tooth profile is considered at the lubricating pumps. Further, it shows how it is possible to describe equidistant of the peritrochoidal profile in parameter form. The profile in contact is the equidistant of the conjugate external envelope. The procedure of identification of the optimal geometrical parameters of the trochoidal gearing at the actual pump is also given in the paper. Described methodology is illustrated on the example of the actual pump and obtained solution is proposed as optimal for given conditions. Based on theoretical considerations, physical models of gears were made and simulation experiment performed, with the results of the analysis of the functional characteristics of the pump and described methodology were entirely verified.

1 Introduction

The main goal of the contemporary projecting of the gear pumps, including the gerotor pumps, which are the subject of the study in this paper, is to provide a high level functional characteristics of the pump and to preserve competitiveness. Basic requirements set for the hydraulic system pumps are to provide needed fluid flow, pressure and endurance, with minimal pump weight and volume. In order to provide the best construction, it is necessary – in the process of design - to observe influence of various parameters on the functional characteristics of the pump.

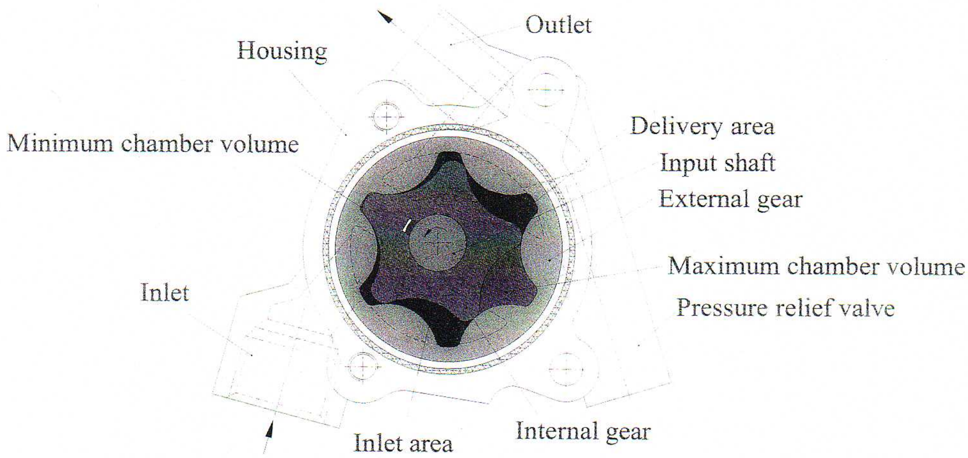


Figure 1: The pump model

For these reasons, the methodology of optimal design of the shape of trochoidal teeth profile was formulated on the basis of a mathematical model given in the literature [1]. The developed mathematical model was used for optimal construction, with the purpose of correction of the existing construction solution on the lubrication pump gears (3120.216.89), produced in PPT, Trstenik. The scheme of the pump model is given in the Fig.1. The main part of the gerotor pump is a gear pair which is assembled to the main shaft by a gear key. The pump body has fluid outlets which are connected to pump suction line and delivery line. The cover is on the front side. Both gears rotate counterclockwise.

The functional characteristics of the pump are significantly influenced by the gear pair parameters. This paper will analyse the influence of geometrical and kinematic parameters of the gear pair profile on the pump functional characteristics.

2 Mathematical model of gearing

The basic geometrical relations during unmodified and modified peritrochoid gearing generation are illustrated by Fig.2. Peritrochoid is presented by the point *D,* fixed in a circle plane of radius r_a, which is rolling inward side on the outside of an immovable circle of the radius r_t. The equations of the peritrochoid are defined in the coordinate system of the trochoid $O_t x_t y_t$, according to Fig.2.

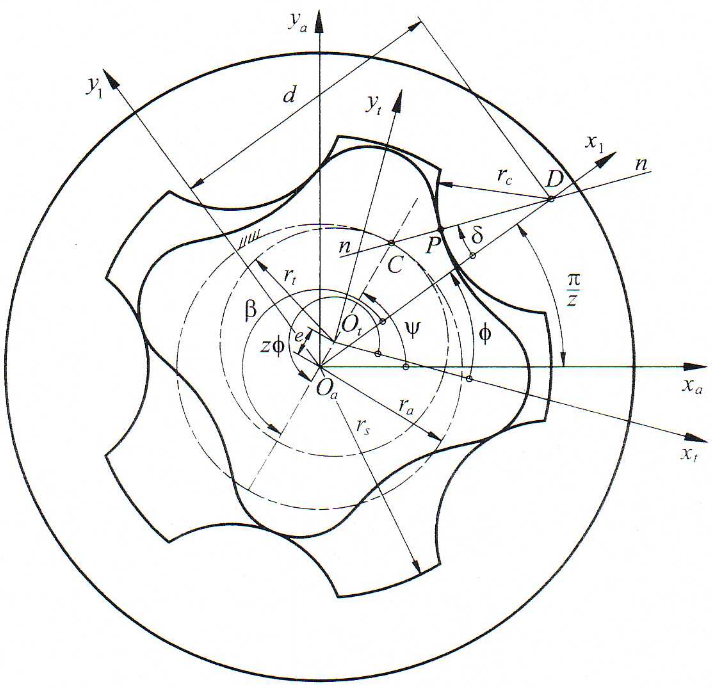


Figure 2: Generating of trochoid gearing

Figure 2 illustrates that during the relative moving of kinematic circles, point *D* generates peritrochoid, while point *P* generates equidistant. The angle specified as δ is an angle between the normal *n-n* and radius vector of the point *D*, and can be defined as leaning angle. Coordinates of the contact point *P* in the coordinate system of trochoid can be written as:

$$\vec{r}_t^{(t)} = \begin{bmatrix} x_t^{(t)} \\ y_t^{(t)} \\ 1 \end{bmatrix} = \begin{bmatrix} e[(\cos z\phi + \lambda z \cos\phi) - c\cos(\phi + \delta)] \\ e[(\sin z\phi + \lambda z \sin\phi) - c\sin(\phi + \delta)] \\ 1 \end{bmatrix} \tag{1}$$

where:

- is coefficient of trochoid, which defines relation between the values of the trochoid radius and the radius of moving circle, $\lambda = d/ez$,
- c is coefficient of equidistant, which defines relation between values of trochoid radius and eccentricity, $c = r_c/e$

Based on geometrical relations from the Fig.1, the formula for determination of angle δ can be obtained:

$$\delta = \arctan\frac{\sin(z-1)\phi}{\lambda + \cos(z-1)\phi} \tag{2}$$

Establishing geometrical relations between angles of rotation in different coordinate systems and applying transformation of coordinates, equations can be derived which describe meshing gear profiles and their contact line.

3 Formulating methodology for identification of optimal gear tooth profile

The structure of algorithm for identification of optimal gear tooth profile is based on theoretical principles of optimal design. To apply the established methodology, it is necessary to comply with the following procedure:

- Definition of the problem,
- Definition of the parameter and functional limits,
- Choice of alternative solutions,
- Definition of the criteria for evaluation of the solutions,
- Choice of best solution,
- Experimental verification.

Above phases of the procedure will be described in the following text.

3.1 Defining the problem

Prior to definition of the problem it is necessary to define starting values of the projected gear pair. Geometrical characteristics which remain constant and are not subject to optimization and are defined by needed external dimensions of the gear pair are: gear width b, eccentricity e and radius of the outer gear root circle, denoted by the parameter S_{fa} of the conjugate envelope, [1]. Values of coefficients λ and c must be determined to define optimal shape of gear tooth profile, which means establishing the best indicators, where the same or bigger volumetric capacity of the pump is provided. It is possible to vary number of gear teeth. It means that the investigation is conducted for z, λ and c, while e and S_{fa} are fixed. The main aspect of the choice of the optimal gear profile is the minimal contact stress at the critical point of contact, and consequently better resistance to tooth sides damaging.

3.2 Parameter and functional limits

To define domain of practical use of geometrical parameters for the gear pair of trochoid pump, the following limits are set:

- Condition for existence of basic prolate perirtochoid envelope, as well as condition that trochoid has no undesirable cusps $\lambda > 1$;

- Analysis of scientific-technical literature, [2], set a condition which defines the range of desirable values of coefficients λ and c with pumps: a) $1 < \lambda < 2$, b) $c>2$;
- From the aspect of realization of a larger contact pattern, in normal working conditions, smaller values of trochoide coefficient λ are advised.
- Criteria for the choice of equidistant radius, to eliminate the phenomenon of tooth profile undercutting, $c < \rho_{t3}/e$;
- Condition for elimination of interference of neighboring tooth profiles of external gear, $c \leq z\lambda \sin\dfrac{\pi}{z}$;
- Condition for elimination of interference of meshing gears profile $c = \lambda z + 2 - S_{fa}$;
- Minimization of maximal equivalent curve of the contact point profile;
- Condition for equal wear of meshing profiles:
  - at the point with highest sliding velocity (Diagram 1. in the Fig.4)
  - at the point with largest curve of trochoid profile (Diagram 2. in the Fig.4)

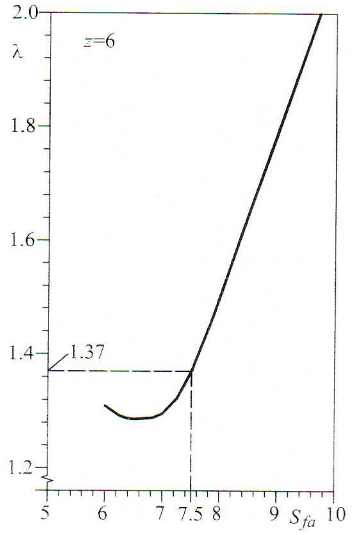


Figure 3: Optimal parameters from aspect $(K_{ekv})_{\min}$

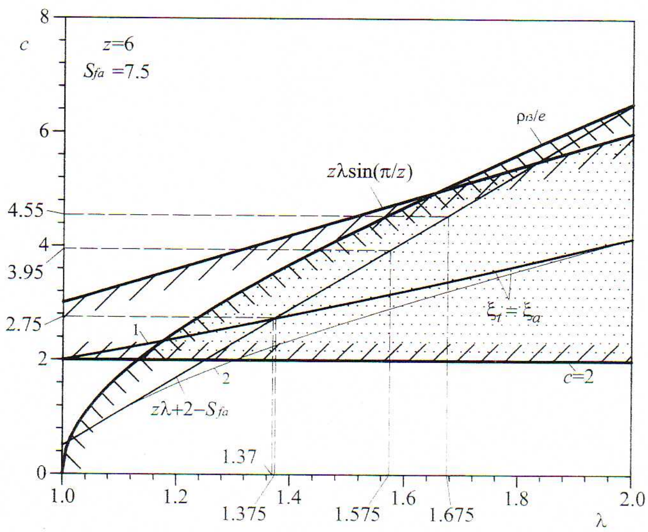


Figure 4: Defining domain of practical use of geometrical parameters

Based on diagram shown by Fig.3, for the given values of the number of teeth z and parameter S_{fa}, defined is the optimal value of trochoid coefficient, according to the criteria of minimal curve in the critical point of contact profile, $(K_{ekv})_{\min}$. Further, based on the corresponding diagram given in Fig.4, it is checked whether a certain value meets other criteria for modification of trochoid profile, and according to that the final decision is brought about choice of value of the parameter λ.

3.3 The choice of alternative solutions

According to the results of the analysis of geometrical parameters of trochoidal gearing and kinematic conditions, solutions are suggested, the basic parameters of which are given in the Tab.1.

Table 1: Parameters of various pump models

Pump parameters	Sign	Number of pump chambers			
		z=6			z=5
Trochoid coefficient [-]	λ	1.375	1.575	1.675	1.85
Equidistant radius coefficient [-]	c	2.75	3.95	4.55	3.75
Calculated volumetric capacity [cm^3/rev]	q	14.003	13.997	13.986	13.675

It can be concluded that for the given design parameters a solution cannot be established with z>6 because it is not within the allowed area. The chosen values of parameters are further investigated according to the criteria set in order to evaluate solutions.

3.4 Criteria for evaluation of the solutions

The quality of the designed pumps can be determined by a number of indicators, when certain criteria are accepted; the criteria which describe different pump characteristics, such as:

- sliding velocity of the trochoidal gear pair tooth profile,
- rolling velocity of the meshing gear tooth profile,
- specific sliding of profile at the point with highest value,
- flow rate irregularity,
- equivalent radius of profile curve at the contact point,
- contact stress of meshing gear at the contact point,
- leakage flow due to technological gaps.

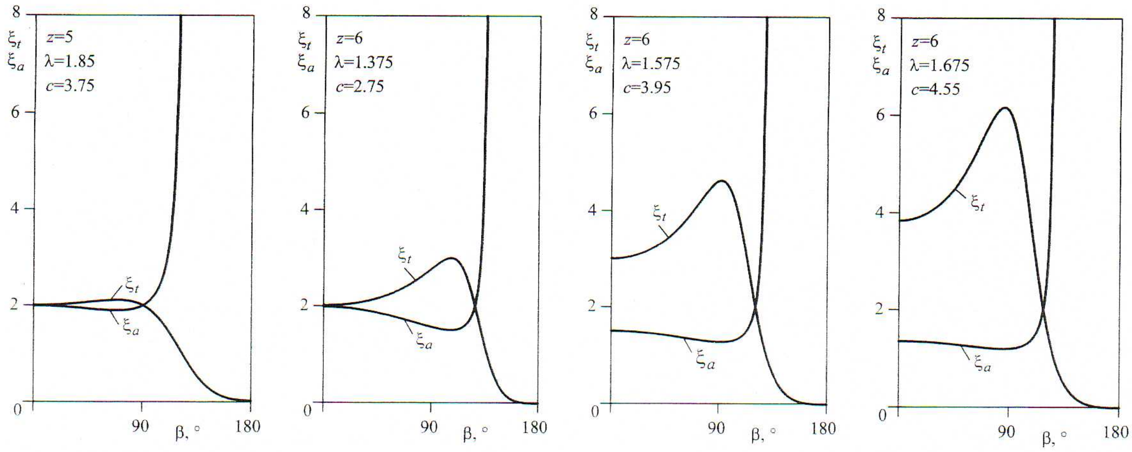
Starting with the chosen criteria for evaluation of solutions and considering the parameters which need to be calculated, the values of parameters needed for calculation are adopted. All initial data are given in Tab.2.

Table 2: Initial data

Parameter	Sign	Value
Needed volumetric capacity [m^3/rev]	q	$14 \cdot 10^{-6}$
Working pressure [bar]	p	6
Number of revolutions of the pump shaft [rpm]	n_t	1500
Angular velocity of the pump shaft [s^{-1}]	ω_t	50π
Coefficient of dynamic viscosity of hydraulic fluid [kg/ms]	η	0.02
Density of hydraulic fluid [kg/m^3]	ρ_f	900
Width of gear pair [mm]	b	16.46
Eccentricity of pump [mm]	e	3.56
Real radius of the root circle of external gear [mm]	r_s	26.94
Theoretical radius of the root circle of external gear [mm]	r_{fa}	26.702
Technological gap [mm]	ε	0.07
Young modul of elasticity of gear material [N/m^2]	E	$2 \cdot 10^{11}$
Poisson's coefficient of gear material [-]	ν	0.3

3.4.1 Kinematical parameters

This section will present some of the results of the kinematical analysis of trochoidal gearing on the concrete examples of gear pairs of the investigated pump models.



(a) model with five chambers **(b) optimal model** **(c) commercial model** **(d) comparative model**

Figure 7: Diagrams of specific sliding trochoid profiles

Figure 7 illustrates diagrams of specific sliding profile, on the basis of which the following conclusions can be drawn:

- Specific sliding on the convex part of trochoidal profile significantly decreases at the chosen solutions, while with the circular profiles of external gears it increases at the top part, at the chosen solutions.
- It is verified that the conditions for even wear of profiles at the point with the highest sliding velocity are met at the chosen solutions, from the aspect of equality of specific sliding.

3.4.2 Functional characteristics

In order to obtain functional dependency which would provide projecting of the pump gear pair, based on the given starting data, a mathematical model of trochoidal gearing pump capacity characteristics has been developed. First to be considered was distribution of working fluid and definition of characteristic phases in the pump operating cycle, followed by description of the methods for defining working capacity and actual rate of delivery of a pump [1], [3]. Based on the analysis of results, relevant values were identified which have influence on pulsation of the rate of delivery and uneven flow.

In order to create conditions which reduce contact forces, and thus reduce wear, analyzed were forces and moments which influence the gear pair of the trochoidal rotational pump. Starting with the specific conditions in which the load is transmitted simultaneously at a number of contact points, considered were fluid thrust force which affect the sides of the gear teeth, contact forces and contact stress, [1]. To check analytical methods and determine error which occurs when applying above noted postulations, the finite element method was applied.

It should be pointed out that in the described investigations the theoretical trochoidal profile was considered, with presumptions on ideally precise geometrical characteristics. Starting from the fact that the real profiles are made with inevitable technological gaps, this paper considers modelling of real meshing profiles, [1], [4]. When characteristics of profiles with technological gaps were considered, their influence on the loss of pump capacity was identified.

3.5 Experimental verification

After the analytical check-up of the theoretical models, an experimental verification of results was conducted. The main goal of the investigation was to check whether the experimental values obtained for the gear pair from optimization results with sufficient precision match with the calculation. Measuring was conducted on four different models of gear pairs in the laboratory PPT Hidraulika, with simulation of real conditions of pump exploitation. Models for experimental investigation were cylindrical trochoidal gears. One of them was a gear pair made in PPT factory, shown in Fig.8, and the other three gear pairs, whose profiles were derived from calculations, are made in Unior Components Ltd. according to appropriate technical documentation. Illustration of the gear pair with suggested optimal solution is given in Fig.9.

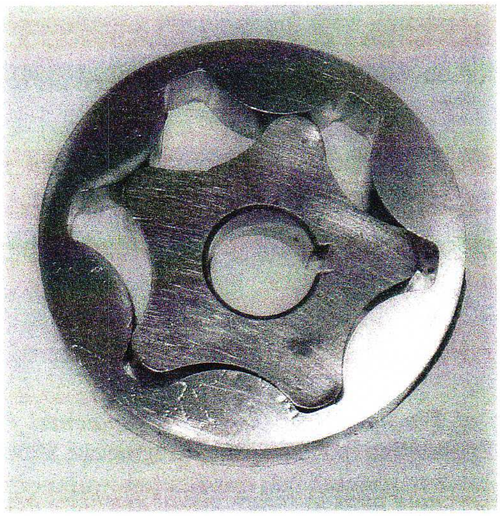


Figure 8: Commercial model

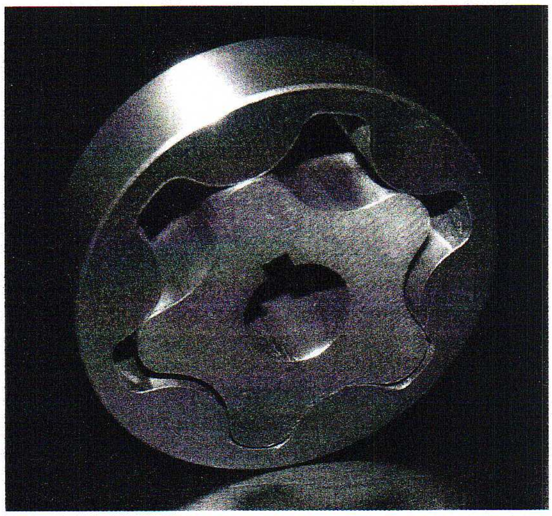


Figure 9: Optimal model

Figure 10 shows diagrams of conditionality of pressure and distribution which a pump makes during the investigation of optimal model of gear pair, while the Fig.11 shows diagrams of volumetric efficiency. Variability of values was implemented on the number of revolutions of the pump main shaft.

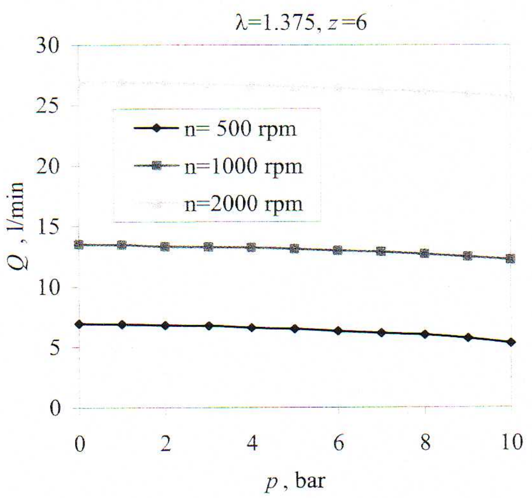


Figure 10: Diagram on conditionality of pressure and flow rate

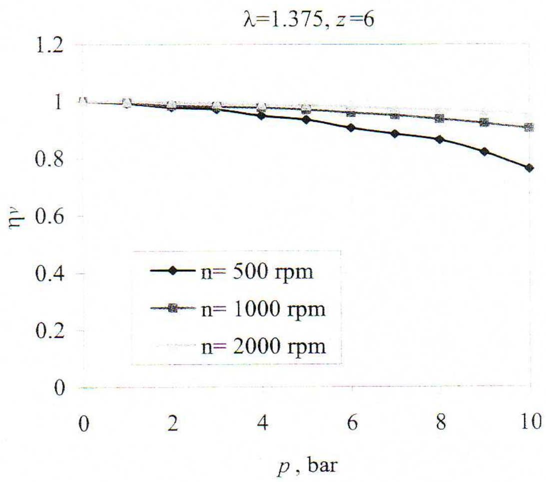


Figure 11: Diagram on volumetric efficiency

Taking into account all results, the chosen optimal model verified through experimental investigation gives one of the potential designed solutions for gerotor pumps with newly generated gearing profile.

4 Conclusions

Based on the results and comparative analysis the following conclusions are drawn:

- Different types of interference were eliminated, which provided regular functioning and mounting of the gear pair;
- Maximal equivalent profile curve was minimal;
- One of the conditions for equal wear of meshing profiles was fulfilled;
- Maximal working capacity of the pump was realized, both theoretically and practically;
- Pulsations of distribution are decreased for an odd number of pump chambers, and smaller values of trochoid coefficient λ;
- Maximal contact stress appear in the meshing zone where contact surfaces have minimal or similar equivalent curve radius.

One of the ideas for the future investigations is development of the derived algorithms and programs with the aim to implement them in practice.

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