USING OF GAP ELEMENT FOR CONTRACTION JOINTS MODELING IN SEISMIC ANALYSIS OF CONCRETE ARCH DAMS

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Abstract

The main goal of this paper is the analysis of the gap element and contact element application in the seismic analysis of contraction joints on concrete arch dams. The goal is to justify the use of gap element for contact modeling because they are more efficient in terms of computation time than contact elements. For the purpose of verification, comparative analysis obtained using the non-linear gap element and contact element between the cantilevers (verification models) in software Simcenter Femap with NX Nastran. For consideration of the set problem, two verification models with the same dimensions, loads, and type of finite elements (FE) and FE mesh density, but with differently modeled contraction joint between cantilevers were analyzed. The boundary conditions and loads are the same in both considered verification FE models. Cantilevers are modeled using 3D 8-node finite elements. Absolute and relative displacements in the contact regions in verification models were considered in dependence of the contact model. After the verification of the contraction joint modeling, the same type of analysis was performed using a real model. For a real 3D model, the concrete arch dam was used, with given loads and boundary conditions. The modeling of the contraction joint using gap element between the cantilevers of the concrete arch dam was done primarily due to the savings of computation time.

Key words: Contact element; gap element; finite element method; dynamic analysis; concrete arch dam

1. Introduction

Contact mechanics is part of solid mechanics, and the deformation of solids that touch each other at one or more points is the main study of it. When two engineering structures physically rest on each other and are not rigidly connected, and when they transmit external forces to each other through a common contact surface, they are said to be in contact. The general distinction in contact mechanics is between stresses that act perpendicular to the contacting bodies' surfaces and frictional stresses that act tangentially between the surfaces.

A commonly used finite element solution procedure in contact mechanics is the assembly of gap elements across the interface. Gap elements were first introduced in 1979 [1] and represent one of the easiest and the most efficient ways of applying contact conditions in the finite element (FE) model. Forming a contact through a gap element across the interface is, also, the fastest way to solve many contact problems in engineering.

A concrete arch dam is designed so that the force of the water which presses against it, known as hydrostatic pressure, causes the arch to straighten slightly. An arch dam is the most suitable solution for narrow canyons or gorges with steep walls of stable rock to support the structure and stresses. Since concrete arch dams are thinner than any other dam type, they require much less construction material, making them economical, very safe, and practical in remote areas.

In the paper [2] the authors proved that the following parameters have a significant influence on the analysis of the arch dam during an earthquake: the semi-unbounded size of the reservoir and foundation-rock domains, wave absorption at the reservoir boundary, dam-water interaction, dam-foundation rock interaction, water compressibility and spatial variations in ground motion at the dam-rock interface. In the paper [3], the influence of rock mass on the stability of a concrete arch dam was investigated. The influence of dynamic loads on the stability of the arch dam is significant. In the paper [4] the authors proved that updating the parameters of the dam configuration and the correct distribution of concrete blocks improves the stability of arch dams. The dynamic analysis of arch dams plays an important role in the earthquake design of new dams and the earthquake safety evaluation of existing dams. The earthquake damage is simulated based on the actual conditions during the earthquake to verify the developed analysis model. In the paper [5] the joint opening and concrete cracking are qualitatively reproduced, wherein the ground motion excitation is spatially defined based on the acceleration records at the dam–rock interface. The influence of joints behavior on arch dam operation during earthquakes is investigated [6].

For solving all these contact problems with complex conditions, authors in their studies used different types of contacts and different combinations of FE with different FE mesh densities. This paper has for aims to confirm the advantage of using gap elements for contact modeling in regard to the contact elements through verification models. After that is proven, investigation of the influence of earthquake of the concrete arch dam with contact modeled with gap elements between cantilevers is conducted, primarily due to the saving of computation time when analyzing certain types of problems in Simcenter Femap with NX Nastran solver [7].

2. Theoretical basis

A gap element is a nonlinear element that can have different stiffnesses under pressure, tension, and shear load. It is used to model surfaces or points that can be separated, joined, or sliding relative to each other. The element has three degrees of freedom in each node: translations in x, y, and z-direction [8].

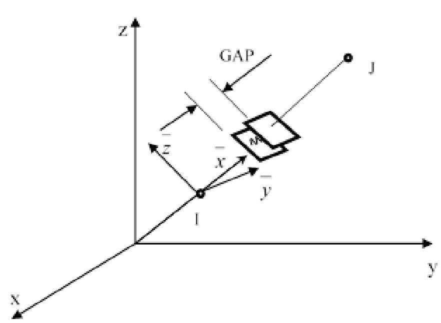

Fig. 1. Gap element

The geometry of the gap element, the position of the nodes in the global and local coordinate system are shown in Fig. 1. The element is defined with two nodes, three stiffnesses (K_n , K_{n_z} and

K_s), an initial gap (GAP), and two coefficients of friction $\mu_{\bar{y}}$ and $\mu_{\bar{z}}$ in the local $\bar{y}$ and $\bar{z}$ direction. The contact surface is normal to the direction of the local axis $\bar{x}$.

The coordinate system of the element has a coordinate origin in node I and the $\bar{x}$ axis is directed towards node J or defined by a direction vector. The contact surface is parallel to the $\overline{yz}$ plane and is defined so that the positive normal displacement of node J relative to node I (in the coordinate system of the element) tends to increase the gap. The gap defines two states of the element: if it is positive there is a gap, and if it is negative there is a penetration [8].

Normal stiffness K_n is based on the stiffness of the surfaces in contact and is used when the gap is $\overline{U_n} \le 0$. Stiffness K_{n_z} is used when the gap is $\overline{U_n} > 0$. Shear stiffness K_s is stiffness in the tangential direction when there is friction. The coefficients of friction $\mu_{\bar{y}}$ and $\mu_{\bar{z}}$ are the characteristics of the material and are taken at the mean temperature of the gaps. Stiffness can also be calculated from the maximum expected force divided by the maximum allowable relative displacement.

If the joint is closed and there is no sliding, there is normal stiffness K_n and shear stiffness K_s. If the joint is closed but sliding, there is normal stiffness K_n and constant friction force μF_n. When the normal force F_n is negative, the joint is in contact and acts as a linear stiffness spring K_n. If the normal force becomes positive depending on the value of the tensile stiffness K_{n_z}, the joint can be parted ($K_{n_z} = 0$) or it can act as a linear spring ($K_{n_z} > 0$).

The normal direction of the gap ($\bar{i}$) can be defined in three ways: 1) through the nodes that define the gap element; 2) via the loaded direction vector; 3) over the initial normal vector to the shell in node I.

The vectors $\bar{j}$ and $\bar{k}$ can be any two mutually perpendicular vectors lying in a tangential plane that is perpendicular to the unit vector $\bar{i}$ of the gap direction $\bar{x}$. For example, we can take:

$$\bar{k} = \frac{\bar{i} \times j}{\left\| \bar{i} \times j \right\|} \tag{1}$$

where $\bar{j}$ is the unit vector of the *y*-axis. If j is parallel to $\bar{i}$, we can take that $\bar{k} = i$. Then, $\bar{j}$ we get as:

$$\bar{j} = \bar{k} \times i \tag{2}$$

Displacements in the tangential plane in the directions of the local axes $\bar{y}$ and $\bar{z}$ are determined as:

$$U_{s_{\bar{y}}} = \left(U_{s_{\bar{y}}}\right)_J - \left(U_{s_{\bar{y}}}\right)_I \text{ and } U_{s_{\bar{z}}} = \left(U_{s_{\bar{z}}}\right)_J - \left(U_{s_{\bar{z}}}\right)_I \tag{3}$$

Shear forces are calculated as:

$$F_{s_{\bar{y}}} = K_s U_{s_{\bar{y}}} \text{ and } F_{s_{\bar{z}}} = K_s U_{s_{\bar{z}}} \tag{4}$$

There are three states of the gap element: 1) closed and fixed (in the contact there is no sliding and the joint acts as a linear spring, $F_n < 0$ and $F_s < \mu\left|F_n\right|$); 2) closed and mobile (sliding contact, $F_n < 0$ and $F_s = \mu\left|F_n\right|$) and 3) the gap is open ($F_n \ge 0$ and $F_s = 0$) [8].

For the aforementioned element states, the stiffness matrices and force vectors in the local coordinate system of the element are given:

1) closed $F_n < 0$ and fixed $F_s < \mu\left|F_n\right|$ gap:

$$\overline{K_e} = \begin{bmatrix} K_n & 0 & 0 & -K_n & 0 & 0 \\ 0 & K_s & 0 & 0 & -K_s & 0 \\ 0 & 0 & K_s & 0 & 0 & -K_s \\ -K_n & 0 & 0 & K_n & 0 & 0 \\ 0 & -K_s & 0 & 0 & K_s & 0 \\ 0 & 0 & -K_s & 0 & 0 & K_s \end{bmatrix} \tag{5}$$

$$\overline{F_e} = \begin{bmatrix} F_n \\ F_{s_{\bar{y}}} \\ F_{s_{\bar{z}}} \\ -F_n \\ -F_{s_{\bar{y}}} \\ -F_{s_{\bar{z}}} \end{bmatrix} = \begin{bmatrix} K_n U_n \\ K_s U_{s_{\bar{y}}} \\ K_s U_{s_{\bar{z}}} \\ -K_n U_n \\ -K_s U_{s_{\bar{y}}} \\ -K_s U_{s_{\bar{z}}} \end{bmatrix} \tag{6}$$

2) closed gap $F_n < 0$ and mobile in both local directions $F_{s_{\bar{y}}} = \mu_{\bar{y}} |F_n|$ and $F_{s_{\bar{z}}} = \mu_{\bar{z}} |F_n|$:

$$\overline{K_e} = \begin{bmatrix} K_n & 0 & 0 & -K_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -K_n & 0 & 0 & K_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

For $\mu_{\bar{y}} \neq \mu_{\bar{z}}$:

$$\overline{F_e} = \begin{bmatrix} F_n \\ F_{s_{\bar{y}}} \\ F_{s_{\bar{z}}} \\ -F_n \\ -F_{s_{\bar{y}}} \\ -F_{s_{\bar{z}}} \end{bmatrix} = \begin{bmatrix} K_n U_n \\ \mu_{\bar{y}} |F_n| \dfrac{U_{s_{\bar{y}}}}{|U_{s_{\bar{y}}}|} \\ \mu_{\bar{z}} |F_n| \dfrac{U_{s_{\bar{z}}}}{|U_{s_{\bar{z}}}|} \\ -K_n U_n \\ -\mu_{\bar{y}} |F_n| \dfrac{U_{s_{\bar{y}}}}{|U_{s_{\bar{y}}}|} \\ -\mu_{\bar{z}} |F_n| \dfrac{U_{s_{\bar{z}}}}{|U_{s_{\bar{z}}}|} \end{bmatrix} \tag{8}$$

If the sliding occurs in only one of the local directions $\bar{y}$ or $\bar{z}$ the stiffness matrix and the force vector are formed analogously to the equations (5) – (8).

For $\mu_{\bar{y}} = \mu_{\bar{z}} = \mu$ the resulting shear force is $F_s = \mu |F_n|$:

$$\overline{F_e} = \begin{bmatrix} F_n \\ F_{s_{\bar{y}}} \\ F_{s_{\bar{z}}} \\ -F_n \\ -F_{s_{\bar{y}}} \\ -F_{s_{\bar{z}}} \end{bmatrix} = \begin{bmatrix} K_n U_n \\ \mu|F_n|\frac{U_{s_{\bar{y}}}}{|U_s|} \\ \mu|F_n|\frac{U_{s_{\bar{z}}}}{|U_s|} \\ -K_n U_n \\ -\mu|F_n|\frac{U_{s_{\bar{y}}}}{|U_s|} \\ -\mu|F_n|\frac{U_{s_{\bar{z}}}}{|U_s|} \end{bmatrix} \tag{9}$$

where:

$$U_s = \sqrt{U_{s_{\bar{y}}}^2 + U_{s_{\bar{z}}}^2} \tag{10}$$

3) open gap and existing stiffness in the direction of tightening $F_n > 0$:

$$\overline{K_e} = \begin{bmatrix} K_{n_z} & 0 & 0 & -K_{n_z} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -K_{n_z} & 0 & 0 & K_{n_z} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

$$\overline{F_e} = \begin{bmatrix} F_n \\ 0 \\ 0 \\ -F_n \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} K_{n_z} U_n \\ 0 \\ 0 \\ -K_{n_z} U_n \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

3. Contact types and test FE models

In contact problems, there are three types of contact: surface, line, and point contact. In this paper, contact between two bodies on the surface was examined.

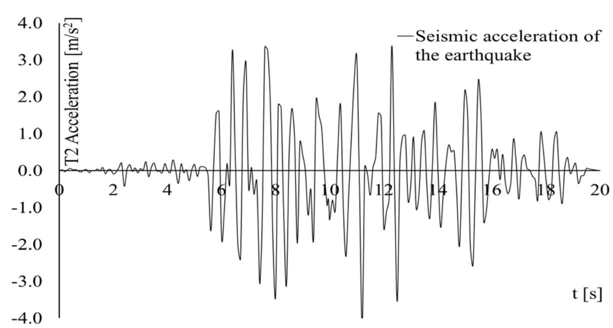

Fig. 2. Seismic acceleration

The main goal of this paper is to prove that contact modeled with gap contact elements and with contact elements will have the same results, and also to monitor node displacements and acceleration in contact regions and to present obtained results. For this study, two geometry models were tested. The earthquake acceleration shown in Fig. 2 is given in both geometry models in the Y direction.

3.1 Verification model

The first FE model consists of two concrete cantilevers jointed using two types of contact elements. The dimensions of the left cantilever are 50x10x10 m, and for the right cantilever, dimensions are 50x20x10 m. Both concrete cantilevers are modeled with 3D hexahedral eight nodes finite elements with corresponding boundary conditions and loads as shown in Fig. 3. The size of one finite element is 2.5x2.5x2.5 m. A constant pressure of 1440 Pa is set on the sides of the cantilevers. The nodes at the bottom of the model are fixed, and the acceleration in the Y direction is applied in them using the function given in Fig. 1. Material characteristics of concrete cantilevers are considered as linear elastic and shown in Table 1.

Type of material	E [GPa]	ρ [t/m3]	ν
Concrete	30.00	2.44	0.18

Table 1. Material characteristics of FE model 1

As already mentioned, models with two geometries have been prepared to monitor node displacement and acceleration in contact regions using different types of contact elements and different FE mesh densities.

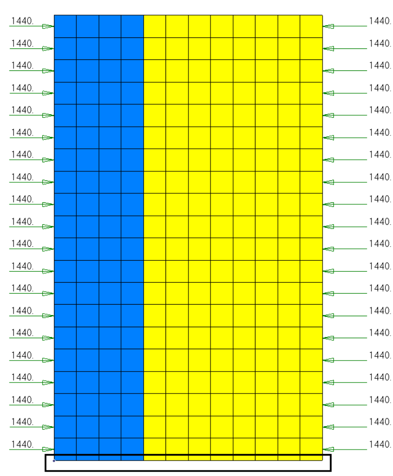

Fig. 3. FE model 1 with given loads and constraints

Two cases of the first FE model were analyzed. For the FE model with the joint between cantilevers with contact elements, dynamic analysis was performed. For the FE model with gap contact elements between cantilevers, linear dynamic analysis was performed.

3.2 Results and discussion

The results of dynamic analysis of cantilevers obtained by using 3D hexahedral 8-node elements with gap contact elements were compared with the results of 3D hexahedral 8-node elements with contact elements between cantilevers. For nodes on top of the cantilevers, diagrams of total and relative displacement in the Y direction are shown for the case of the model with contact and model with gap contact in Fig. 4 and Fig. 5.

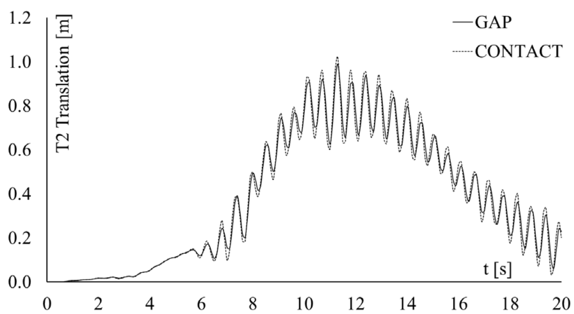

Fig. 4. Ty translation

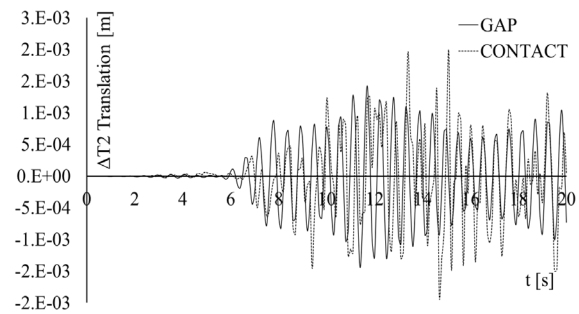

Fig. 5. Relative displacement in the Y direction

According to displacement in Fig. 4 and Fig. 5 satisfactory matching in obtained results for two considered FE models can be noticed. Fig. 4 and Fig. 5 show displacement fields in the Y direction, for the most critical step of the analysis for both considered models: FE model with gap contact elements between cantilevers and FE model with contact elements between cantilevers. Maximum values of node displacement in the Y direction are given in Table 2.

Figure	Maximum relative displacement Ty [m]
Fig. 6	0.86
Fig. 7	0.83

Table 2. Maximum displacement in the Y direction

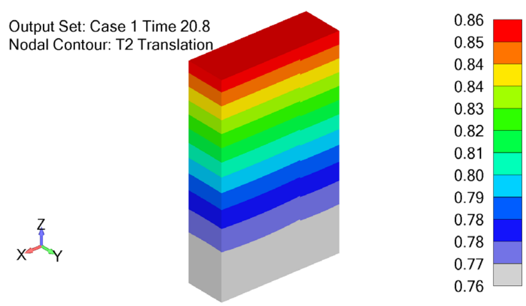

Fig. 6. FE model with gap contact

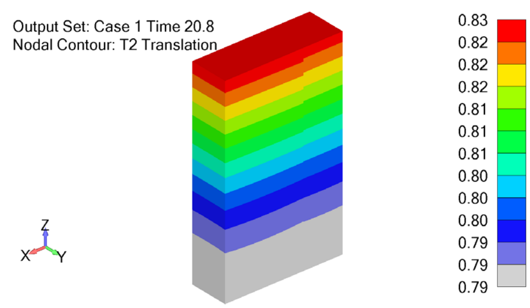

Fig. 7. FE model with contact

According to displacement fields in a Y direction shown in Fig. 6 and Fig. 7 satisfactory matching in obtained results for two considered FE models can be noticed. Through the verification model, it was concluded that the analysis of problems with gap contact elements requires significantly less time than the analysis of problems with contact elements (the calculation on the model with gap contact elements takes 10 times shorter).

3.3 Concrete arch dam model

The second FE model which is used as a case model is shown in Fig. 8. The concrete arch dam consisted of eight cantilevers with different geometries is shown in Fig. 9. The impact of an earthquake on a concrete arch dam with given appropriate constraints has been investigated.

Contact between concrete cantilevers on the arch dam is modeled with gap elements. This model also consists of perimeter and rock mass. Concrete cantilevers, perimeter, and rock mass are modeled with 3D tetrahedral 10-node finite elements. Natural constraints (base nodes of rock mass are fixed and constraints in perpendicular direction are set) were used for the arch dam. Different colors on the rock mass show different types of materials. Material characteristics of the concrete arch dam with perimeter and rock mass are shown in Table 3.

Type of material	E [GPa]	ρ [t/m3]	ν
Concrete arch dam	30.00	2.44	0.18
Rock mass 1 (green color)	45.00	2.62	0.20
Rock mass 2 (blue color)	26.00	2.65	0.22
Rock mass 3 (yellow color)	14.00	2.59	0.24

Table 3. Material characteristics of FE model 2

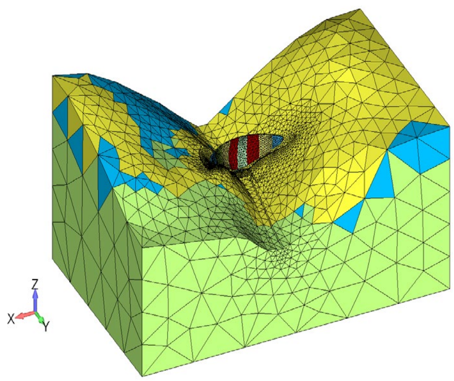

Fig. 8. FE model 2

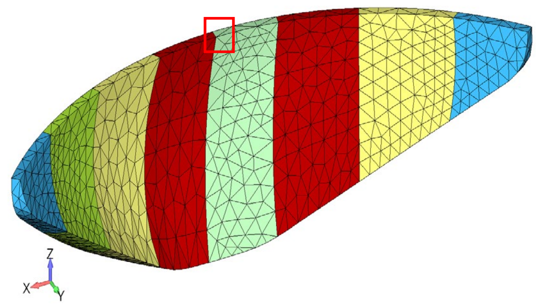

Fig. 9. Concrete arch dam

Based on the results obtained during the analysis of verification models, where the justification of the use of gap elements in modeling the contact was shown, between the cantilevers on the concrete arch dam, the contact was modeled with gap elements.

For nodes on top of the cantilevers (indicated in Fig. 9), diagrams of total and relative displacements in the Y direction are shown in Fig. 10 and Fig. 11.

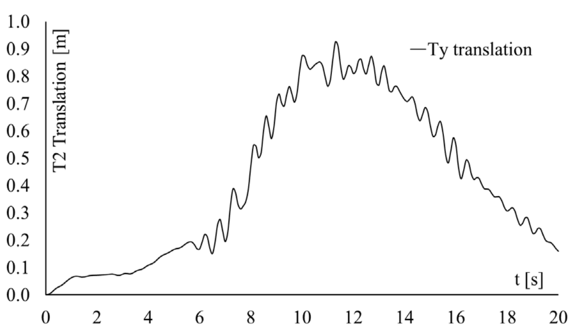

Fig. 10. Ty translation

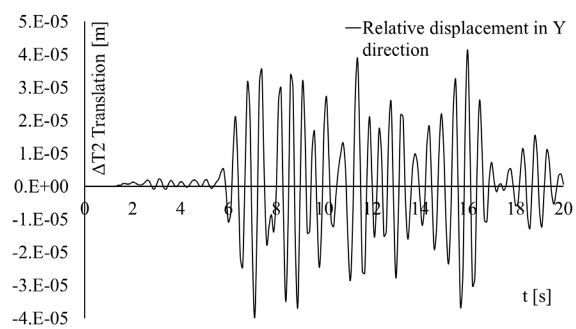

Fig. 11. Relative displacement in the Y direction

Fig. 12 shows displacement fields in the Y direction, for the most critical step of the analysis for the considered model. Maximum values of the displacements in the Y direction are given in Table 4.

Figure	Maximum relative displacement Ty [m]
Fig. 12	0.926

Table 4. Maximum relative displacement in the Y direction

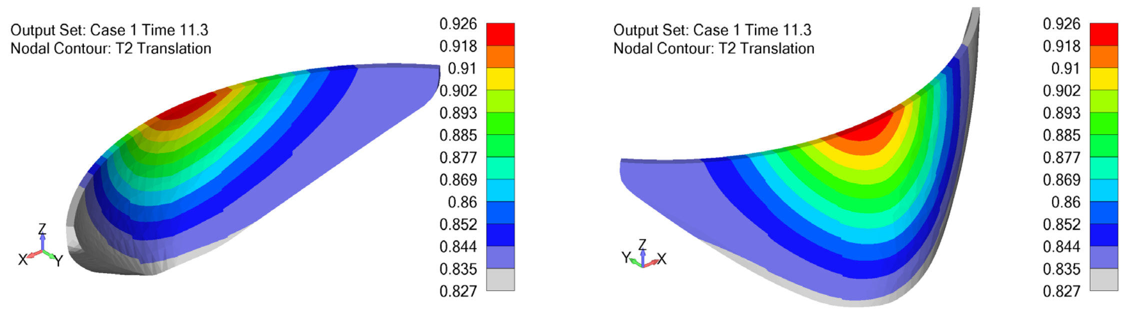

Fig. 12. Displacement in Y direction of concrete arch dam

Fig. 13 shows the fields of major principal stress distribution for the most critical step of the analysis for the considered model under seismic acceleration of the earthquake.

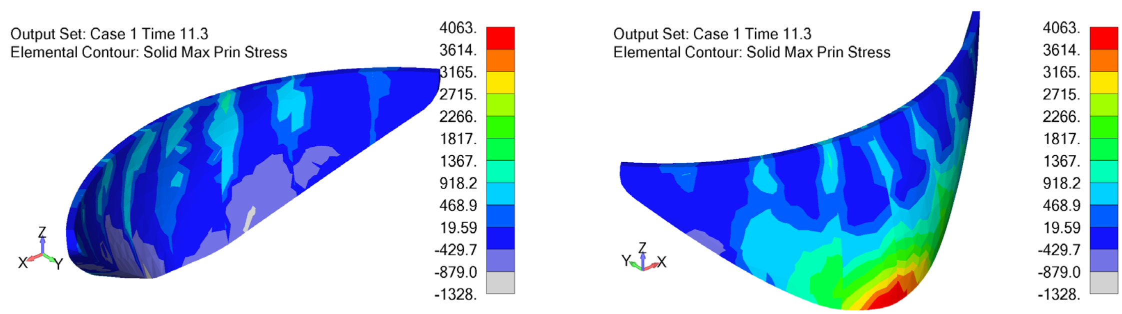

Fig. 13. Major principal stress distribution

Fig. 14 shows the fields of vertical stress distribution for the most critical step of the analysis for the considered model under seismic acceleration of the earthquake.

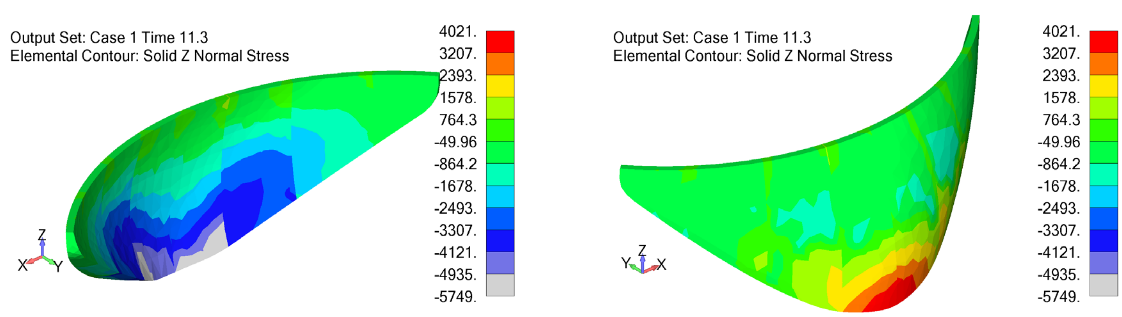

Fig. 14. Vertical stress distribution

Maximum values of the major principal stress and vertical stress distribution for the considered model (concrete arch dam) under seismic acceleration of the earthquake are given in Table 5.

Figure	Major principal and vertical stress distribution [kPa]
Fig. 13	4063
Fig. 14	4021

Table 5. Maximum values of the major principal and vertical stress distribution

4. Conclusions

Based on comparative analysis of results obtained using the non-linear contact element and gap element for contact modeling between the two cantilevers in verification models in software Simcenter Femap with NX Nastran with the same type of elements and FE mesh density the following conclusions were deduced: satisfactory matching in absolute and relative node displacement for two considered FE models can be noticed; maximum relative displacement for the most critical step of analysis for both considered FE models are almost the same; maximum absolute nodal displacement in contact regions for both FE models have a good match; the solving times of the simulations with contact gap elements and with contact elements are highly different. The results clearly show that the models containing contact gap elements (and the element iterative solver enabled) have significantly shorter solving time than the models which have contact modeled with contact elements.

The concrete arch dam is modeled using finite elements and its interaction with the reservoir and foundation is considered in the seismic analysis. The inertia and damping of the concrete arch dam are considered along with its stiffness. For computational cost and post-processing time saving, gap elements for contraction joints modeling in seismic analysis of concrete arch dams were used. A complete joint constitutive model that can simulate both the opening-closing and shear sliding non-linear effects, as well as the shear key effects, is formulated and is subsequently used in a finite element program to study the non-linear effects of contraction joints displacements on the seismic response of a typical arch dam.

Based on the facts presented in the paper, it can be concluded that it is quite legitimate to use contact modeled with gap elements when analyzing certain types of problems.

Acknowledgements

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