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ADDED MASS METHOD APPLICATION FOR DAM-ACCUMULATION INTERACTION ANALYSIS

Dragan M. Rakić[1], Lidija Z. Jelić[1], Miroslav M. Živković[1], Nikola J. Milivojević[2], Milan Lj. Bojović[1]

[1] University of Kragujevac
Faculty of Engineering, Sestre Janjić 6, 34000 Kragujevac
e-mail: lidija.jelic@uni.kg.ac.rs, drakic@kg.ac.rs, zile@kg.ac.rs, mbojovic@uni.kg.ac.rs
[2] Jaroslav Černi Water Institute, Jaroslava Černog 80, 11226 Pinosava, Belgrade
e-mail: nikola.milivojevic@jcerni.rs

Abstract

The aim of this paper is to analyse the influence of modelling the interaction between the dam and accumulation in case of seismic loads using two approaches. A direct transient response analysis was used to solve this type of problem. Theoretical basis of direct transient response analysis as well as the basic equations of Westegaard's added mass and pseudo hydro-dynamic forces method for simulation of the dam-accumulation interaction are shown in the paper. The method of added masses is used to analyse the interaction between dam and accumulation, taking into account that a certain part of the water moves with the upstream face of the structure while the rest of the volume of water remains inactive. The concept of added mass greatly simplifies the analysis procedure considering hydrodynamic effects during earthquakes.

The interaction between the dam and accumulation was modelled with two approaches, as pseudo hydro-dynamic forces and added mass. To verify the procedure, simple model of vertical concrete cantilever was used, after which obtained results were compared. Verified procedure of dam-accumulation interaction using pseudo hydro-dynamic forces and added mass method was applied on concrete arch dam "Grančarevo".

Results shows that the cantilever response under seismic load is delayed in case of added mass as the cantilever has larger mass compared to cantilever without using hydrodynamic effect. Displacement response of the cantilever for pseudo hydro-dynamic approach in comparison to results for added mass, has the same character of displacement but with increased values. Obtained result of dam crest are consistent with results from simple example. Using added mass method, the dynamic effects that occur during seismic loading are taken into account as opposed to the approach of pseudo hydro-dynamic forces in which the change of acceleration that affect the structure is not taken into account.

Key words: direct transient response analysis, added mass method, FSI, concrete arc dam

1. Introduction

The phenomenon that occurs during the seismic effect on the dam was first physically explained and mathematically formulated by Westegaard [1]. The first dynamic analyses, taking into account hydrodynamic pressures that occur as a result of earthquakes, were based on the application of the method of added masses. The method of added mass is considered the simplest way to deal with the hydrodynamic effects of water as an incompressible fluid, as demonstrated in [2, 3].

During earthquakes, accumulation causes additional pressure on the dam's upstream face. Forces that occur due to hydrodynamic water pressure on the dam operate in both directions. Water pressure that operates on the dam's upstream face and moves along with the dam is considered an additional mass.

The aim of this work is to analyse the impact of earthquakes on the dam and compare two ways of defining the impact of water from the accumulation on the dam's structure, by defining pseudo hydro-dynamic forces and the defining added masses.

In the first part of the paper, the theoretical basis of direct transient response analysis is given, which is one of the basic methods for analysing the impact of changing dynamic loads on structures. Also given the theoretical basis of Westegaard's formulation as well as the process of calculating the added masses for dam-accumulation interaction analysis.

In the second part of the paper, direct transient response analysis of the concrete cantilever was performed. The concrete cantilever was subjected to seismic load and the interaction of the dam and accumulation was analysed using pseudo hydro-dynamic forces and method of added masses, based on previously shown theoretical basis.

To perform described analysis Finite element method (FEM) [4, 5] was used with realistic Finite Element (FE) model of the Grančarevo dam, and acceleration signal of the Petrovac 1979 earthquake [6]. Model preparation, numerical simulation and post processing of the results was performed in the software package Femap with Simcentar Nastran [7].

2. Theoretical basics

2.1 Direct transient response analysis

Direct transient response analysis is the most general dynamic method for considering system behaviour that is exposed to a variable dynamic load. One such load is seismic. The seismic load is a virtually unknown stochastic load whose effect on projected objects is taken into account through monitoring the history of earthquakes that occurred and assumptions about a possible earthquake. The record of acceleration due to earlier earthquakes is used as a burden when designing objects to study their behaviour under the influence of possible earthquakes.

Direct transient response analysis gives the system response under the influence of seismic load. The balance equation in this type of analysis is the motion equation that defines the balance of the system at all times. The following equation is a linear differential second-class equation that takes into account all forces that act on the system at any time and represents the basic dynamic equation of system movement [8].

$$\mathbf{M}\ddot{\mathbf{U}}(t)+\mathbf{B}\ddot{\mathbf{U}}(t)+\mathbf{K}\mathbf{U}(t)=\mathbf{F}(t) \tag{1}$$

First member, $\mathbf{M}\ddot{\mathbf{U}}(t)$ of equation (1) is an inertia force that is proportionate to the product of mass and acceleration. Second member, $\mathbf{B}\ddot{\mathbf{U}}(t)$ is a viscous damping force that is proportionate to the product of the damping and acceleration coefficient. Idealized, after the initial movement $\mathbf{U}(0)$, the system will oscillate infinitely with a masked amplitude and will never stop, which is not a realistic case. The effect in which free oscillations gradually reduces their amplitude is called damping. The energy of the oscillation of the system is dispersed by different mechanisms. The damping coefficient $\mathbf{B}$ is selected so that the oscillation energy is wasted equivalent to all existing mechanisms of actual construction, and therefore it is called the equivalent viscous damping. The viscous damping force expressed in the circular frequency function may be presented with the following equation [8]:

$$F_V(t)=i\omega\mathbf{B}\mathbf{U}(t) \tag{2}$$

The basic dynamic equation of movement when only structural damping is present is the following form:

$$\mathbf{M}\ddot{\mathbf{U}}(t)+(1+iG)\mathbf{KU}(t)=\mathbf{F}(t) \tag{3}$$

where G is structural damping coefficient.

The force of the structural damping expressed in the circular frequency function is given by the following equation:

$$F_S = iG\mathbf{KU}(t) \tag{4}$$

On Fig. 1 the structural force of the damping and the viscous force of the damping are shown, depending on the system's own angular frequency. The viscous damping force is proportional to the angular frequency, while the structural force of the dampener is independent of the angular frequency [8].

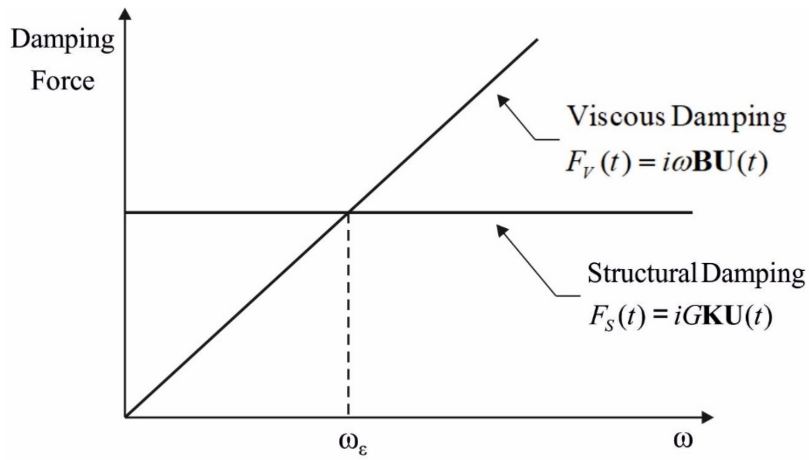

Fig. 1 Structural and viscous dampening depending on circular frequency

Third member $\mathbf{KU}(t)$ is called elastic force, which is a function of system movement and rigidity. This force is linear for small deformations, but it becomes nonlinear for large ones.

The right side of the basic dynamic motion equation refers to external loads defined in the time function. The aforementioned load is independent of the construction it operates on, while its impact on different structures is different for example, the same signal of earthquakes on different structures will not cause the same response.

2.2 Westegaard's formulation of added mass method

The method of added masses is used to simulate dam-accumulation interaction, making sure that a certain part of the water moves with the upstream face of the dam to which it operates while the rest of the water volume remains inactive.

In order for Westegaard's formulation of added masses to be applied to the concrete arch dam, it is necessary to modify Westegaard's basic assumption. First, it should be taken into consideration that when the water on the upstream face of the dam is dynamically operated, a certain part of the water moves with the dam while the rest remains inactive. It should also be considered that the effect of earthquakes is not normal on the dam face, as the upstream face of the dam is curved. The orientation of the dam's face relative to the dam's high varies from point to point [9, 10].

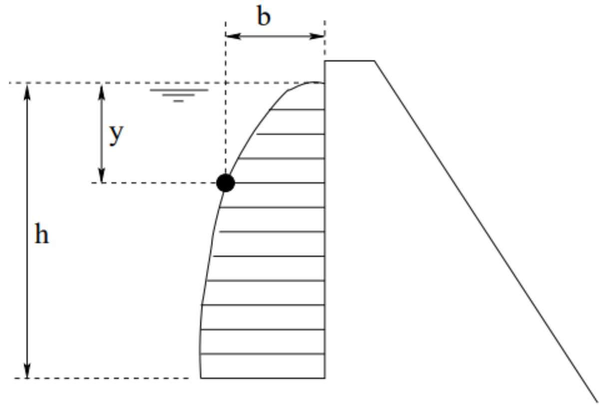

Fig. 2 Westegaard's added mass concept

Distribution of the mass of water that is oscillating together with the dam in the effect of seismic load was shown in Fig. 2. In that figure, h is the depth of the water, y is the depth of the corresponding point at which the extra mass affects, and b is the distance from the dam's face to the corresponding point on the parable. At depth y, the value of the corresponding mass per unit of surface on the upstream face of the dam is determined by the following equation [9]:

$$y_{lump} = \frac{b\rho_W}{g} \tag{5}$$

where the ρ_w unit water density and g gravitational acceleration. Distance b is defined by the following equation:

$$b = \frac{p}{\alpha\rho_W} \tag{6}$$

In the previous equation, p is hydrodynamic pressure that acts on the dam, α is a coefficient describing the intensity of earthquakes (a fraction of g). The aforementioned hydrodynamic pressure, presumed to have a parabolic effect on the face of the dam, is defined by the following equation:

$$p = C\alpha\sqrt{hy} \tag{7}$$

where the C constant is defined by an expression:

$$C = \frac{K}{\sqrt{1 - \frac{16\rho_W h^2}{gkT^2}}} \tag{8}$$

K is Westegaard's constant with value $K = 8{,}011.4 N/m^3$, k is an elastic mode of water and T is the period of horizontal vibrations of the lowest mode of its own oscillation of construction.

By applying the given relationships, the final equation for calculating the value of added mass per unit of surface, due to the dynamic effect of water on the upstream face of the dam, is given by following equation:

$$\gamma_{lump} = \frac{K\sqrt{hy}}{g} \frac{1}{\sqrt{1-\frac{16\rho_w h^2}{gkT^2}}} \tag{9}$$

3. Methodology verification

Using simple model, which represents a simplified vertical cantilever of a concrete arch dam, Westegaard's formulation of added masses and pseudo hydro-dynamic forces was applied. Using the prepared FE model, the dynamic transient response of the structure under the influence of seismic load was analysed, assuming that the material of the cantilever is linear elastic.

The material characteristics of the concrete used in the analysis are given in Table 1. The values of elastic material parameters are adopted from literature [11].

Young's modulus, [kPa]	$30\cdot10^5$
Density, [kg/m^3]	2448.7
Poisson's ratio	0.18

Table 1 Material characteristic of concrete

The FE model consists of one cantilever dimension 50x10x6m. The cantilever is modelled with linear hexahedra finite elements size 1.25x1x1m. The model consists of 2840 elements and 3157 nodes. Prepared FE models with appropriate constraints and loads are shown in Fig. 3.

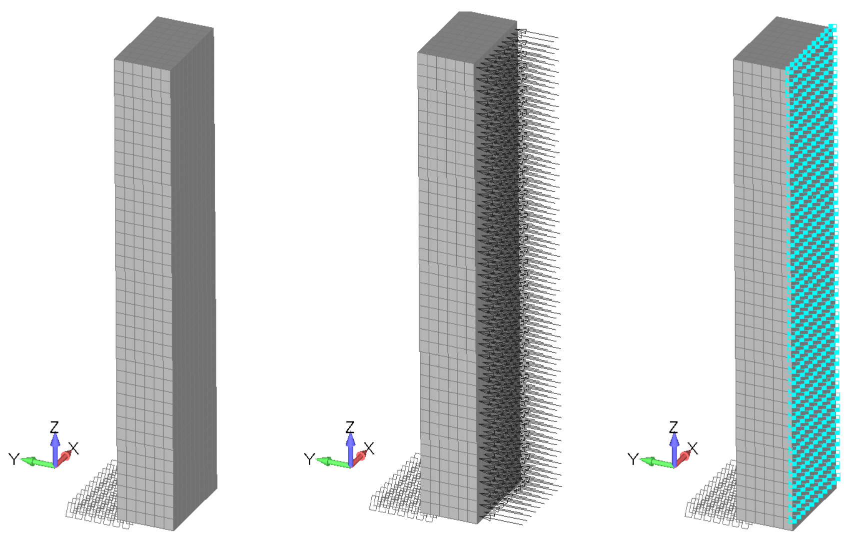

Fig. 3 FE model of cantilever a) without interaction b) with pseudo hydro-dynamic forces and b) with added mass

To perform verification three cases where analysed:

- Case 1 No interaction
- Case 2 Pseudo hydro-dynamic forces
- Case 3 Added masses

Pseudo hydro-dynamic forces and added masses are applied on upstream face of cantilever according to equation (7) and (9) respectively. For case 2 load is defined as constant value, with initial ramp defined in interval of 0-1s in order to reduce and avoid impact loading.

Nominal static load includes cantilever deadweight and dynamic load defined as acceleration in the Y direction in all cases. Seismic acceleration is defined on the bottom nodes of cantilever as shown on Fig. 3 and is applied as time function in dynamic analysis. Earthquake of Petrovac (Montenegro) 1979 was used as seismic acceleration load (Fig. 4). Nodes at bottom of cantilever are constrained in all direction.

Results from Case 1 are used as reference values for comparison with results from two other methods of defining water influence on structure.

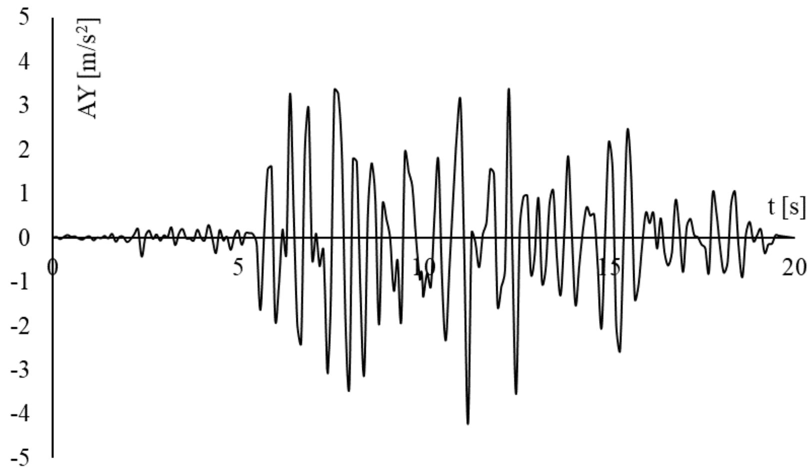

Fig. 4 Seismic acceleration – Earthquake of Petrovac 1972. (Montenegro)

Calculated values of added mass as a function of water depth are shown on Fig. 5. The mass function has a shape of a parable whose tip is on the surface of the accumulation.

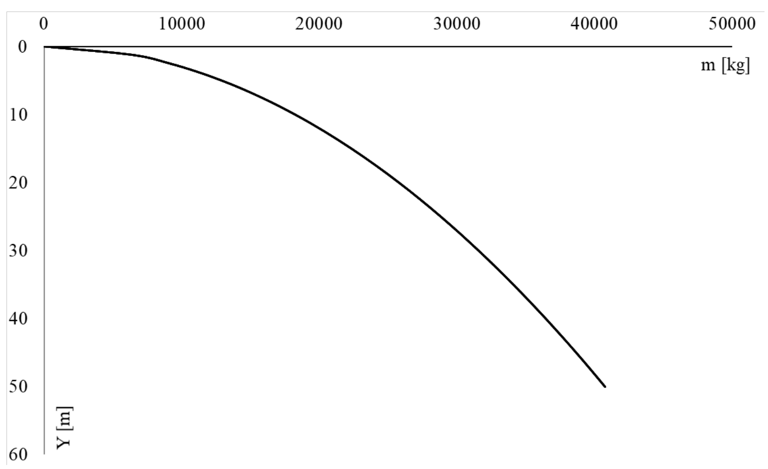

Fig. 5 Added masses in the function of water depth

The earthquake signal is the dominant load that causes a large dynamic system response. In order to reduce its amplitude for the cantilever free response, the appropriate value of proportional damping was used. Viscous damping of the structure is determined based on cantilever eigenvalue modes. Based on modal frequency analysis, the eigenvalue modes of the cantilever for the first five modes have been obtained.

As stated in the previous chapter, the lowest eigenvalues frequency is adopted for system damping frequency. The lowest eigenvalue frequency is ω =1.34 Hz, while the value of the structural damping coefficient is adopted to 0.5.

Cantilever prepared as previously described are used to track the displacement and acceleration of nodes in order to determine the effects of earthquakes without a complicated geometry.

Comparison of top nodes displacement in Y direction for all cases is shown on Fig. 6. In case 2, dam-accumulation interaction defined as pseudo hydro-dynamic forces, result shows that the displacement values are highest for any value of time (Fig. 6) in comparison to no interaction and added mass model results. This is a consequence of force defined as a constant, on which the change of acceleration that affect the structure is not taken into account.

Displacement values for added mass show similar character overall with some local deviation in comparison to no interaction. Furthermore, cantilever response to the seismic load is delayed because of the increased mass of the cantilever compared to results from case 1 with no interaction.

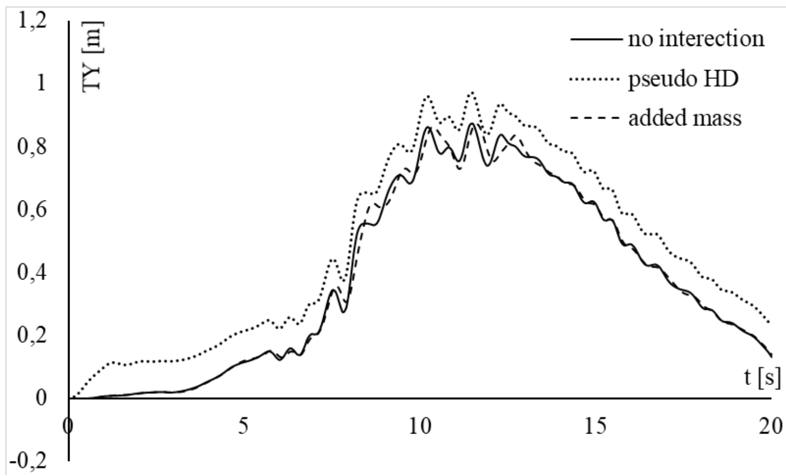

Fig. 6 Displacement of top of cantilever for all cases

Comparison of Y component of acceleration for the selected node at the top of the cantilever is shown on Fig. 7. Based on the showed results, the acceleration in case of added masses has lower amplitude in comparison to the cantilever without added masses due to higher value of total mass and response is delayed.

As for the case 2 with pseudo hydro-dynamic forces, acceleration curve for the observed node is identical in amplitude in comparison to case 1, with exception to the first two seconds as a result of 1s ramp load.

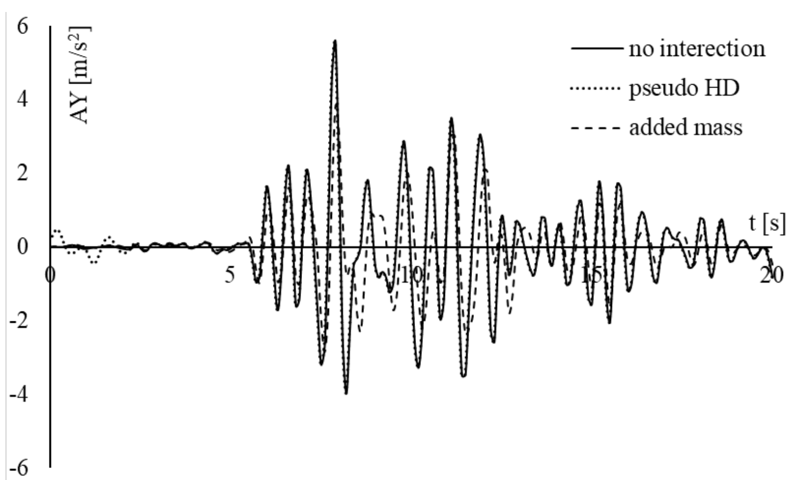

Fig. 7 Acceleration of the top of cantilever for all cases

4. Fluid structure interaction (FSI) between the dam and accumulation

The methodology of pseudo hydro-dynamic forces and added masses methods shown in the previous chapter were applied to the model of the concrete arch dam "Grančarevo". Dam model without interaction was used as reference model as based values for comparison with pseudo hydro-dynamic and added masses methods. Material characteristics of concrete [11] used in the analysis of the dam are shown Table 2.

Young's modulus, [kPa]	$40 \cdot 10^5$
Density, [kg/m^3]	2448.7
Poisson's ratio	0.17

Table 2 Material characteristic of concrete arch dam

The model consists of multiple solids representing the basic elements of the structure, which are the terrain, grout curtain, the perimeter, and the concrete dam. The dimensions of the model are 1200x750x360m. The finite element mesh is made with parabolic tetrahedral finite elements with midside nodes. The model consists of 86346 elements and 123564 nodes. FE model of the concrete arch dam "Grančarevo" and surrounding rock-mass is shown on Fig. 8.

Different colors of finite elements shown on Fig. 8 represent different geological zones defined based on in-situ testing.

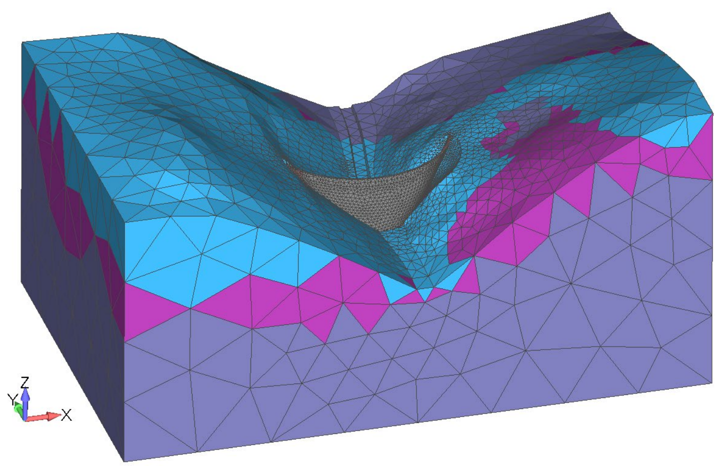

Fig. 8 FE analysis model of concrete arch dam and surrounding rock-mass

Nominal static loads of model include dam self-weight and hydrostatic pressure, while the dynamic load is defined as acceleration function in the direction of the Y axis. Same seismic signal of Petrovac 1979 earthquake (Fig. 4) used for verification model was used for the seismic load of the dam. Seismic acceleration is defined at the boundaries of the model (vertical side and bottom of the model).

On the nodes that belong to vertical surfaces of the terrain representing the extreme boundaries of the model, boundary conditions of symmetry were defined that is, translation in normal direction to surface was constrained. As for the nodes on the bottom horizontal surface of the terrain translation in all direction was constrained.

Displacement result of the selected node on the dam crest for three analysed cases are shown on Fig. 9. Dam-accumulation interaction analysed as pseudo hydro-dynamic forces show that the maximal displacement values are higher (Fig. 6) in comparison to added mass results at the beginning and end of seismic load. This can be result of dam complex shape and asymmetric support on terrain.

The dam response to the seismic load for case 3 is delayed as a result of the increased mass of the dam compared to results from pseudo hydro-dynamic forces method.

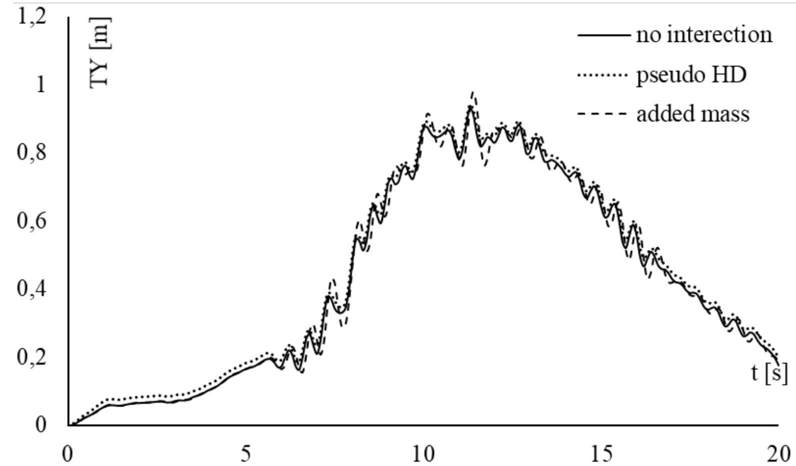

Fig. 9 Displacement of top of arch dam for all cases

Comparison of Y component of acceleration for the node at the top of the dam is shown in Fig. 10. Based on the numerical results, pseudo hydro-dynamic forces do not have an impact on the change of the acceleration of dam in comparison to referenced values of dam with no interaction. For added mass, as expected, small delay in dam response has occurred, because of increased mass on dam's upper face and oscillation amplitudes are higher in comparison to referenced case 1.

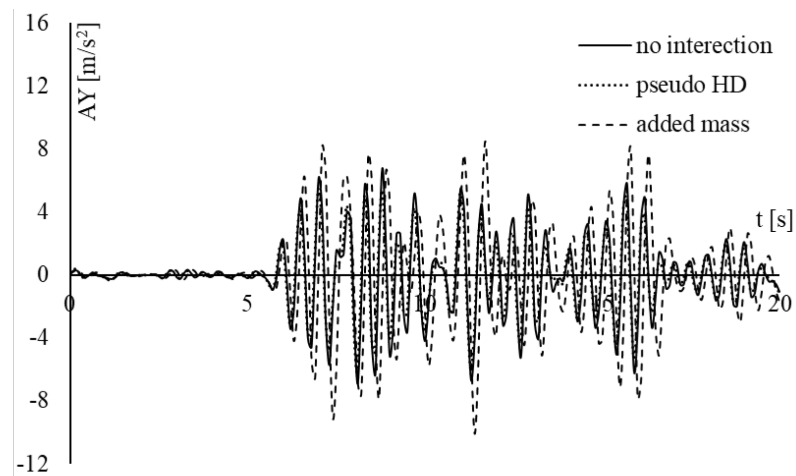

Fig. 10 Acceleration of top of arch dam for all cases

5. Conclusion

The added mass method is an approach that considers the dynamic effects of dam-reservoir interaction that are occurring during earthquake. The volume of water moving with the dam is determined by equating inertia forces with pressures that act on the upstream dam's surface. It is precisely this water that represents the added masses that are defined on the dam face.

The main advantage of the method of added mass relative to the method of pseudo hydro-dynamic forces is that the method of added mass has a variable load that depends on acceleration. In the case of pseudo hydro-dynamic forces, the effects of acceleration change due earthquakes are not taken into account as the given load is a constant and does not change during the analysis. Therefore, the specified forces push against the dam even when the impact of earthquake acceleration changes direction which does not represent real situation that happens on physical construction. Due to this key disadvantage, construction displacements are significantly larger in the direction of pressure acting, as beside hydrostatic pressure, dam is additionally loaded with constant hydrodynamic load. Comparing displacement values of added mass and pseudo hydro-dynamic forces, deviation between those two results is more emphasized for simple cantilever model then for real model of dam. This is because cantilever model has simple geometry, clear loading, and boundary conditions while the dam model is more complex due to dam and terrain real geometry.

For pseudo hydro-dynamic case there is no change in mass of model, so the acceleration responses for the no interaction and pseudo hydro-dynamic models are the same. For added mass method added masses are defined on upstream face to simulate the effect of water movement, so accelerations of the complete construction are lower due to model increased mass.

Based on the above, pseudo hydro-dynamic forces method is not suitable to be used for direct transient response analysis as it gives large deviations in comparison to added mass method, but this approach of interaction calculation is still used for performing quasi static analysis in case of earthquake.

Acknowledgements

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