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Analyzing the Transition Rates of the Ionization of Atoms by Strong Fields of a CO_2 Laser Including Nonzero Initial Momenta

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Abstract—Here, the method of including nonzero initial momenta for ejected electrons in strong infrared laser fields is further developed [8]. It has been shown that, apart from being natural, including the nonzero initial momenta enables one to go into a deeper analysis of the process of tunnel ionization of atoms in strong laser fields (intensity up to 10^{16} W/cm^2). This is due to looking closely at Fig. 2, which indicates that all electrons that could be ejected, under the circumstances, are ejected at a field intensity ~10^{13} W/cm^2, and that the effect of ionization after that is strongly diminished, which can be seen from the slope of the plates on Figs. 2 and 4. This also explains the saturation effect for fields up to 10^{16} W/cm^2 [1, 4, 5, 7], and probably this saturation goes on until the fields raising relativistic effects ~10^{18} W/cm^2 [7]. Opposite to what was believed earlier [7], the atomic field intensities could be increased to values over 10^{17} W/cm^2 only when more than 10 electrons are ejected from the atom, it is shown that the properly calculated ionization of 9 electrons increases the atomic field intensity to ~10^{18} W/cm^2.

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1. INTRODUCTION

There are several approaches to the problem of multiphoton ionization, and especially to the tunneling regime, when low-frequency lasers are involved [1]. But, in our opinion, the closest approach to the phenomenological picture which underlies the theoretical model is the one that leans on the assumptions based on the Keldysh approximation [2]. First, the internal potential of the atoms does not affect the energy of the final state of the ejected electron when it leaves the atom, because the electron is far enough from the nucleus (the short-range potential). Second, the potential of the external field does not influence the initial energy of the electron (for this, the external laser field should be smaller than the atomic field intensity ~10^{16} W/cm^2). Thus, the main effect of the external field was the speeding up of the ionized electrons. The next step was to treat the Coulomb potential of the electromagnetic field as a perturbation of the final state energy, which was essential in the ADK theory [3]. Yet, when constructing the ADK theory, the Coulomb interaction was not included in the calculations of the turning point τ, which, when revised, lead to the corrected ADK theory, or the cADK theory [1, 4–7]. But, it was always assumed that the ionized electrons are leaving the atom with zero initial momenta, which is not a natural assumption. In [8, see also references therein], we were interested in how the nonzero initial momentum influences the transition probability of the tunnel ionization. Now, using a more precise expression for the momentum of the ejected electrons [9], we discuss the results that emerged during this new research: the downshift in the probability maximum, its dependence on the momenta of the ejected electrons, and, above all, the indicative result that gives one the insight into the process of the tunnel ionization of atoms in a strong laser field (up to 10^{16} W/cm^2) (see comment under Fig. 2).

2. CALCULATING NONZERO INITIAL MOMENTUM

Now, we shall obtain the exact expression for the momentum that an electron possesses when leaving the atom. In order to do this, we shall introduce parabolic coordinates $\xi = r + z$, $\eta = r - z$, and $\phi = \arctan(y/x)$. The atomic unit system $e = \hbar = m_e = 1$ will be used throughout this paper. Now, following [9], we shall begin with a stationary Schrödinger equation for a charged particle in the Coulomb field

$$\left(-\frac{1}{2}\nabla^2 - \frac{Z}{r} + Fz - E\right)\Psi = 0. \quad (1)$$

In parabolic coordinates, the Laplace's operator is given by the expression [10]

$$\nabla^2 = \frac{4}{\xi+\eta}\left[\frac{\partial}{\partial\xi}\left(\xi\frac{\partial}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(\eta\frac{\partial}{\partial\eta}\right)\right] + \frac{1}{\xi\eta}\frac{\partial^2}{\partial\phi^2}, \quad (2)$$

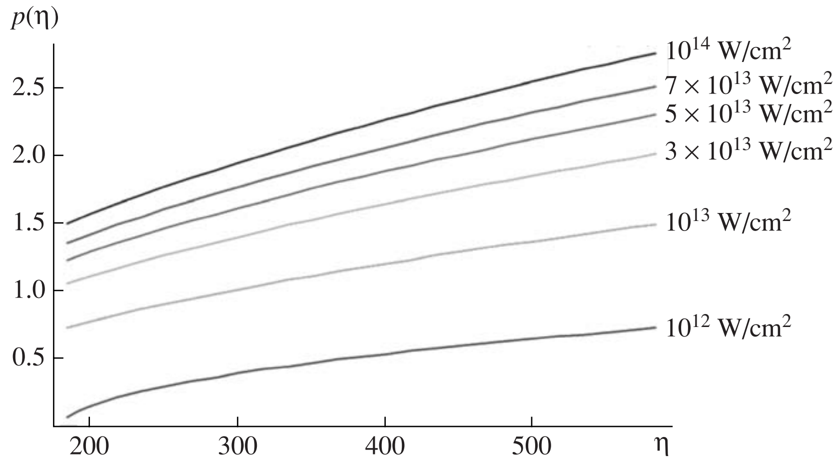

Fig. 1. Momentum plotted against η = 185–586.

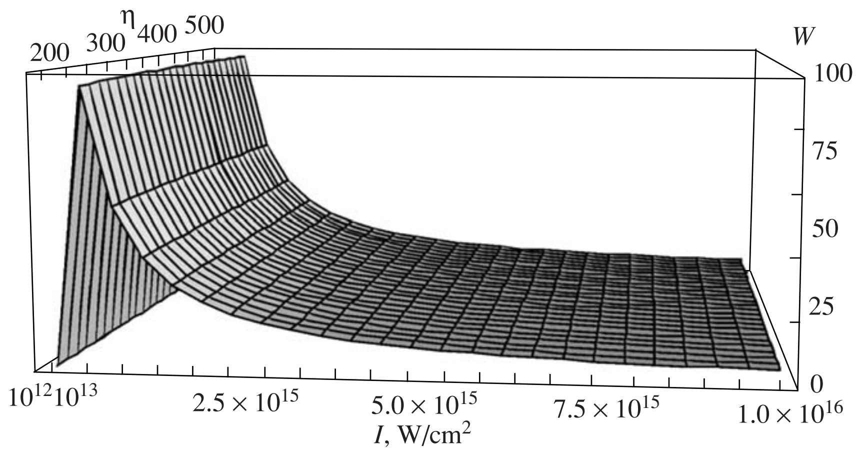

Fig. 2. W_{cADK} and W_{pcADK} transition rates plotted together. The scale for the field intensities is not linear.

so that the Scrödinger Eq. (1) gets the following form in the parabolic coordinates:

$$\frac{4}{\xi+\eta}\left[\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\Psi}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\Psi}{\partial\eta}\right)\right] + \frac{1}{\xi\eta}\frac{\partial^2\Psi}{\partial\phi^2}+2\left(E+\frac{2}{\xi+\eta}\right) = 0. \tag{3}$$

After choosing the appropriate functions for a solution and bringing in the adequate separating constants $\beta_1 + \beta_2 = 1$, after some calculations, one obtains

$$\frac{\partial}{\partial\xi}\left(\xi\frac{\partial f_1}{\partial\xi}\right)+\left(\frac{E\xi}{2}-\frac{F\xi^2}{4}-\frac{m^2}{4\xi}+\beta_1\right)f_1 = 0, \tag{4a}$$

$$\frac{\partial}{\partial\eta}\left(\eta\frac{\partial f_2}{\partial\eta}\right)+\left(\frac{E\eta}{2}-\frac{F\eta^2}{4}-\frac{m^2}{4\eta}+\beta_2\right)f_2 = 0. \tag{4b}$$

As can be seen from Eq. (1), ionization occurs in the $-z$ direction, i.e., along the $\eta = r - z$ coordinate. Hence, we are interested in the second of the two above equations, namely (4b).

Taking into account conditions [9] $m = 0$, $\beta_2 = 1/2$, and $E = -1/2$, one obtains for Eq. (4b)

$$-\frac{1}{2}\left(\frac{\partial^2 f_2}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial f_2}{\partial\eta}\right)-\frac{1}{4\eta}f_2-\frac{F}{8}\eta f_2 = -\frac{1}{8}f_2. \tag{5}$$

Introducing function $f_2(\eta) = \chi(\eta)/\sqrt{\eta}$, and after straightforward but rather cumbersome calculations, Eq. (5) transforms into

$$-\frac{1}{2}\frac{d^2\chi}{d\eta^2}-\frac{1}{2}\left(\frac{1}{2\eta}+\frac{1}{4\eta^2}+\frac{1}{4}F\eta\right)\chi = -\frac{1}{8}\chi. \tag{6}$$

In Eq. (6), the second term on the left side represents the potential, which we will denote as

$$V(\eta) = -\frac{1}{2}\left(\frac{1}{2\eta} + \frac{1}{4\eta^2} + \frac{1}{4}F\eta\right). \quad (7)$$

Since the momentum is generally defined as $p(\eta) = \sqrt{2[E - V(\eta)]}$, one finds that the momentum corresponding to (7) is given by the expression

$$p(\eta) = \sqrt{-\frac{1}{4} + \frac{1}{2\eta} + \frac{1}{4\eta^2} + \frac{F}{4}\eta}. \quad (8)$$

Developing Eq. (8) into a power series results in

$$p(\eta) = \frac{1}{2}\left(\sqrt{F\eta - 1} + \frac{1}{\eta\sqrt{F\eta - 1}} + \ldots\right)$$
$$\text{outside barrier} \quad \eta > \frac{1}{F}. \quad (9)$$

It is obvious that $\eta_L = 1/F$ is a certain limit depending on the field intensity (atomic unit system): $\eta_L = 1/F[10^{12}] = 185.455$. We have chosen as the lowest field intensity of 10^{12} W/cm^2, at which we shall begin our evaluations of the transition rate for the ejected electrons from potassium atoms in a strong field of a CO_2 laser.

3. ESTIMATING THE TRANSITION PROBABILITIES WITH NONZERO MOMENTUM INCLUDED

First, we shall analyze the dependence of momenta of ejected electrons on the coordinate η. It can be seen from Fig. 1 that the momentum of the ejected electrons is gaining in its strength as the intensity of the field increases and this growth is not linear, i.e., the space between the curves is changing, and also the slope of the curves and the rate of growth of the momentum is greater as the field gets stronger. This we shall discuss in more details after showing the transition rates for cADK and also something we shall call pcADK (which represents the transition rate for the case of the cADK theory with nonzero initial momenta included). But, here, we should mention that, naturally, the stronger the laser field is, the more energy is transferred to ejected electrons.

In [1, 4, 5], it was shown that the transition rate in the cADK case is given by expression

$$W_{\mathrm{cADK}} = \left[\frac{4Z^3e}{Fn^{*4}}\frac{1}{1 + \dfrac{2ZF}{(p^2 + 2E_i)^2} + \dfrac{Z^2F^2}{2E_i(p^2 + 2E_i)^3}}\right]^{2n^* - 1} \quad (10)$$
$$\times \exp\left(-\frac{2Z^3}{3Fn^*}\right);$$

in [8], the transition rate for the cADK with a correction for the nonzero initial momenta was also obtained

$$W_{\mathrm{pcADK}} = \left[\frac{4Z^3e}{Fn^{*4}}\frac{1}{1 + \dfrac{2ZF}{(p^2 + 2E_i)^2} + \dfrac{Z^2F^2}{2E_i(p^2 + 2E_i)^3}}\right]^{2n^* - 1} \quad (11)$$
$$\times \exp\left(-\frac{2Z^3}{3Fn^*} - \frac{p^2\gamma^3}{3\omega}\right).$$

Thus, the two transition rates, given by (10) and (11), when plotted together on a 3D graph for fields 10^{12}–10^{16} W/cm^2, and for η ranging from 185–585, and arbitrary units for W, produce the following scheme:

Figure 2 does not make us smarter in relation to the differences between the W_{cADK} and W_{pcADK} variants of the theory, as the differences between the two are too small to be observable for the range chosen. But, it gives us the opportunity to better understand the phenomenology of the process of tunnel ionization. Notice that the scale on Fig. 2 for the field intensities is not linear. Figure 3 is plotted for the same objects, but in the range that shows only the peak of the graph in Fig. 2 making the differences obvious, which we shall discuss later on. Now, we learn from Fig. 2 that, at the laser field intensity ~10^{13} W/cm^2, the transition rate has a maximum, which indicates that all electrons available are ejected. In the case of potassium, in the low frequency strong field of CO_2 laser, it is 1 valent electron and assuming the 8 electrons of the first closed shell, which makes 9 available electrons for ionization. Their depletion leads to a freeing of the electrical charges of the atomic nuclei, making the intensity of its electrical field on the order of the magnitude of 10^{18} W/cm^2.[1] This enables us to use the cADK theory in the entire range of field intensities we are working with, i.e., for 10^{12}–10^{16} W/cm^2.

Figure 3 is plotted for the same objects, but in the range that only shows the peak of the graph on Fig. 2 and makes the differences obvious. Hence, at the laser field intensity ~10^{13} W/cm^2, the transition rate has a maximum, which indicates that all electrons available are ejected. In the case of potassium, in the low-frequency strong field of a CO_2 laser, it is 1 valent electron assuming 8 electrons of the first closed shell, which makes 9 available electrons for ionization with, i.e., for 10^{12}–10^{16} W/cm^2. Their depletion leads to freeing electrical charges of the atomic nuclei, creating an intensity of its electrical field on the order of the magnitude of 10^{18} W/cm^2. This enables us to use the cADK theory

[1] Carefully examining the procedure of transferring from standard units (CGS) to the atomic unit system, one obtains that the gain in 9 electric charges results in a higher value for the atomic field intensity opposite the expected value of 10^{17} W/cm^2.

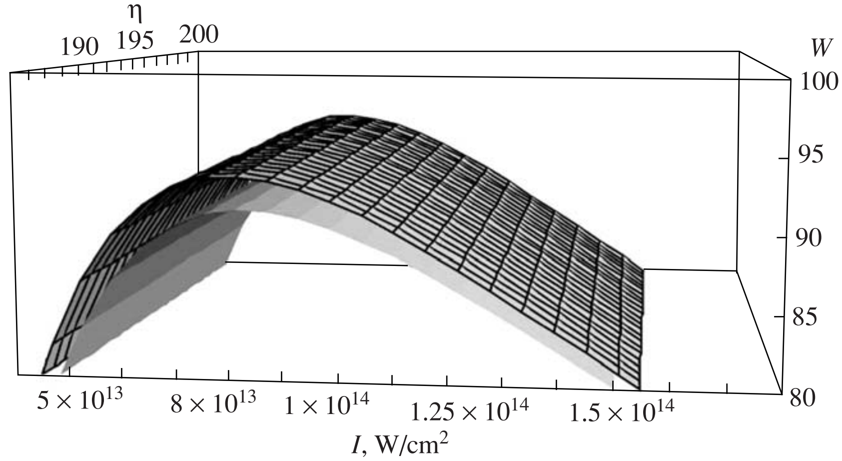

Fig. 3. The peak of the W_{cADK} and W_{pcADK} transition rates plotted together. The lower curved plate represents W_{pcADK}.

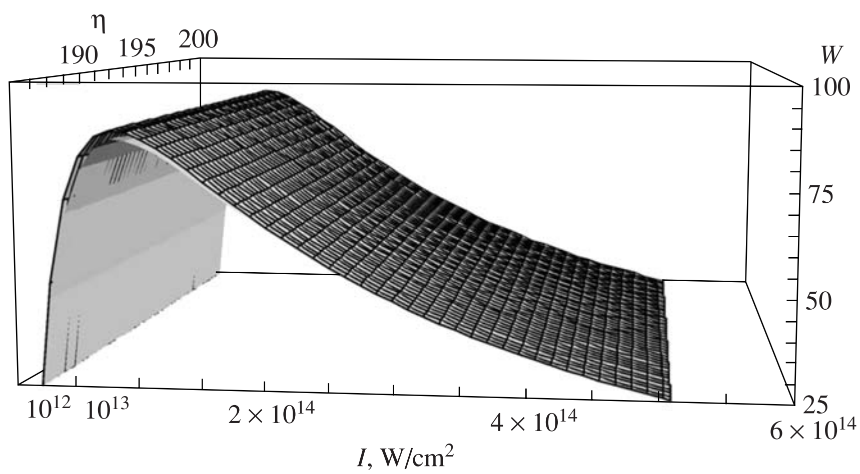

Fig. 4. The W_{cADK} and W_{pcADK} transition rates plotted together with a greater range than in Fig. 3. The barely visible lower curved plate also represents W_{pcADK}.

across the entire range of field intensities with which we are working.

A close examination of Fig. 3 tells us that the transition rate in the W_{pcADK} case is a bit smaller than in the W_{cADK} case. This maximum was calculated to yield the following values: $W_{\text{cADK}}^{\max} = 7.9 \times 10^{13}$ W/cm^2, $W_{\text{pcADK}}^{\max} = 8.2 \times 10^{13}$ W/cm^2, and the differences between the two follow from the effect of transferring the energy of quanta from the laser beam to the momentum gain of the ejected electrons. Thus, the effect of the electrons ejected with nonzero momenta can be detected, although it is not so large, and, as can be seen in Fig. 4, is not long lasting.

Namely, this effect proves once again that, at the laser field intensity ~10^{13} W/cm^2, the transition rate has a maximum, which indicates that almost all available electrons are ejected. As the effect of ionization after the maximum is strongly diminished—that is, shown by the slope of the surfaces on Figs. 2 and 4, and there are fewer electrons which are taking nonzero momenta with them—the surfaces for W_{cADK} and W_{pcADK} are merged (see also Fig. 2). This explains the saturation effect for the fields up to 10^{16} W/cm^2 [1, 4, 5].

It could be expected that this saturation goes further until the relativistic effects emerge at field intensities of ~10^{18} W/cm^2 [6].

An illustration of the above discussion could also be seen in Fig. 5, where on a 2D graph, W_{cADK} (blue) and W_{pcADK} (red) are plotted against the laser field intensity. Hence, the probability maximum is definitely down shifted when the effect of the nonzero momentum is included, and it is also moved to the left. Of course, for the range of field intensities used in Fig. 5, the conver-

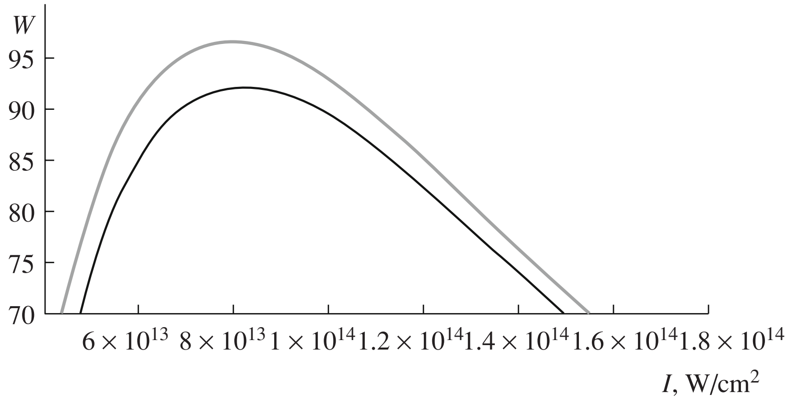

Fig. 5. W_{cADK} (blue) and W_{pcADK} (red) are plotted against the laser field intensity.

gence of the two curves such as that seen in Figs. 2 and 4 is not observable.

Once again, the difference between the curves representing W_{cADK} and W_{pcADK} emerge because of the effect of transferring the energy of the quanta from the laser beam to the momentum gain of the ejected electrons, and, thus, the maximum of the transition rate of electrons ejected from the atom is obviously diminished.

4. CONCLUSIONS

We shall end this by reminding the reader of our analysis of Fig. 2, i.e., that, at the laser field intensity ~10^{13} W/cm^2, the transition rate has a maximum, which indicates that most of the available electrons are ejected. In the case of potassium, in the low frequency strong field of a CO_2 laser, which we have chosen as the typical case, there are 9 electrons that, after being depleted, release the electrical charges of the atomic nuclei, thus resulting in an electrical field on the order of the magnitude of 10^{18} W/cm^2. This is opposite to what was believed earlier, whereby atomic fields could be increased to values of over 10^{17} W/cm^2 only when more than 10 electrons are ejected from the atom. Thus, we can use the cADK theory for the entire range of field intensities with which we are working, i.e., for 10^{12}–10^{16} W/cm^2).

As mentioned earlier, it was always assumed that the ionized electrons are leaving the atom with zero initial momenta. This assumption, being unnatural, has forced us to examine how the nonzero momentum influences the transition probability of the tunnel ionization, and we are discussing results that emerged in this new research: the downshift of the probability maximum, and its dependence on the momenta of ejected elections. Also, we discuss the saturation effect during the ionization of potassium atoms by a low-frequency field of a CO_2 laser (see comments after Figs. 3 and 4).

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