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Research paper

DISCRETE VARIABLE TRUSS STRUCTURAL OPTIMIZATION USING BUCKLING DYNAMIC CONSTRAINTS

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Abstract: *Using continuous variables in truss structural optimization results in solutions which have a large number of different cross section sizes whose specific dimensions would in practice be difficult or expensive to create. This approach also creates optimal models which if varied, even slightly, result in structures which do not meet constraint criteria. This research proposes the discretization of cross section sizes to standard sizes of stock produced for the particular cross section and material, and a 1mm precision for node location when using shape optimization. Additionally, Euler buckling constraints are added to all models in order to achieve optimal solutions which can find use in practical application. Several standard test models of trusses from literature, which use continuous variables, are compared to the discrete variable models under the same conditions. Models are optimized for minimal weight using sizing, shape, topology, and combinations of these approaches.*

Key words: *truss, structural optimization, buckling, dynamic constraints.*

1. INTRODUCTION

Truss structural optimization for minimal weight is a complex problem which can consider one or more aspect of the construction for optimization. Most studies in this field optimize cross-section dimensions, which is called sizing optimization, while fewer consider topology and shape optimization, and even fewer a combination of two or all three. Continuous variables for truss sizing produce optimal solutions with decimal precisions with such specific cross-section dimensions that such bars would be hard or impossible to produce. In order to achieve usable results the sizing aspect of truss optimization needs to be done using discrete variables. Proper constraining of models is also very important. By adding dynamic constraints for Euler buckling the resulting structure becomes practically applicable.

The majority of work published to date considers truss optimization problems with just stress and displacement constraints using various heuristic methods. Few papers consider buckling constraints when solving truss structural optimization problems [1, 2]. Only in the last year or so has the addition of buckling constraints started to appear in research.

In [3] Madah and Amir have optimized the geometric nonlinear response, instead of by imposing a large number of constraints, in order to consider buckling, ensuring local and global stability without actually imposing any buckling constraints. In their research, Grande et al [4] proposed a new approach to optimization of grid shell structures based on a mixed sizing/topologic process, specifically accounting for the global buckling behaviour introducing local and global buckling phenomena opportunely. Sizing and topology truss optimization using dynamic and static constraints was also conducted in [5] imposing a critical buckling load as a static constraint, and adding dynamic natural frequency constraints to avoid deconstructive resonance. Assimi et al [6] also considered a static critical buckling load constraint for sizing and topology optimization using genetic programming, even applying it to a 10 bar truss problem. A combination of sizing, shape, and topology optimization of truss structures using Jaya algorithm was used in [7] with considering dynamic constraints for buckling in analysed models. Optimization was done with discrete sizing and shape variables, also considering simplified topology optimization.

Authors in [8] compared the implementation of Euler buckling constraints to sizing optimization problems and found that examples from literature which do not include this constraint have solutions which do not meet buckling criteria, while the addition of the constraint increased the overall weight of tested models by a small percentage. This problem was further explored in [9] where authors compared structural optimization for sizing, topology, shape, and their combinations with and without buckling constraints on a standard 10 bar truss example.

As previous research shows, the addition of buckling constraints increases complexity, requires longer calculation times, and results in constructions of greater weight than their counterparts which do not. Such an approach, however, ensures practical applicability of attained results. This paper aims to show the change in solutions when discrete sizing variables are used on a standard 10 bar truss problem, as it has been observed that solutions using continuous variables give results with expensive or impossible precision. The motivation behind this research is the creation of a comprehensive structural optimization method for trusses which can produce realistic optima.

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2. OPTIMIZATION PROBLEM FORMULATION

Optimization is the process of finding solutions from a group of alternative possible solutions. These solutions impose better characteristics of the construction, while simultaneously decreasing invested efforts and expended costs. Truss structural optimization, based on discrete design variables, entails simultaneous sizing, topological, and shape optimization. In many practical problems, the optimization of all three of these aspects is not always applicable for various reasons. This means that optimization models should consider the application of any one of these types of optimization as well as possible combinations of any two or all types, simultaneously. The objective function for this problem is to find optimal solutions, for all seven possible cases, with a minimal weight of the construction. For typical truss optimization found in literature the minimum weight design problem can be defined as:

$$\begin{cases} minW\left(A,\ n,\ l\right)=\sum_{i=1}^{i=n}\rho_i\ A_i l_i \text{ with } A=\left(A_1,\dots,A_n\right) \\ \text{subjected to}\begin{cases} A_{min}\le A_i\le A_{max} & \text{for } i=1,\dots,n \\ \sigma_{min}\le \sigma_i\le \sigma_{max} & \text{for } i=1,\dots,n \\ u_{min}\le u_j\le u_{max} & \text{for } j=1,\dots,k \end{cases} \end{cases} \tag{1}$$

In (1) the number of nodes, l_i is the length of the i^{th} element, A_i is the area of the i^{th} element cross section, σ_i is the stress of the i^{th} element, u_j is displacement of the j^{th} node. Depending on the case the objective function criteria changes accordingly, however the constraints are unchanged for all cases.

Trusses have elements subjected to compression forces which need to be lower than critical buckling values for each cross section. Since the Euler critical buckling load equation (3) considers axial compression force, cross sectional characteristics, and bar length, buckling needs to be checked for all bars for each iteration. To avoid surplus calculations axial compression forces are compared to Euler's critical load, instead of comparing stresses which are derived from these forces and the same area value. The proposed Euler buckling constraint defined by Euler's critical load is given in the following expressions:

$$\left|F_{Ai}^{comp}\right|\le F_{Ki}\ \text{ for } i=1,...,n \tag{2}$$

$$F_{Ki}=\frac{\pi^2\cdot E_i\cdot I_i}{l_i^2} \tag{3}$$

In (2) and (3), F_{Ai}^{comp} is the axial compression force, F_{Ki} is Euler's critical load, E_i is the modulus of elasticity, and I_i is the minimum area moment of inertia of the cross section of the of the i^{th} element. For the purposes of this research the condition from equation (1) is added to the existing constraints. Since the buckling constraint changes with each iteration, this constraint is considered a dynamic constraints, and its calculation significantly increase the complexity of the optimization problem.

2.1. Optimization Method and Algorithm

The optimization method selected for the purposes of this research is genetic algorithm (GA) because of its favorable characteristics and availability. GA is a heuristic optimization method whose operation is based on imitating natural processes [10]. As this research is not focused on the algorithm characteristics, other algorithms are not considered, however the same principles would apply for use in with other optimization algorithm.

The genetic algorithm consists of three elementary operators: selection, crossover, and mutation (figure 1). The process of transferring genetic information through generations is called selection. Crossover represents the operations between two parents, where an exchange of genetic information is conducted, and new generations are created. A random change in the genetic structure of some individuals for overcoming early convergence is created by the mutation operator.

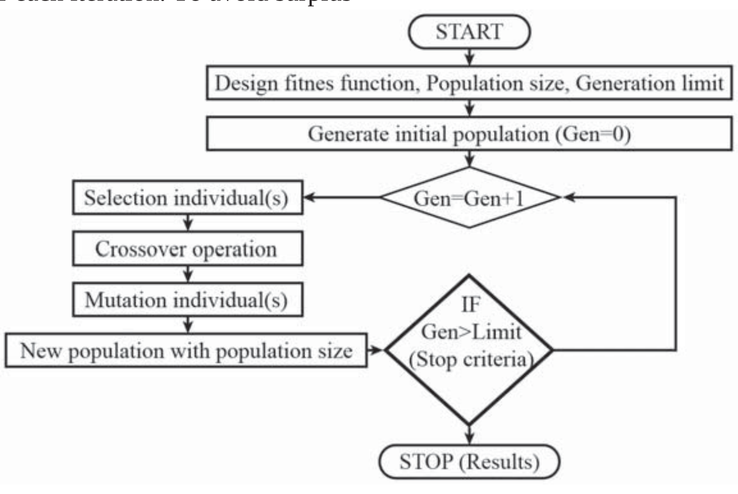


Fig.1. Genetic algorithm

Algorithm operation is based on survival of the fittest individuals through evolution which exchange genetic material. Selection is used to rank individuals in the population using values from the fitness function, which defines the quality of the individual.

The parametric model and optimization in this research are all done in Rhinoceros 5.0 software using Grasshopper, Galapagos optimization, and Karamba plugins as well as using operators programmed in Visual Basic. An original files were created in this program which allows for the choice of optimization type, and/or combination of types, as well as the choice of constraints used. Galapagos optimization uses GA as its optimization method.

2.2. The 10 Bar Truss Problem

One of the most frequently appearing examples in literature, when it comes to truss optimization, is the 10 bar truss (Figure 2). This cantilever truss has 10 independent sizing variables (cross section diameters), 4 shape variables (*x* and *y* coordinates for nodes *3* and *4*), and 10 topology variables (bars). The material of the truss elements is Aluminium 6063-T5 whose characteristics are: Young modulus 68947MPa, and a density of 2.7g/cm3. The point load is F=444.82kN, in nodes 2 and 4, as shown in figure 2. The model is limited to a maximal displacement of ±0.0508m of all nodes in all directions, and axial stress of ±172.3689MPa for all bars.

Discrete variables for cross section diameters are taken from various vendors, and the compiled list of available diameters for this material stock is : 12, 16, 20, 25, 30, 34, 35, 40, 45, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 140, 145, 150, 152, 160, 165, 170, 175, 178, 180, 190, 200, 210, 220, 230, 240, 250, 254, 260, 270, 278, 280, 300, 305, 356, given in mm.

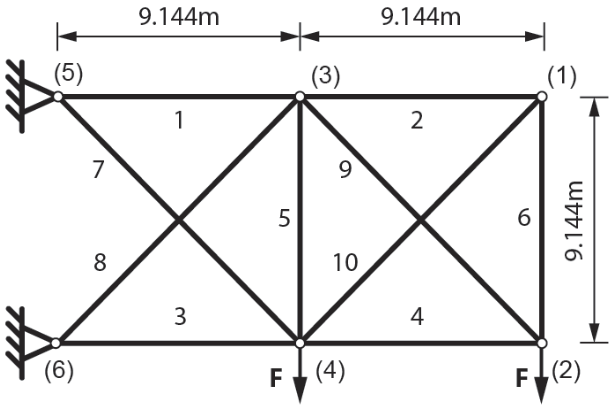


Fig.2. Configuration of the 10 bar truss problem

The initial cross section area for all calculations is 45238.932mm^2 (240mm diameter). It is calculated by optimizing the initial model which would have the same diameter of all bars and a minimal weight in such a configuration. This is also taken as the cross section area of all bars for all examples which do not consider sizing. The initial model with these bars has a weight of 13089.2614kg. In order to allow for shape optimization coordinates of nodes 1 and 3 are variables in examples which optimize this aspect of the truss [9]. Node 5, as it is a support is not set as a variable, as found in [1]. Topology optimization is limited to the removal of at most 6 bars. A 1mm precision for node location is set when using shape optimization.

3. ANALYSIS AND RESULTS

Optimization was conducted according to the parameters set in the previous section and compared to results from [9]. Table 1 shows the comparison of results between continuous variable, and discrete variable models. Since the cross sections of the optimizations which do not consider sizing use the same cross-section as in [9], the optimization results are the same, and therefore not given in Table 1.

Table 1. Comparison of bar cross-section areas, displacements, and optimal weights for continuous and discrete optimization models.

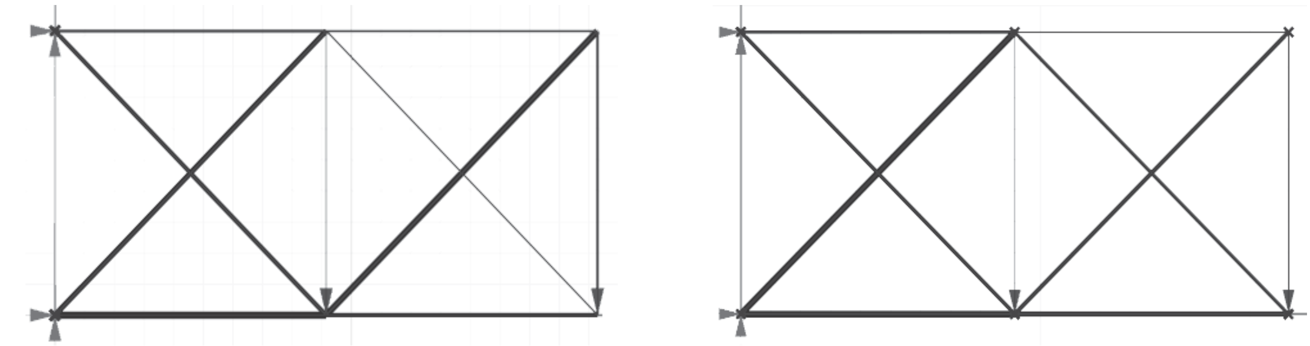
Area of bar (cm^2)	**Sizing**		**Sizing and topology**		**Sizing and shape**		**Sizing, topology and shape**	
	Continuous [9]	Discrete	Continuous [9]	Discrete	Continuous [9]	Discrete	Continuous [9]	Discrete
1	74.58352	78.540	89.08818	240.528	179.1678	78.540	125.6289	165.130
2	52.71413	15.904	-	-	10.34957	44.179	-	-
3	425.1333	415.475	370.3749	490.874	368.7339	415.476	454.3156	490.874
4	157.3157	240.528	262.2361	283.529	330.5622	283.529	308.5585	380.133
5	0.741299	1.131	-	283.529	28.70441	1.131	102.5278	56.745
6	61.05257	15.904	-	-	11.99239	23.758	-	-
7	169.4642	122.718	36.44082	113.097	33.7946	122.71.8	74.89887	86.590
8	267.758	415.47.6	441.3566	-	339.8973	314.159	-	-
9	27.82731	103.869	111.2438	226.980	116.4499	95.033	215.1459	165.130
10	352.5366	181.458	-	-	0.690409	95.033	-	-
Weight (kg)	4759.458	**4795.734**	3838.440	**4416.674**	3715.950	**3968.862**	3172.868	**3460.028**
Displacement (m)	0.0508	**0.0498**	0.0508	**0.0507**	0.0508	**0.0508**	0.0508	**0.0512**

Optimal node coordinates of nodes 1 and 3 for sizing and shape, and sizing topology and shape are given in table 2 for both continuous and discrete variable models. These are the only two cases in which the results of node positioning vary between the two examples.

Table 2. Optimal node coordinates

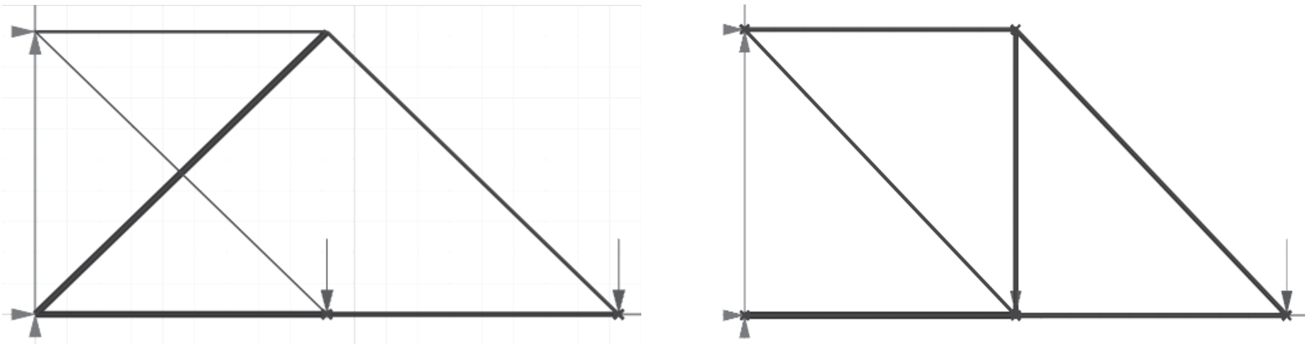
Area of bar (cm^2)	**Sizing and shape**		**Sizing, topology and shape**	
	Continuous [9]	Discrete	Continuous [9]	Discrete
Node 1 (x, y) [m]	(12.079, 3.887)	**(8.120, 8.186)**	-	-
Node 3 (x, y) [m]	(8.954, 5.879)	**(12.560, 6.668)**	(12.799, 3.966)	**(8.875, 5.012)**

Figures 3 to 6 show the visual differences between optimal continuous and discrete models for sizing, sizing and topology, sizing and shape, sizing, topology and shape respectively.



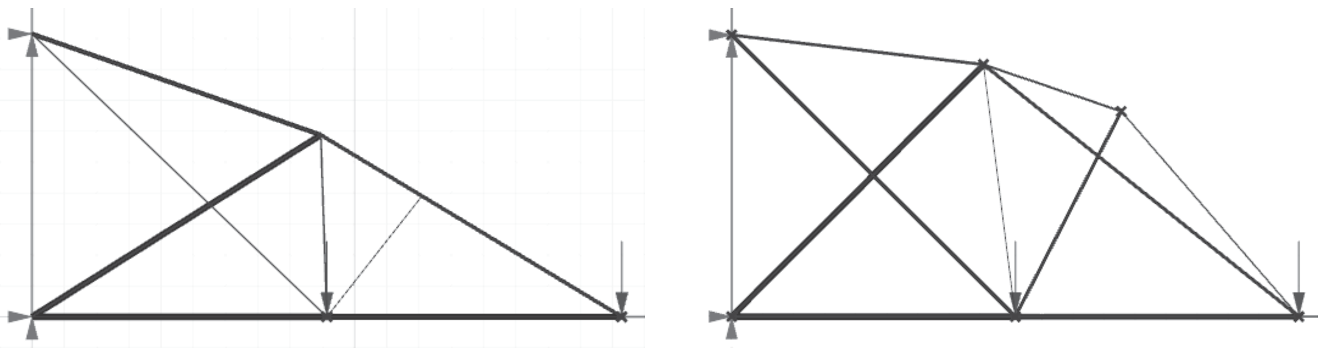
a) **b)**

Fig.3. Optimal sizing models using a) continuous [9], and b) discrete variables.



a) **b)**

Fig.4. Optimal sizing and topology combination models using a) continuous [9], and b) discrete variables.



a) **b)**

Fig.5. Optimal sizing and shape combination models using a) continuous [9], and b) discrete variables.

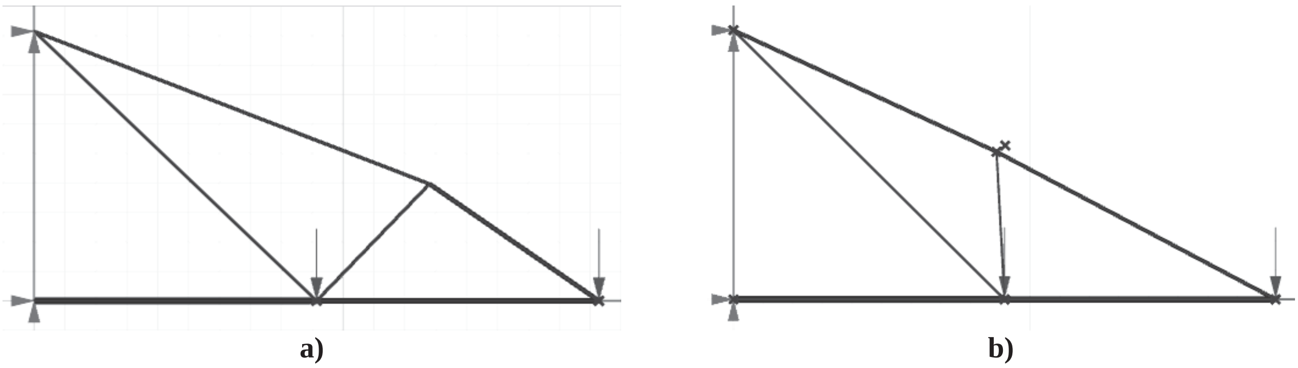


Fig.6. Optimal sizing, topology and shape combination models using a) continuous [9], and b) discrete variables.

4. CONCLUSION

Structural optimization of trusses, especially with the addition of dynamic constraints for Euler buckling, is an intricate process. Implementation of discrete variables additionally complicates the problem definition. This must be conducted in a single stage optimization approach to ensure the best solution combination is achieved. This increase in complexity however results in optimal models which can find practical application. The resulting list of specific cross-sections can be ordered directly from vendors, cut to length and assembled. Furthermore the more of the same diameter stock which can be ordered the more the overall price can be decreased. The differences in just topology, just shape, and the topology and shape combination optimizations are not considered in this paper, as the discretization of cross-section parameters does not influence them since the diameter of 240mm (area of 452.389cm^2) is one of the discrete diameters listed, meaning that the optimal results from literature are the same.

After comparing optimal solutions which use discrete variables to those from literature which use continuous cross-section variables, it can be found that the weights do not vary significantly. The introduction of discrete variables has influenced all aspects of optimization in the results, most notably the layout in sizing and topology combination, and sizing and shape combination models, as their models differ from their continuous variable counterparts.

The differences in weight between the continuous and discrete optimal models are 0.762% for sizing, 15.064% for sizing and topology combination, 6.806% for the sizing and shape combination, and 9.0505% for the complete structural optimization. These differences are negligible compared to the benefits of having optimal models with cross-sections which can be ordered from stock and implemented in practical application. There is also the added benefit of having the same cross sections for at least two bars in each model.

The optimal model which uses continuous variables can achieve results right on the upper limit of displacement. Optima achieved with discrete variables have displacement values of 1.2mm less for sizing, and 1mm less for sizing and topology. The more complex sizing and shape, as well as the complete structural optimizations can give results right on the displacement limit due to the length of elements being discretised to 1mm increments.

It can be concluded that truss structural optimization using discrete cross-section variables, thought more complex, gives more practically applicable optima. Using this approach on larger scale models is expected to give even smaller variances in optimal mass, and will be the subject of further research in this field. The goal is to create a process which would create results which can be built and applied in practice. The next step will also consider the influence of tolerances of cross-sectional diameters on optimal solutions.

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