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The energy at which the maximum number of photoelectrons are observed during the ionization of potassium and xenon atoms

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Abstract
The two formulae for the energy at which the maximum number of ejected photoelectrons are detected are examined in this paper; one formula is $E_{\max}$ with non-zero momentum included in both the exponent and the pre-exponent part of the expression, and the other formula is $E_{\max\mathrm{App}}$, which represents a previous formula, but approximated for higher-intensity laser fields. We examine which formula gives numerical values that are closest to the experimental values obtained in this area of research concerning tunnel ionization of K and Xe atoms by linearly polarized laser fields. It is found that the formula $E_{\max}$ gives satisfactory results in a wide range of laser field intensities, for both atoms. For an intensity of $4 \times 10^{12}\,\mathrm{W\,cm^{-2}}$, $E_{\max} = 9.046\,\mathrm{eV}$ in the case of the K atom. In the case of the Xe atom, this $E_{\max}$ value occurs at an intensity of $5 \times 10^{13}\,\mathrm{W\,cm^{-2}}$. In contrast, in experiments, it has been determined that this $E_{\max}$ value occurs at an intensity of $7.5 \times 10^{13}\,\mathrm{W\,cm^{-2}}$. The formula for $E_{\max\mathrm{App}}$ is applicable for higher laser intensities; it is consistent with the formula for $E_{\max}$ in a range of field intensities 10^{15}–$10^{16}\,\mathrm{W\,cm^{-2}}$.

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(Some figures may appear in colour only in the online journal)

1. Introduction

In an intense laser field, the potential barrier of an atom is distorted considerably. As the intensity increases, the length of the barrier that electrons have to pass through decreases and the electrons can escape from the atom easily. This process is known as tunnelling ionization, and analytic expressions exist for tunnelling rates of atoms [1–4] and for the energy and angular electron spectra in a strong low-frequency laser field [5–9]. In particular, the energy spectra of electrons in the tunnelling ionization of atoms and ions of xenon and potassium by the radiation of a carbon dioxide laser have been studied experimentally and theoretically [4], and it was found that the Ammosov, Delone and Krainov (ADK) theory [2, 3] fits the data very well.

The regime in which tunnelling ionization takes place is determined by the smallness of the Keldysh parameter [1] (atomic units are used throughout this paper), $\gamma = \omega\sqrt{2E_{\mathrm{i}}}/F \ll 1$, where E_{i} is the atomic ionization potential and ω angular laser frequency. In this limit an analytical expression for the total ionization rate of atoms and positive ions was found in [2], and in [10] it was corrected (for non-zero initial value of ejected electron linear momentum p) in the following form:

$$W^{p}_{\mathrm{ADK}} = \left(\frac{3\,\mathrm{e}}{\pi}\right)^{3/2} \frac{Z^2}{n^{*9/2}} \left(\frac{16\,\mathrm{e}\,E_{\mathrm{i}}^2}{F\,Z}\right)^{2n^{*}-3/2} \times \mathrm{e}^{-\frac{2(2E_{\mathrm{i}})^{3/2}}{3F} - \frac{\gamma^3}{3\omega}p^2}, \tag{1}$$

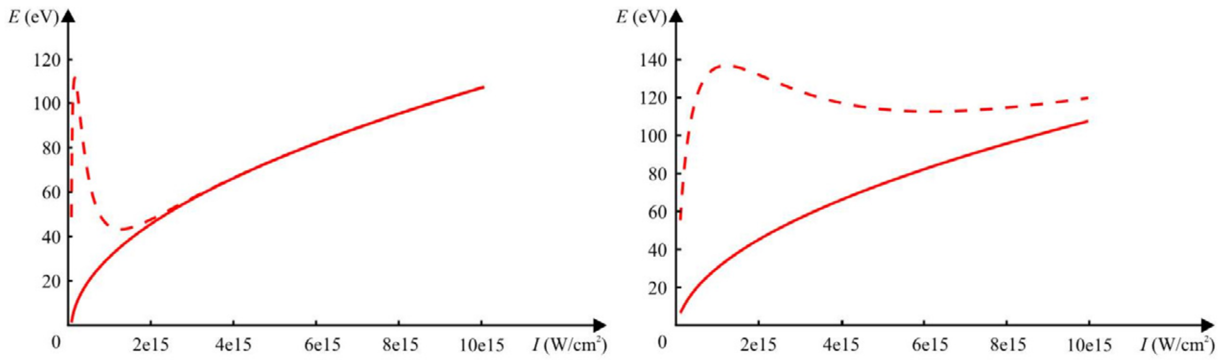

Figure 1. $E_{\max}$ and $E_{\max\mathrm{App}}$ of the maximum number of ejected electrons for the potassium atom: $E_{\mathrm{i}}^{\mathrm{K}} = 4.3407\,\mathrm{eV}$; the solid line represents $E_{\max}$ and the dashed line represents $E_{\max\mathrm{App}}$.

where $n^* = Z/\sqrt{2E_{\mathrm{i}}}$ is effective quantum number, Z is the charge number and $\mathrm{e} = 2.718\,28$; the term $p^2\gamma^3/3\omega$ determines to what extent the transition rate depends on the kinetic energy of ejected electrons [11].

It was also assumed that the spatiotemporal distribution of laser radiation has a Gaussian form

$$F(\rho, t) = F\,\mathrm{e}^{-\frac{\rho^2}{2R^2} - \frac{t^2}{2t_1^2}} \tag{2}$$

where ρ represents the axial cylindrical coordinate (in the direction perpendicular to the propagation of the laser beam), R is the radius of the laser beam, t is the emerging time of the ejected electron, t_1 is the laser pulse duration and F is the amplitude of laser field strength.

2. The formula for the energy at which the maximum number of ejected photoelectrons are observed

In order to compare our results with experiments, we need to calculate the theoretically expected value of the energy at which the maximum number of ionized electrons is observed.

To this end, we substitute (2) into (1) and perform some transformations, obtaining

$$W_{\mathrm{ADK}}^{p\,\mathrm{Gauss}} = \left(\frac{3\,\mathrm{e}}{\pi}\right)^{3/2} \frac{Z^2}{n^{*9/2}} \left(\frac{16\,\mathrm{e}\,E_{\mathrm{i}}^2}{F\,Z}\right)^{2n^*-3/2} \mathrm{e}^{-\frac{2(2E_{\mathrm{i}})^{3/2}}{3F}}$$
$$\times\,\mathrm{e}^{\left[\frac{2n^*-3/2}{2} - \frac{(2E_{\mathrm{i}})^{3/2}}{3F}\right]\left(\frac{\rho^2}{R^2} + \frac{t^2}{t_1^2}\right) - \frac{\gamma^3}{3\omega}p^2}. \tag{3}$$

Taking into account formula (1) and the fact that the first term can be neglected, because $(2n^* - 3/2)/2 \ll (2E_{\mathrm{i}})^{3/2}/3F$, it follows that

$$W_{\mathrm{ADK}}^{p\,\mathrm{Gauss}} = W_{\mathrm{ADK}}\,\mathrm{e}^{-\frac{(2E_{\mathrm{i}})^{3/2}}{3F}\left(\frac{\rho^2}{R^2} + \frac{t^2}{t_1^2}\right) - \frac{\gamma^3}{3\omega}p^2}. \tag{4}$$

If the saturation effect of ionization is included [10, 11], then a focal radius ρ can be determined to a high degree of accuracy from the condition

$$\int_{-\infty}^{\infty} W_{\mathrm{ADK}}^{p\,\mathrm{Gauss}}(\rho)\,\mathrm{d}t = 1. \tag{5}$$

After substituting (4) into (5), the following formula is obtained:

$$W_{\mathrm{ADK}}\,\mathrm{e}^{-\frac{(2E_{\mathrm{i}})^{3/2}}{3F}\frac{\rho^2}{R^2} - \frac{\gamma^3}{3\omega}p^2} \int_{-\infty}^{\infty} \mathrm{e}^{-\frac{(2E_{\mathrm{i}})^{3/2}}{3F}\frac{t^2}{t_1^2}}\,\mathrm{d}t = 1. \tag{6}$$

The solution of Poisson's integral is $\int_{-\infty}^{\infty} \mathrm{e}^{-\beta x^2}\mathrm{d}x = \sqrt{\pi/\beta}$, so after integrating (6), it follows that

$$W_{\mathrm{ADK}}\,\mathrm{e}^{-\frac{(2E_{\mathrm{i}})^{3/2}}{3F}\frac{\rho^2}{R^2} - \frac{\gamma^3}{3\omega}p^2} \sqrt{\pi\,\frac{3F\,t_1^2}{(2E_{\mathrm{i}})^{3/2}}} = 1. \tag{7}$$

From (7), ρ can be obtained; it represents the radius of the region where saturation of the ionization probability occurs: inside the region all atoms are ionized, while outside it none are ionized:

$$\rho^2 = \frac{3\,F\,R^2}{(2E_{\mathrm{i}})^{3/2}}\left[\ln\left(W_{\mathrm{ADK}}\,t_1\sqrt{\pi\,\frac{3F}{(2E_{\mathrm{i}})^{3/2}}}\right) - \frac{\gamma^3 p^2}{3\omega}\right]. \tag{8}$$

The formula for the final energy of ejected electrons can be simplified by taking into account the condition $t \sim t_1\sqrt{3F/(2E_{\mathrm{i}})^{3/2}} \ll t_1$; the following is obtained:

$$E = \frac{p^2}{2} + \frac{F^2}{4\omega^2}\,\mathrm{e}^{-\frac{\rho^2}{R^2}}. \tag{9}$$

After substituting (8) into formula (9) for the final energy of ejected electrons, one obtains

$$E_{\max} = \frac{p^2}{2} + \frac{F^2}{4\omega^2}\left(W_{\mathrm{ADK}}\,t_1\sqrt{\pi\,\frac{3F}{(2E_{\mathrm{i}})^{3/2}}}\right)^{-\frac{3F}{(2E_{\mathrm{i}})^{3/2}}} \mathrm{e}^{-\frac{\gamma^3 p^2 F}{\omega(2E_{\mathrm{i}})^{3/2}}}. \tag{10}$$

When the strength of the laser field is equal to the atomic field strength: $F = F_{\mathrm{at}}$, the formula for ionization rate (1) is approximated, according to the condition $2(2E_{\mathrm{i}})^{3/2}/3F = 1$, with the formula

$$W_{\mathrm{ADK}}^{\mathrm{sf}} = \left(\frac{3\,\mathrm{e}}{\pi}\right)^{3/2} \frac{Z^2}{n^{*9/2}} \left(\frac{16\,\mathrm{e}\,E_{\mathrm{i}}^2}{F\,Z}\right)^{2n^*-3/2} \mathrm{e}^{-1}, \tag{11}$$

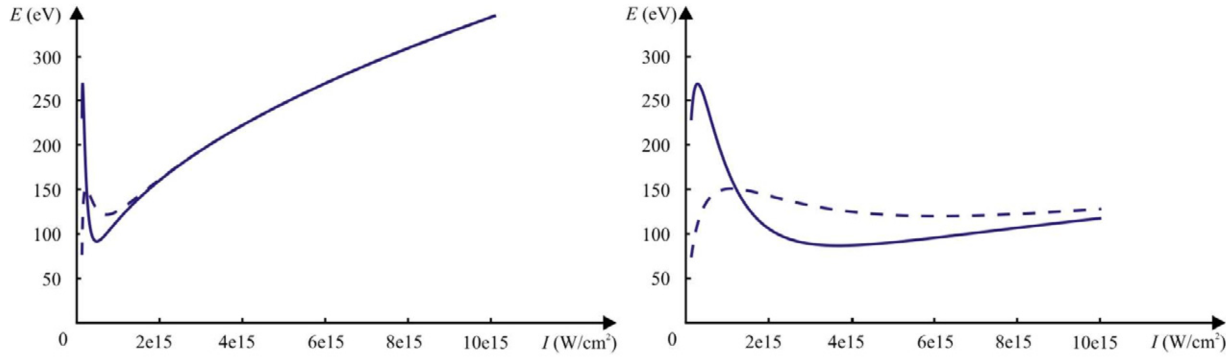

Figure 2. $E_{\max}$ and $E_{\max\mathrm{App}}$ of the maximal number of ejected electrons for the xenon atom: $E_{\mathrm{i}}^{\mathrm{Xe}} = 12.5\,\mathrm{eV}$; the solid line represents $E_{\max}$ and the dashed line represents $E_{\max\mathrm{App}}$.

while the formula for the energy at which the maximum number of ionized electrons are observed becomes

$$E_{\max\mathrm{App}} = \frac{p^2}{2} + \frac{F^2}{4\omega^2}\left(W_{\mathrm{ADK}}^{\mathrm{sf}}\, t_1\sqrt{\pi\,\frac{2F}{F_{\mathrm{at}}}}\right)^{-\frac{2F}{F_{\mathrm{at}}}} \mathrm{e}^{-\frac{2\gamma^3 p^2 F}{3\omega F_{\mathrm{at}}}}. \quad (12)$$

Note that in the atomic units system, the strength of an atomic field is set equal to 1. This was used for figures 1 and 2.

Formulae (10) and (12) will now be compared in order to see which one better describes the experimental results concerning tunnelling ionization.

3. The energy at which the maximum number of ejected photoelectrons of potassium and xenon atoms are observed

We now examine which of the two formulae (10) and (12) gives numerical values that best describe experimental results obtained during the tunnelling ionization of potassium and xenon atoms. Both the cases when medium and high laser intensities were applied were analysed. Therefore, the range of laser intensities 10^{13}–$10^{16}\,\mathrm{W\,cm^{-2}}$ is examined.

The formula for $E_{\max}$ gives results that are in good agreement with the experimental ones in a broad range of laser field intensities, for both atoms. The numerical value of $E_{\max}$ for the potassium atom is in good agreement with that obtained in [5]. For a field intensity of $4\times10^{12}\,\mathrm{W\,cm^{-2}}$, that value is $E_{\max}^{\mathrm{K}} = 9.046\,\mathrm{eV}$.

In the case of xenon atoms, the obtained results are compared with the results of [5] in the high-energy spectrum (200 eV and higher). It is concluded that the values of kinetic energy of the ejected electrons are in good agreement with the experimental values: $E_{\max}^{\mathrm{Xe}} = 220$–$270\,\mathrm{eV}$. The numerical value of the energy at which the maximum number of ionized electrons are detected is determined at an intensity of $5\times10^{13}\,\mathrm{W\,cm^{-2}}$, while in [5], it was determined at $7.5\times10^{13}\,\mathrm{W\,cm^{-2}}$.

For $E_{\max\mathrm{App}}$ the results are in good agreement with the results from the formula for $E_{\max}$ only at laser fields 10^{15}–$10^{16}\,\mathrm{W\,cm^{-2}}$; for potassium, equivalent numerical values of energy occur at $5\times10^{15}\,\mathrm{W\,cm^{-2}}$ (figure 1), while in the case of xenon that energy is determined at $10^{15}\,\mathrm{W\,cm^{-2}}$ (figure 2). The fact that ejected electrons can have a non-zero value of initial momentum was taken into account by including it in both the exponential and the pre-exponential part of both the expressions for energy.

4. Conclusion

The main results of this work can be summarized as follows: compared with the experimental results, the formula for $E_{\max}$ shows very good agreement in the case of both atoms. By including non-zero initial momentum in both the exponential and the pre-exponential part of the expressions for energy, the obtained results were $E_{\max}^{\mathrm{K}} = 9.046\,\mathrm{eV}$ and $E_{\max}^{\mathrm{Xe}} = 250\,\mathrm{eV}$. We also concluded that the formula for $E_{\max\mathrm{App}}$ is useful in the case of strong fields (ones that are comparable with atomic field strength).

We stress that the numerical results stated above were obtained when the external field strength was small compared to the atomic field strength. However, in experiments nowadays the laser field strength can be of the order of the atomic field strength or even higher, so the formula $E_{\max\mathrm{App}}$ was examined in order to gain insight into its area of applicability as far as processes of tunnel ionization of atoms are concerned.

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