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ANALISYS OF STRESS STATE IN A PLANE ANISOTROPIC FIELD WEAKENED BY A CIRCULAR HOLE

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Abstract

It is the fact that mechanical parts with holes are often encountered in practice. A knowledge of stress distribution is of paramount importance for engineering practice. Therefore, the objective of this paper analyzes the impact of the circular hole as a source of stress concentration on the stress state of uniaxial strained orthotropic plate since the practice has shown that the largest number of structures is weakened by the hole shape. In this paper for obtaining the results of stress distribution in a plane anisotropic field weakened by a circular hole, numerical method was used. The methodology was applied enabling the determination of the values of the stress at each point of plate, as well as at points at the hole contour, which is based on the application of the basic equations of the theory of elasticity.

Key words: circular hole, finite element method, orthotropic plate, stress concentration, stress distribution.

1. INTRODUCTION

A large number of mechanical structures in order to ensure minimum weight or other structural requirements are made of parts that contain notches, grooves, chamfers, rounding, vents and holes. The same represent sources of stress concentration. The holes in machined parts, whether transverse or longitudinal, are typical sources of stress concentration. We find them in the areas of production machines and tools, construction, mining, transport and agricultural machinery, cranes, steel support structures, etc. In addition, they are predicted for the purpose of reducing weight, exercising interconnection of elements, conducting lubricants or other reasons. And precisely the holes are the places where there comes to the stress increase, and in the case that the opening is in the vicinity of the second opening or a source of stress concentration, the resulting stress can be higher or lower than the individual one, depending on the shape and position of the source of stress concentration and the type of strain. Due to it and the effect of loads in parts of structures cracks are created and developed which in most cases lead to fracture.

In engineering practice, of great importance, is the knowledge of structural analysis that is performed via the simplified mechanical models. Analyzing simplified mechanical models were researched by the great amount of investigators in this field. So, (Ahmed et al., 2018) are used finite elements for analyzing stress concentration for composite laminate member with central circular. (Bathe, & Wilson, 1976) are introducing in structural analysis numerical methods and the finite element method. In analysis of supporting structure tool machine of composite materials, (Ćirković et al., 2015) are using the finite element method. For analysis stress concentration factor for composite materials (Makki, & Chokri, 2017) they use experimental, analitical and numerical methods. As numerical methods involve the use of computers and related program packages here for generation of finite element mesh was used software package (MSC/NASTRAN). Basic problems of the theory elasticity was researched by (Mushelishvili, 1964). (Nagpal, Jain, & Sanyal, 2012) are studing the stress concentration and they are suggesting its mitigation techniques for flat plate with singularities. Results for stress distribution in anisotropic plane weakened by an eliptical hole, we can find in the article (Nikolić et al., 2015). Influence of polygonal cut-out of complex geometry on the stress concentration in an infinite orthotropic plate are researched by (Patal, & Sharma, 2017). The stress strain conditions in the zones of geometric discontinuites of mechanical constructions elements it's the object of studing (Radojković, 2008). Fundimentaly equations of the theory of elasticity which are resolved similar problems, they can be found in (Rašković, 1985), and equations for the stress calculation in isotropic and anisotropic field which are weakened by holes different shapes (Savin, 1968).

In this paper, special attention will be devoted to the study of stress distribution in parts of the type of plate weakened by the circular hole, made of anisotropic or orthotropic materials and exposed to static load.

2. OVERVIEW OF THE EQUATIONS OF LINEAR THEORY OF ELASTICITY IN THE FINITE ELEMENT METHOD

Considering the type of problem discussed here, the equations related to solving the problem of the flat stress state will be given (Radojković, 2008).

The components of surface forces will be the components of the vector F_n:

$$F_n = \begin{Bmatrix} F_{nx} \\ F_{ny} \end{Bmatrix}. \tag{1}$$

At any point of the observed body, the displacement vector s with the displacement components u and v in the direction of the coordinate axes x and y, is shown as:

$$s = \begin{Bmatrix} u \\ v \end{Bmatrix}. \tag{2}$$

The links between the displacement vector s and the deformation vector ε can be represented by Cauchy's kinematic equations in the following form:

$$\varepsilon_x = \frac{\partial u}{\partial x},\ \varepsilon_y = \frac{\partial v}{\partial y},\ \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \tag{3}$$

The Cauchy's tensor of relative deformations for small deformations is represented by a symmetric matrix, in the form:

$$\varepsilon = \begin{bmatrix} \varepsilon_x & \varepsilon_{yx} \\ \varepsilon_{xy} & \varepsilon_y \end{bmatrix}. \tag{4}$$

If it is known that gliding or shearing is γ_{xy}, is equal to the double value of the component of the tensor deformation ε_{xy}, the relative deformation tensor matrix can now be written in the following form:

$$\varepsilon = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y \end{bmatrix}. \tag{5}$$

Thus, the state of deformation of the elastic body around some point is determined by three components: two dilatations (ε_x, ε_y) and one glide γ_{xy}. The deformation tensor, taking into account the symmetry, i.e. that $\varepsilon_{xy} = \varepsilon_{yx}$ can be written as a vector, i.e. as a column matrix with three elements, in the form:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}. \tag{6}$$

Dependence Eq. 5, with respect to Eq. 2, can be represented in the matrix form:

$$\varepsilon = d\ s. \tag{7}$$

The matrix of the differential operator d and its transposed matrix d^T have the following form:

$$d = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix},\ d^T = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}. \tag{8}$$

The stress state at the observed point of the strained body is determined by three component stresses: two normal (σ_x, σ_y) and one tangential ($\tau_{xy} = \tau_{yx}$) that act in that point. The stress tensor can be written in the form:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}. \tag{9}$$

The conditions of balance between the internal and external forces on the contour segment where the contour conditions are given by the surface forces are given by Cauchy's equations

(Cauchy's boundary conditions):

$$d_s^T \sigma = F_n, \tag{10}$$

where: d_S^T - is a transposed matrix of the d_S matrix whose elements are the cosine of the angles that the normal n covers at the points of the contour surface with x and y axes. The d_S matrix has the form:

$$d_s = [d_s] = \begin{bmatrix} \cos(n,x) & 0 \\ 0 & \cos(n,y) \\ \cos(n,y) & \cos(n,x) \end{bmatrix} = \begin{bmatrix} n_x & 0 \\ 0 & n_y \\ n_y & n_x \end{bmatrix}. \tag{11}$$

The general form of the constituent equations, that is, the connection between the matrix components of the stress tensor and the matrix components of the deformation tensor for the elastic material is given by the following expression:

$$\sigma = D\,\varepsilon, \tag{12}$$

which represents the generalization of the well-known Hooke’s law, where: D - a stiffness matrix of material which, in the case of homogeneous isotropic materials, is expressed through Young's modulus of elasticity E and Poisson’s coefficient μ, hence the name for this matrix "matrix of elastic constants or elasticity tensor matrix". Its form in this case is:

$$D = \frac{E}{2(1+\mu)} \begin{bmatrix} \frac{2(1-\mu)}{1-2\mu} & \frac{2\mu}{1-2\mu} & 0 \\ \frac{2\mu}{1-2\mu} & \frac{2(1-\mu)}{1-2\mu} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(13).

3. RESULTS OF STRESS DISTRIBUTION OBTAINED NUMERICALLY

The results reached in this paper were obtained by using the finite element method (FEM) and relate to the distribution of stress σ_{max} in the uniaxial strained orthotropic plate. The plate dimensions in the examples were 120 × 120 × 1 m. The same were weakened by the circular hole in the middle of the plate, of radius $r = 2$ m. The load was uniaxial and acted in the form of surface tensile forces whose intensity in the examples was $p = 1$ N/m^2. The material from which the test plate were made was birch veneer with values of the elasticity modules in the directions of the main axes $E_{max} = 1,2 \times 10^{10}$ N/m^2 and $E_{min} = 0,6 \times 10^{10}$ N/m^2, the glide module $G = 0,07 \times 10^{10}$ N/m^2 and Poisson's ratios in these directions $v_{max} = 0,071$ and $v_{min} = 0,036$. The tests were carried out so that, in the examples, the main directions of elasticity of the material coincided with the directions of the coordinate axes x and y, and the load was acting along the x axis. In the above examples four-node 2D finite elements were used, because by checking the accuracy of the selected finite elements using the trial „patch” tests, the justification of their implementation was confirmed (Radojković, 2008). In the figures with the distribution of stress only a detail of stress distribution σ_{max} around the hole was shown, because showing the stress distribution throughout the model would be unclear (unreadable), and that the research shown that the highest stress values were obtained in the vicinity of the hole. In the mentioned figures you can also see the finite elements mesh.

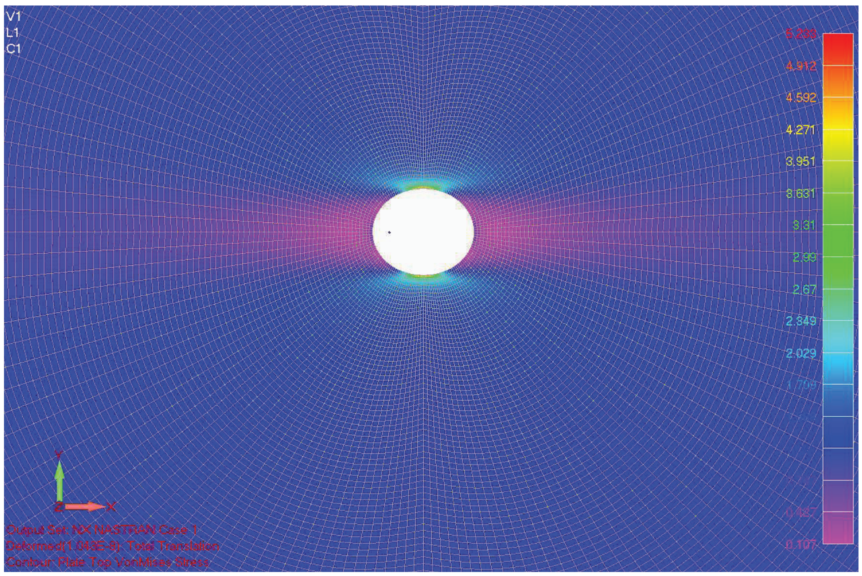

Figure 1. The distribution of stress σ_{max} at straining the plate weakened by the circular hole in the direction of x axis, when the same coincides with the direction corresponding to the maximum value of the modulus of elasticity

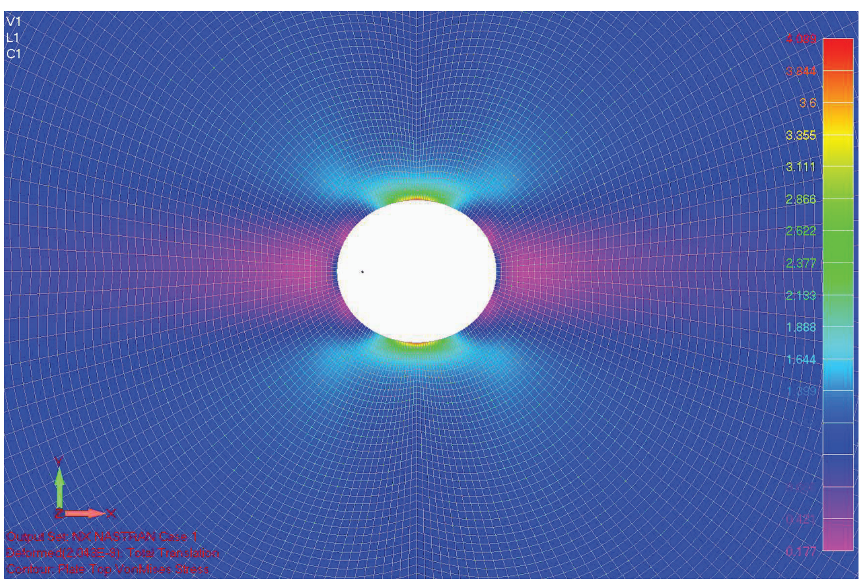

Figure 2. The distribution of stress σ_{max} at straining the plate weakened by the circular hole in the direction of x axis, when the same coincides with the direction corresponding to the lowest value of the modulus of elasticity

Figure 1 shows the stress distribution σ_{max}, in the uniaxial strained orthotropic plate of birch veneer weakened by the circular hole, when it is strained by the surface forces along the axis *x* and when the *x*-axis direction corresponds to the direction for which the modulus of elasticity of the material has the highest value ($E_x = E_{max}$), and the *y*-axis corresponds to the direction for which the modulus of elasticity of the material has the lowest value ($E_y = E_{min}$). The highest obtained value of the maximum strain stress in this case is σ_{max} = 5,233 N/m^2. When the orthotropic plate of birch veneer weakened by the circular hole is strained by the surface forces along the *x* axis, and when the *x*-axis corresponds to the direction for which the modulus of elasticity of the material has the lowest value ($E_x = E_{min}$), and *y*-axis corresponds to the direction in which the modulus of elasticity of the material has the highest value ($E_y = E_{max}$), the highest obtained value of the maximum strain stress is σ_{max} = 4,089 N/m^2 (Figure 2).

4. ANALYSIS OF RESULTS OBTAINED NUMERICALLY

Based on the obtained values for the maximum strain stress σ_{max}, with the uniaxial strained anisotropic, i.e. orthotropic plate weakened by the circular hole, it can be seen that the lowest values of the stress are obtained at straining in the direction for which the modulus of elasticity of the material has the lowest value which should be considered during the design and construction of parts of the type of thin sheets weakened by the circular hole having a certain reservation because different anisotropic, or orthotropic materials behave differently to stress concentration.

5. CONCLUSIONS

This paper has shown that, to determine the stress distribution in mechanical structures composed of elements of type of uniaxial strained anisotropic (orthotropic) plates weakened by the circular hole, numerical methods can be used, and that the results obtained by the finite elements method (FEM) justify the application of it. By the analysis of the results obtained for the uniaxial tensioned orthotropic plate of birch veneer, weakened by the circular hole a number of conclusions can be derived that may be valid for other anisotropic materials, but with some reservations because different anisotropic materials may behave differently in terms of stress concentration.

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