

# Quantum Correlations Relativity for Continuous Variable Systems

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**Abstract:** It is shown that a choice of degrees of freedom of a bipartite continuous variable system determines amount of non-classical correlations (quantified by discord) in the system's state. Non-classical correlations (that include entanglement as a special kind of correlations) are ubiquitous for such systems. For a quantum state, if there are not non-classical correlations (quantum discord is zero) for one, there are in general non-classical correlations (quantum discord is non-zero) for another set of the composite system's degrees of freedom. The physical relevance of this 'quantum correlations relativity' is emphasized also in the more general context.

Key words: entanglement, non-classical correlations, quantum discord, tensor product structure.

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## 1. Introduction

The promise of the quantum information processing is the promise of the quantum-information resources [1]. To this end, some surprising results and observations are possible and even expectable. The discovery of non-classical (quantum) correlations not necessarily including entanglement, as quantified by quantum discord [2, 3], opens a new avenue in quantum information processing; for recent reviews see [4, 5, 6]. A search for quantum information resources and the ways of their operational use is at the core of the current theoretical and experimental research [4, 5, 6, 7, 8] (and the references therein).

Entanglement Relativity is a corollary of the universally valid quantum mechanics that states [9, 10, 11, 12, 13, 14, 15]: for a composite (e.g. bipartite) system, there is entanglement for at least one structure (one set of the degrees of freedom) of the composite system. The structures being mutually related by the proper (e.g. the linear canonical) transformations of the composite system's degrees of freedom; paradigmatic are the composite system's center-of-mass and the "relative (internal)" degrees of freedom. In practice, it means: if a quantum state is separable (no entanglement), just change the degrees of freedom and entanglement will appear [10, 12, 13]. Quantum entanglement is ubiquitous as a quantum information resource.

In this paper we consider the continuous variable (CV), including open, quantum systems with an emphasis on their bipartitions. Based on Entanglement Relativity, we point out relativity, i.e. structure (degrees of freedom) dependence, of the more general non-classical correlations quantified by quantum discord. Likewise entanglement, the more general non-classical (quantum) correlations are also structure-dependent and ubiquitous in quantum systems.

So we conclude: There are non-classical correlations (not necessarily including entanglement) for practically every quantum state of the systems relative to some structures.

In Section 2, we briefly outline Entanglement Relativity. In Section 3 we derive our main result. Section 4 is discussion placing our considerations in a more general context and we conclude in Section 5.

## 2. Entanglement relativity

We consider a composite system  $C$  that can be decomposed as  $1 + 2$ , or  $A + B$ . The continuous degrees of freedom of the subsystems being mutually related by the proper linear canonical transformations (LCTs) [10, 12, 13, 14]. Then, by a definition, the  $C$ 's Hilbert state space,  $\mathcal{H}_C$ , can be factorized

as  $\mathcal{H}_1 \otimes \mathcal{H}_2$  as well as  $\mathcal{H}_A \otimes \mathcal{H}_B$ ;  $\mathcal{H}_1 \otimes \mathcal{H}_2 = \mathcal{H}_C = \mathcal{H}_A \otimes \mathcal{H}_B$ . The two decompositions of  $C$  represent the two different "structures" of  $C$ . There is a one-to-one relation between the "structure (decomposition)" and the composite system's state space factorization. In general, every subsystem (1 or 2, or  $A$  or  $B$ ) can be of arbitrary number of the degrees of freedom.

Entanglement Relativity establishes [9, 10, 11, 12, 13, 14, 15]: if a (pure) state is separable for one structure (one set of degrees of freedom), it typically becomes entangled for another structure of the composite system:

$$|i\rangle_1|j\rangle_2 = \sum_{\alpha,\beta} C_{\alpha\beta}^{ij} |\alpha\rangle_A |\beta\rangle_B, \quad C_{\alpha\beta} \neq a_\alpha b_\beta, \quad \sum_{\alpha,\beta} C_{\alpha\beta}^{ij} C_{\alpha\beta}^{i'j'} = \delta_{ii'} \delta_{jj'}. \quad (1)$$

The proof of Eq.(1) easily follows, cf. e.g. Ref. [13], with the use of the covariance function  $C_f = \langle \hat{A}_A \otimes \hat{B}_B \rangle - \langle \hat{A}_A \rangle \langle \hat{B}_B \rangle$ . Given a separable state for 1 + 2 structure, e.g.  $|i\rangle_1|j\rangle_2$ , and for a pair of the observables  $\hat{A}_A, \hat{B}_B$  of the subsystems  $A$  and  $B$  respectively, the condition  $C_f = 0$  is necessary in order the state can bear separable (tensor-product) form *also* for  $A + B$  structure. The condition  $C_f = 0$  is not yet sufficient for the separability.

The exceptions from Eq.(1) are known—there are the states of the separable form for both structures [13, 16]. Nevertheless, as the number of entangled states is by far larger than the number of the tensor-product states, the number of states satisfying Eq.(1) is incomparably larger than the number of states not satisfying Eq.(1). This is the reason in practice these exceptional cases are usually neglected.

So, one may say: there is entanglement for all pure quantum states relative to certain structure—i.e. entanglement is a matter of a composite-system's structure and *is* present for one structure, at least. Certainly, for a fixed structure, a pure quantum state is either separable (tensor-product) or entangled.

Now, it is natural to wonder if something analogous can be expected for the mixed quantum states. Of course, now the question refers not only to quantum entanglement, but also to the more general quantum, i.e. non-classical, correlations as quantified by quantum discord [2,3]. In the next section we provide a generalization of Entanglement Relativity that directly includes open quantum systems: [likewise entanglement itself] the more general non-classical correlations are subject to relativity, i.e. are ubiquitous regarding the continuous variable bipartite systems.

### 3. Quantum correlations relativity

Quantum Discord is a common term for different measures of non-classical correlations in composite (e.g. bipartite) quantum systems [1-8]. Historically

the first and probably the best known is the so-called "one-way" discord. For completeness, we give the formal definitions of both one-way and two-way discord.

One-way quantum discord for the  $S + S'$  system,  $D_S = I(S : S') - J_S \geq 0$ , and von Neumann entropy of a state  $\rho$ ,  $\mathcal{S} = -\text{tr} \rho \ln \rho$ . Both the total mutual information,  $I(S : S') = \mathcal{S}(S) + \mathcal{S}(S') - \mathcal{S}(S, S')$ , and the classical correlations,  $J_S = \mathcal{S}(S) - \inf_{\{\Pi_{S'i}\}} \sum_i |c_i|^2 \mathcal{S}(\rho_S | \Pi_{S'i})$ —where  $\rho_S | \Pi_{S'i} = I_S \otimes \Pi_{S'i} \rho I_S \otimes \Pi_{S'i}$  is the state remaining after a selective quantum measurement defined by the projectors  $\Pi_{S'i}$ —are non-negative. Two-way discord,  $D = \max\{D_S, D_{S'}\}$ , where  $D_{S'}$  is one-way discord referring to the  $S'$  system.

In this section, we proceed with considering the bipartite structures of a composite system  $C$  with arbitrary number of the continuous degrees of freedom.

### 3.1 Quantum correlations relativity

Let us consider the  $1 + 2$  structure of the composite system  $C$ . The one-way discord  $D_1$  equals zero if and only if the composite system's state,  $\hat{\rho}_C$ , is of the form [4, 5, 6]:

$$\hat{\rho}_C = \sum_k p_k |k\rangle_1 \langle k| \otimes \hat{\rho}_{2k}, \quad \sum_k p_k = 1 \quad (2)$$

and analogously for the other  $D_2$  discord. It is easy to prove for  $\hat{\rho}_C$  in Eq.(2) that both the one-way discord  $D_2$  (in general) and the covariance function  $C_f$  (Section 2) are nonzero. Of course, the later is a consequence of the classical correlations in  $1 + 2$  decomposition. Introducing  $\hat{\rho}_{2k} = \sum_l \omega_l^k |\chi_l^k\rangle_2 \langle \chi_l^k|$ ,  $\sum_l \omega_l^k = 1, \forall k$  into Eq.(2) gives

$$\hat{\rho}_C = \sum_{k,l} p_k \omega_l^k |k\rangle_1 \langle k| \otimes |\chi_l^k\rangle_2 \langle \chi_l^k|. \quad (3)$$

Now, with the use of entanglement relativity Eq.(1), we introduce the alternate structure,  $A + B$ , into considerations. Let us first, in disagreement with Eq.(1), the states for both structures are tensor-product,  $|k\rangle_1 |\chi_l^k\rangle_2 = |k\rangle_A |\phi_l^k\rangle_B, \forall k$ . Then the form Eq.(2) of the composite system's state is valid also for the  $A + B$  structure:

$$\hat{\rho}_C = \sum_k p_k |k\rangle_A \langle k| \otimes \hat{\rho}_{Bk}, \quad \sum_k p_k = 1, \quad (4)$$

i.e. the one-way discord  $D_A = 0$  (in general,  $D_B \neq 0$ ) while again  $C_f \neq 0$ .

However, Entanglement Relativity Eq.(1) leads to the following form of Eq. (3):

$$\hat{\rho}_C = \sum_{k,l,\alpha,\beta,\beta'} p_k \omega_l^k C_{\alpha\beta}^{kl} C_{\alpha\beta'}^{kl*} |\alpha\rangle_A \langle\alpha| \otimes |\beta\rangle_B \langle\beta'| + \sum_{k,l,\alpha \neq \alpha',\beta,\beta'} p_k \omega_l^k C_{\alpha\beta}^{kl} C_{\alpha'\beta'}^{kl*} |\alpha\rangle_A \langle\alpha'| \otimes |\beta\rangle_B \langle\beta'|. \quad (5)$$

Clearly from Eq.(5): in order for some basis  $|\alpha\rangle_A$  to provide  $D_A = 0$ , the second term on the rhs of Eq.(5) must vanish. Due to the linear independence of the terms  $|\alpha\rangle_A \langle\alpha'| \otimes |\beta\rangle_B \langle\beta'|$ , this can happen only if:

$$\sum_{k,l} p_k \omega_l^k C_{\alpha\beta}^{kl} C_{\alpha'\beta'}^{kl*} = 0, \forall \alpha \neq \alpha', \forall \beta, \beta'. \quad (6)$$

Eq. (6) is actually a set of the simultaneously satisfied equalities. The number of the equalities is equal to the number of the combinations for the indices  $\alpha \neq \alpha'$  and  $\beta$  and  $\beta'$ . Clearly, for the continuous variable systems, this number is infinite. On the other hand, there is some freedom in choice of  $p_k$  and  $\omega_l^k$ , as well as the normalization conditions (cf. Eq.(1)),  $\sum_{\alpha,\beta} C_{\alpha\beta}^{kl} C_{\alpha\beta}^{k'l'*} = \delta_{kk'} \delta_{ll'}$ . This may reduce the number of the expressions Eq.(6) that should be simultaneously satisfied. So, we cannot exclude that there exist some states of the form Eq.(4) for both structures,  $1+2$  and  $A+B$ . Nevertheless, for every combination of the coefficients  $C_{\alpha\beta}^{kl}$  satisfying Eq.(6), there is the infinite number of variations of the coefficients  $p_k$  and  $\omega_l^k$  that do not satisfy Eq.(6). In practice, it means one may forget about the states fulfilling Eq.(6), i.e. bearing zero one-way discord for a pair of structures,  $1+2$  and  $A+B$ .

Regarding the two-way discord ( $D$ ) for the  $1+2$  structure, the condition  $D = 0$  (i.e.  $D_1 = 0 = D_2$ ) can be satisfied if and only if the composite system's state can be written as [4, 5, 6]:

$$\hat{\rho}_C = \sum_{kl} p_{kl} |k\rangle_1 \langle k| \otimes |l\rangle_2 \langle l|, \sum_{k,l} p_{kl} = 1. \quad (7)$$

Such states are now commonly termed classical-classical (CC) states. The presence of only classical correlations in CC states [3] is revealed by  $C_f \neq 0$ , which is straightforward to show for Eq.(7). Again, assuming non-validity of Eq. (1), i.e. assuming equality  $|k\rangle_1 |l\rangle_2 = |k\rangle_A |l\rangle_B, \forall k, l$ , gives directly rise to the form Eq.(7) also for the structure  $A+B$ , i.e.  $D_A = 0 = D_B$ .

However, substitution of Eq.(1) for  $|k\rangle_1 |l\rangle_2$  into Eq.(7) gives rise to:

$$\hat{\rho}_C = \sum_{k,l,\alpha,\beta} p_{kl} |C_{\alpha\beta}^{kl}|^2 |\alpha\rangle_A \langle\alpha| \otimes |\beta\rangle_B \langle\beta| + \sum_{k,l,\alpha \neq \alpha',\beta \neq \beta'} p_{kl} C_{\alpha\beta}^{kl} C_{\alpha'\beta'}^{kl*} |\alpha\rangle_A \langle\alpha'| \otimes |\beta\rangle_B \langle\beta'|. \quad (8)$$

In order Eq.(8) to be a CC state—i.e. to be of the form Eq.(7)—also for the  $A+B$  structure, the following conditions should be satisfied:

$$\sum_{k,l} p_{kl} C_{\alpha\beta}^{kl} C_{\alpha'\beta'}^{kl*} = 0, \forall \alpha \neq \alpha', \forall \beta \neq \beta'. \quad (9)$$

The coefficients  $p_k \omega_l^k$  in Eq.(6) are variants of the more general form  $p_{kl}$  appearing in Eq.(9). In other words: Eq. (6) is a variant of the more general and more stringent equation Eq.(9). So, likewise for Eq.(6), the number of states satisfying Eq.(9) is negligible compared to the number of states not fulfilling Eq.(9). Likewise for the one-way discord, one may practically ignore the possible existence of the states bearing zero two-way discord for a pair of structures,  $1 + 2$  and  $A + B$ .

Thereby, the number of the possible solutions to Eq.(6) as well as to Eq. (9) is by far negligible compared to the number of the states not fulfilling Eq.(6) i.e. Eq.(9). This observation clearly emphasizes Quantum Correlations Relativity (QCR) as a new corollary of the universally valid quantum mechanics: non-classical correlations are ubiquitous for quantum systems. If there are not correlations (quantum discord is zero) for one structure (for one set of the composite system's degrees of freedom), there are certainly non-classical correlations (non-zero discord) for an alternative structure (for an alternative set of the degrees of freedom) of the composite system. Formally, QCR is presented by the following equality:

$$\begin{aligned} \sum_k p_k |k\rangle_1 \langle k| \otimes |k\rangle_2 \langle k| &= \sum_{k,l,\alpha,\beta} p_{kl} |C_{\alpha\beta}^{kl}|^2 |\alpha\rangle_A \langle \alpha| \otimes |\beta\rangle_B \langle \beta| \\ + \sum_{k,l,\alpha \neq \alpha', \beta \neq \beta'} p_{kl} C_{\alpha\beta}^{kl} C_{\alpha'\beta'}^{kl*} &|\alpha\rangle_A \langle \alpha'| \otimes |\beta\rangle_B \langle \beta'|, \end{aligned} \quad (10)$$

and analogously for the one-way discord(s).

So one can say: "non-classical correlations" is not a characteristic of a composite *system*, but is a characteristic of a composite system's *structure*.

### 3.2 Comments

From the previous section we learn: adding a new structure into considerations reduces the number of states bearing the form Eq.(4) for all the structures. Thereby we conclude, although not rigorously prove: every quantum state bears non-classical (quantum) correlations, for at least one structure of the composite system. The lack of the rigorous proof is in intimate relation to the following more general considerations.

While the variables transformations is a universal physical method, we have only recently started to understand its importance in the quantum

mechanical context. This may be a consequence of the fact that the following, easily formulated, task is *open* in the quantum mechanical formalism:

(**T**) *Starting from the left-hand (right-hand) side to obtain the right-hand (left-hand) side of Eq. (1) i.e. of Eq. (10).*

Regarding Eq.(10), the task **T** reads: given a quantum state  $\hat{\rho}_C$  (e.g. Eq.(7) for the  $1 + 2$  structure) to check if the state takes the separable form Eq.(7) for the structure  $A + B$ . This is the task of the Quantum Separability Problem (QUSEP) thoroughly investigated in the literature for the finite-dimensional systems (see e.g. Gharibian [17] and the references therein). Solving the equations (6) and (9) is clearly also an instance of the QUSEP problem, which suggests that the general solutions to Eqs. (6) and (9) are hardly expectable.

#### 4. Discussion

"Quantum discord" is designed to capture all the kinds of non-classical correlations, including "entanglement", in quantum systems. Therefore here formulated Quantum Correlations Relativity (QCR) generalizes Entanglement Relativity, Section 2.

In analogy with [18], QCR can be expressed as "non-classical (quantum) correlations for (practically) *all* quantum states relative to some structures". Thus we go beyond the "almost all quantum states have nonclassical correlations" of Ferraro et al [18]—this "almost" is lost in our conclusion. In terms of the result of Ferraro et al [18], QCR can be readily expressed as follows: if a quantum state falls within the zero-discord set  $\Omega_o$  for one, it is highly improbable to fall within the analogous  $\Omega_o$  set for virtually any other structure of the composite system. While the result of Ferraro et al [18] refers to Markovian open systems, our finding bears universality.

It is worth emphasizing: quantum correlations relativity is not a consequence of the reference-frame change or of the more general relativistic considerations such as e.g. in [19, 20]. The degrees-of-freedom transformations implicit to our considerations cannot be written in a separable form for the unitary operators, i.e. in the form  $U_1 \otimes U_2$  for the  $1 + 2$  structure—such transformations are known to preserve discord [4, 5, 6] (and the references therein). Interestingly enough, some formally trivial variables transformations exhibit QCR also for the finite-dimensional (e.g. qubit) systems.

To illustrate, consider a three-qubit system,  $\mathcal{C} = 1 + 2 + 3$ , and its bipartite structures,  $1 + S_1$  and  $S_2 + 3$ , where the bipartite systems  $S_1 = 2 + 3$  and  $S_2 = 1 + 2$ . As it is well known from quantum teleportation [21], the  $\mathcal{C}$ 's state  $|\phi\rangle_1 |\Phi^+\rangle_{S_1}$ , where  $|\Phi^+\rangle_{S_1} = (|0\rangle_2 |0\rangle_3 + |1\rangle_2 |1\rangle_3)/2^{-1/2}$ , can be re-written as  $\sum_i |\chi_i\rangle_{S_2} |i\rangle_3/2$ , where the  $S_2$ 's states represent the Bell states [1] for the pair

1 + 2. The point is that for the  $1 + S_1$  structure, the state is tensor-product and therefore not bearing any correlations between the 1 and  $S_1$  systems, while there is entanglement in the  $S_2 + 3$  structure.

In our considerations, entanglement relativity (Section 2) is basic to the more general correlations relativity, Section 3. Not surprisingly, as we use entanglement relativity for the pure quantum states, which are the building blocks of the mixed states. The inverse, however, is not in general correct: entanglement relativity (even for the mixed states) does not in general follow from the more-general-correlations relativity.

To see this, let us consider a structure  $1 + 2$  for which  $D_1 = 0$ . Then the quantum state is of the form  $\rho = \sum_i p_i |i\rangle_1 \langle i| \otimes \rho_{2i}$ ;  $\sum_i p_i = 1$ . Now, the correlation relativity suggests there is a structure  $A + B$  for which  $|i\rangle_1 \langle i| \otimes \rho_{2i} = \sum_j \omega_j^i \rho_{Aj} \otimes \rho_{Bj}$ ;  $\sum_j \omega_j^i = 1, \forall i$ . Now, by substituting the later into the initial form for  $\rho$  gives rise to  $\rho = \sum_j \lambda_j \rho_{Aj} \otimes \rho_{Bj}$ ;  $\lambda_j \equiv \sum_i \omega_j^i p_i$ , and  $\sum_j \lambda_j = 1$ . Certainly, for this one easily obtains  $D_A \neq 0, D_B \neq 0, C_f \neq 0$ , but there is not entanglement for  $A + B$  structure.

To support intuition, we express our main result on QCR in "operational" terms: In order to use a composite system's non-classical correlations as information theoretic resource, one need not specifically prepare the system's state. Rather, even if the initial state is short of the non-classical correlations, one can manage by targeting the alternative variables (e.g. degrees of freedom) without any additional/intermediate operations. Some details in this regard can be found e.g. in [12].

## 5. Conclusion

Non-classical (quantum) correlations are a matter of the composite systems's structure, rather than that of the composite systems itself. This Quantum Correlations Relativity is a new corollary of quantum mechanics that is here rigorously established for the continuous variable systems and illustrated for a typical example for a qubits system. Physically, we realize that the quantum information resources are ubiquitous in the bipartitions of the composite quantum systems. From the operational perspective, our observation suggests the quantum information resources can be directly used without specific state preparation.

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