IMK-14 – Research & Developement 18(2012)1, EN19 - 23
UDK 621 ISSN 0354-6829

Stability of Reservoir of Tank-wagon at Longitudinal Impact

Dragan Petrović[1,*] – Milan Bižić[1]
[1] Facuty of Mechanical Engineering Kraljevo, University of Kragujevac, Dositejeva 19, 36000 Kraljevo, Serbia

This paper presents the methodology of determination of dynamic parameters of wagons during the impact using the principles of nonlinear dynamics and the theory of elasticity. The general expression of oscillation of the tank of the tank-wagon has been derived; it is described through nonlinear differential equations connected by common variables. The expression for determination of the change of hydrodynamic pressure on the tank bottom at the impact of wagons has been derived, where the model in which the change of pressure corresponds to propagation of elastic waves of stresses and deformations in the beam subjected to impact has been adopted. In addition to that, a mathematical model for two wagons impact simulation that takes into consideration freight moving has been developed.

Ključne reči: Stability, reservoir, tank-wagon, longitudinal impact.

0 INTRODUCTION

The reservoir (tank) of the tank-wagon (Fig. 1) is made in the form of a circular cylinder with the radius *R*, the constant thickness *h* and the length *l*. It usually leans on the wagon underframe along its edges, so that supports, at one of the ends, most frequently have the freedom of motion in the direction of longitudinal axis. In this way the tank is protected from the action of horizontal axial forces which are, at the impact of wagons, transmitted from the buffer to the underframe. At impact, considerable loads of the tank arise from the action of liquid on the tank bottom.

Fig. 1. *Tank-wagon*

Sudden action of longitudinal load causes radial oscillations of the tank where, if the load *p* is smaller than a certain value, those oscillations will be without the increase of amplitude around the equilibrium position, and vice versa, if the load *p* is greater than that value, than the deflection amplitude rises with the time and, consequently, the tank loses its stability. It means that the values of loads at which the deflection very quickly tends to infinity are critical values. The task is to determine that critical load from which the infinite increase of deflection amplitude arises. For the analysis we shall use general equations of motion of the cylindrical shell given by the following expressions [2]:

$$\frac{D}{h}\nabla^4(w-w_o)=L(w,\Phi)+\nabla_k^2\Phi+\frac{q}{h}-\frac{\gamma}{g}\frac{\partial^2 w}{\partial t^2} \quad (1)$$

$$\frac{1}{E}\nabla^4\Phi=-\frac{1}{2}\left[L(w,w)-L(w_o,w_o)\right]-\frac{1}{R}\frac{\partial^2}{\partial x^2}(w-w_o) \quad (2)$$

where:
D – cylindrical rigidity of the shell
∇^4 – double Laplace operator
w – deflection of the tank of the tank-wagon
w_o – initial deflection of the tank of the tank-wagon
Φ – function of stress
γ – specific gravity of the material
g – gravity acceleration
t – time
q – cross load of the shell

As there are still no exact methods of integration of the above equations, we shall look for their approximate solutions in the form of an order. Let us take the element of the middle surface of the tank and the coordinate axis, as it is shown in Figure 2, and let the following loads act in the general case:
q – cross load which is normal on the middle surface,
p_x – compression or tension load which acts along the direction x,
p_y – compression or tension load which acts along the direction y.

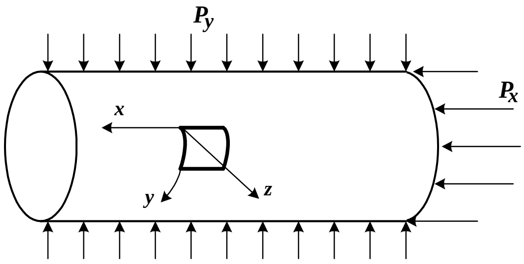

Fig. 2. *Loads of the tank of the tank-wagon*

In experimental tests it has been noticed that deflections of the tank toward the centre and from the centre of the curve are not the same. Deflection directed to the centre of the curve are greater than deflections directed from the centre of the curve. There fore, we shall adopt the following expression for the assumed displacements *w*:

$$w=f(t)\cdot\left(\sin\frac{m\pi x}{l}\cdot\sin\frac{ny}{R}+\psi\cdot\sin^2\frac{m\pi x}{l}\right) \quad (3)$$

where:
$f(t)$ – amplitude of deflection,
l – length of the cylinder,

*Corr. Author's Address: Faculty of Mechanical Engineering Kraljevo, Dositejeva 19, 36000 Kraljevo, Serbia, petrovic.d@mfkv.kg.ac.rs

R – radius of the cylinder,
m – number of semi-waves per the length of the cylinder,
n – number of waves per radius,
ψ – correction of the amplitude (time function).

Depending on the oscillation form of the reservoir, the parameters m and n are changed (Fig. 3). On this basis, structural changes (e.g. installing rings on the tank) can affect the number of waves.

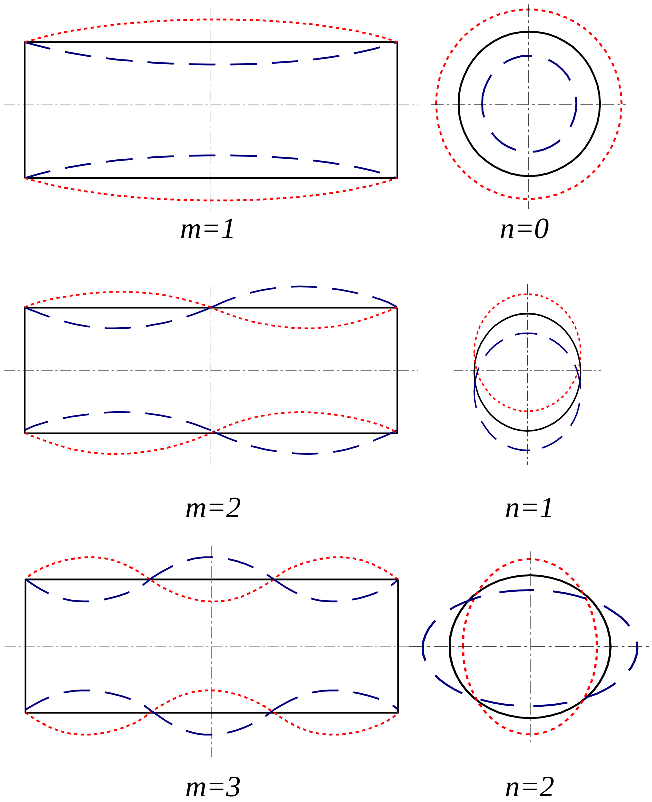

Fig. 3. *Different forms of tank oscillations*

In addition to that, we shall take that the tank has initial deflections, that is irregularities which have the same character as the total deflection w:

$$w_o = f_o(t)\cdot\left(sin\frac{m\pi x}{l}\cdot sin\frac{ny}{R}+\psi\cdot sin^2\frac{m\pi x}{l} \right) \qquad (4)$$

Adopting that:

$$\alpha=\frac{m\pi}{l} \text{ and } \beta=\frac{n}{R},$$

and solving the equations (1) and (2) by applying Bubnov-Galerkin method [1,2], we obtain a system of the second order non-linear differential equations connected by common variables. This system can be solved by applying the Runge-Kut method, bat before that it is necessary to determine the change of load, that is the pressure on the bottom of the tank of the tank-wagon $\overline{p}_x$, at longitudinal hydraulic impact [2].

$$\begin{aligned}
&\rho\ddot{f}+\left[\frac{\alpha^4}{R^2\left(\alpha^2+\beta^2\right)^2}+\frac{h^2\left(\alpha^2+\beta^2\right)^2}{12\left(1-v^2\right)}\right]E\left(f-f_0\right)+\\
&+E\alpha^4\beta^4\left[\frac{1}{\left(\alpha^2+\beta^2\right)^2}+\frac{1}{\left(9\alpha^2+\beta^2\right)^2}\right]\left(f^2-f_0^2\right)f\psi^2+\\
&+\frac{E}{16}\left(\alpha^4+\beta^4\right)\left(f^2-f_0^2\right)f-\frac{E\alpha^4\beta^2}{R\left(\alpha^2+\beta^2\right)^2}\psi\left(2f^2-ff_0-f_0^2\right)-\\
&-\left(\alpha^2\overline{p}_x+\beta^2\overline{p}_y\right)f-\frac{E\beta^2}{4R}\left(f-f_0\right)\psi f=0
\end{aligned} \qquad (5.1)$$

$$\begin{aligned}
&\frac{3}{4}\rho\left(\ddot{f}\psi+\ddot{\psi}f+2\dot{\psi}\dot{f}\right)-\frac{E\alpha^4\beta^2}{2R\left(\alpha^2+\beta^2\right)^2}\left(f-f_0\right)f+\\
&+\left[\frac{E\alpha^4\beta^4}{2\left(\alpha^2+\beta^2\right)^2}+\frac{E\alpha^4\beta^4}{2\left(9\alpha^2+\beta^2\right)^2}\right]\left(f^2-f_0^2\right)f\psi+\\
&\frac{Eh^2\alpha^2}{3\left(1-v^2\right)}\left(f-f_0\right)\psi-\frac{q}{h}+\alpha^2\overline{p}_x(t)f\psi+\frac{\overline{p}_y(t)}{R}-\\
&-\frac{E}{8R}\left[\frac{\beta^2}{2}\left(f^2-f_o^2\right)-\frac{2\left(f-f_o\right)\psi}{R}\right]=0
\end{aligned} \qquad (5.2)$$

1 DETERMINATION OF PRESSURE OF LIQUID ON THE TANK BOTTOM AT LONGITUDINAL IMPACT

At the impact of tank-wagons filled with liquid, there arises hydraulic impact of liquid on the tank bottom. Behaviour of the tank tank can then be completely different from its behaviour at static load. The cause is in inertial forces which arise in a very short time interval. The structure does not succeed in obtaining displacements which correspond to fast changes of load. Such delay causes abrupt deformation of the structure.

In examination of the value of hydraulic pressure on the bottom of the tank-wagon, the model proposed by Euler will be used [2], it refers to the speed of motion of liquid particles which is seen as the function of time t and the coordinates x, y and z of the volume in which the liquid moves.

Let us separate the elementary parallelepiped with the sides dx, dy and dz from the total volume of liquid (Fig. 4). v_x, v_y and v_z denote speeds of liquid particles in that parallelepiped which correspond to the axes x, y and z. Speeds of particles are functions of coordinates and time.

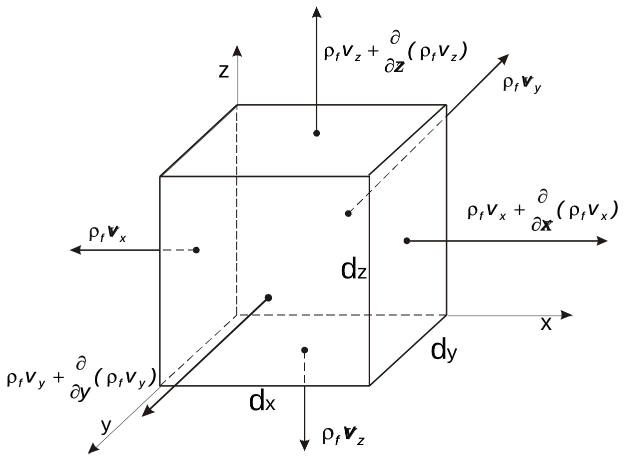

Fig. 4. *Elementary volume of fluid*

By projecting the components from the above figure on the axis x, we obtain:

$$\left\{\left[\rho_f v_x + \frac{\partial}{\partial x}(\rho_f v_x)\right] - \rho_f v_x\right\} dydzdt = \frac{\partial}{\partial x}(\rho_f v_x) dxdydzdt$$

where: $\rho_f = \rho_f(x,y,z,t)$ – density of fluid.

The increase of components along the y and z axes can be found in the same way so that the total change of mass of elementary fluid during the time dt is equal to:

$$\left[\frac{\partial}{\partial x}(\rho_f v_x) + \frac{\partial}{\partial y}(\rho_f v_y) + \frac{\partial}{\partial z}(\rho_f v_z)\right] dxdydzdt \qquad (6)$$

On the other hand, the change of mass of elementary fluid can be expressed through the change of its density:

$$-\frac{\partial \rho_f}{\partial t} dxdydzdt\ ;$$

the sign minus is in the case of decrease of volume.

By equating these two expressions, the equations of continuity (compatibility) of the environment are obtained:

$$\frac{\partial \rho_f}{\partial t} + \frac{\partial}{\partial x}(\rho_f v_x) + \frac{\partial}{\partial y}(\rho_f v_y) + \frac{\partial}{\partial z}(\rho_f v_z) = 0 \qquad (7)$$

Equations of motion will be formed in the same way, with the note that internal frictional forces and forces due to temperature influence are neglected. Equations of motion of elementary fluid in the direction of the axis x. The difference of forces that acts on the plane $dydz$ is:

$\left(\frac{\partial \overline{p}}{\partial x} dx\right) dydz$, where is: $\overline{p} = \overline{p}(x,y,z)$ – pressure.

The inertial force in the direction of the axis x is: $-\rho_f dxdydz \frac{dv_x}{dt}$

With the assumption that the change of liquid density is neglected, dynamic equations of ideal liquid can be written as:

$$\rho_f \frac{\partial v_x}{\partial t} + \frac{\partial \overline{p}}{\partial x} = 0\ ,\ \rho_f \frac{\partial v_y}{\partial t} + \frac{\partial \overline{p}}{\partial y} = 0\ ,\ \rho_f \frac{\partial v_z}{\partial t} + \frac{\partial \overline{p}}{\partial z} = 0 \qquad (8)$$

At constant density of fluid, the equations (7) become:

$$\frac{\partial \rho_f}{\partial t} + \rho_f\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0 \qquad (9)$$

It can be proved [1] that the value:

$$c_f = \sqrt{\frac{\partial \overline{p}}{\partial \rho_f}} \text{ - speed of sound in the fluid;} \qquad (10)$$

then the equation of compatibility gets the form:

$$\frac{1}{c_f^2}\frac{\partial \overline{p}}{\partial t} + \rho_f\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0 \qquad (11)$$

If it is adopted that the potential φ, then the speed of fluid particles can be expressed in the following way:

$$v_x = \frac{\partial \varphi}{\partial x}\ ,\ v_y = \frac{\partial \varphi}{\partial y}\ ,\ v_z = \frac{\partial \varphi}{\partial z} \qquad (12)$$

By replacing the expressions for speed in the equations (8), we obtain:

$$\overline{p} = -\rho_f \frac{\partial \varphi}{\partial t} \qquad (13)$$

When such a defined pressure is put in the equation of compatibility (11), we have that:

$$\frac{\partial^2 \varphi}{\partial t^2} - c_f^2 \nabla^2 \varphi = 0 \qquad (14)$$

The previous equation, according to its structure, corresponds to the equation which describes propagation of waves of stress and deformation in the beam subjected to impact [1]. By observing the wavy motion along the direction of the axis x, it can be written:

$$\frac{\partial^2 \varphi}{\partial t^2} - c_f^2 \frac{\partial^2 \varphi}{\partial x^2} = 0 \qquad (15)$$

Every function $f(x \pm c_f t)$ can be its solution [1]. If only the wave of propagation is observed, it can be written:

$$\varphi = f(x - c_f t)$$

$$v_x = \frac{\partial \varphi}{\partial x} = f'(x - c_f t) \qquad (16)$$

From the equation (13), it follows:

$$\overline{p}_x^d = -\rho_f \frac{\partial \varphi}{\partial t} = \rho_f c_f f'(x - c_f t) \qquad (17)$$

Finally, by replacing the expression for speed v_x (16) in the previous equation, we obtain that the dynamic pressure on the tank bottom of the tank-wagon, at longitudinal impact, is:

$$\overline{p}_x^d = \rho_f c_f v_x \qquad (18)$$

For calculation of the total force of pressure of liquid on the bottom of the tank $\overline{p}_x^u$, at the impact of tank-wagons, hydrodynamic force should be added by hydrostatic force $\overline{p}_x^s$.

$$\overline{p}_x^u = \overline{p}_x^d + \overline{p}_x^s \qquad (19)$$

Real liquids are different from the ideal ones taken in the model by the forces of internal friction and frictional forces between liquids and tank walls. Besides, this model introducer the assumption that the liquid is non-elastic. For the problems studied here, it can be taken that the mentioned influences are neglectable.

For the purpose of determination of the value of hydraulic impact on the tank bottom of the tank-wagon, it is necessary to study the process of impact of two wagons thoroughly. Change of the fluid speed v_x, which arises at impact, will be determined according to the model (Fig. 5) which has been formed on the basis of theoretical and experimental knowledge. Let us consider the case of impact of two wagons of the masses m_1 and m_2 loaded with the freights of masses m_3 and m_4, where there is relative moving of masses in the wagons by x_3, that is x_4. Let the elastic connections of rigidity between the wagon structure and the freight be c_3 and c_4. Besides, let the relative moving of the masses m_3 and m_4 are opposed by the forces of resistance of dry friction ($\mu_3 m_3 g$, $\mu_4 m_4 g$) and the forces of resistance of viscous friction which are proportional to the first degree of speed of the relative moving of the freights $\beta_3 \dot{x}_3$ and $\beta_4 \dot{x}_4$. Also, motion of the first and the second wagons are opposed by the forces of rolling friction $\mu_1 g(m_1+m_3)$ and $\mu_2 g(m_2+m_4)$.

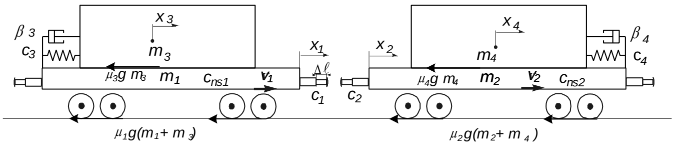

Fig. 5. *Impact of the wagons where there is moving of freight*

By determining kinetic and potential energies as well as the function of dissipation and by applying Lagrange's equations of the second order, we obtain:

$$
\begin{aligned}
&(m_1+m_3)\ddot{x}_1 + m_3\ddot{x}_3 + cx_1 - cx_2 + \mu_1(m_1+m_3)g\cdot sign\dot{x}_1 = 0\\
&(m_2+m_4)\ddot{x}_2 + m_4\ddot{x}_4 + cx_2 - cx_1 + \mu_2(m_2+m_4)g\cdot sign\dot{x}_2 = 0\\
&m_3\ddot{x}_3 + m_3\ddot{x}_1 + \beta_3\dot{x}_3 + c_3x_3 + \mu_3 m_3 g\cdot sign\dot{x}_3 = 0\\
&m_4\ddot{x}_4 + m_4\ddot{x}_2 + \beta_4\dot{x}_4 + c_4x_4 + \mu_4 m_4 g\cdot sign\dot{x}_4 = 0
\end{aligned}
\qquad (20)
$$

This defined system of differential equations takes into consideration moving of freight during the impact of wagons and is suitable for numeric solving. The standard method of Runge-Kutt of the fourth order has been used in solving this system of differential equations.

2 DETERMINATION OF DYNAMIC PARAMETERS AT LONGITUDINAL HYDRAULIC IMPACT

The calculation scheme of the tank of the tank-wagon consists of a closed cylindrical shell which at its ends has membrane partitions (bottoms), which are absolutely rigid in their planes. Theoretically and experimentally seen, the greatest stresses arise in the zone of passing from the cylinder to the bottom. However, in exploitation it is very rare to have a crack at the observed point regardless great stresses obtained by calculation and evident during tests. This can be explained by the fact that the stresses in the zone of passing from the cylinder to the bottom whose values exceed the elasticity limit are limited by the field of stresses which have considerably lower level, and according to the Saint-Venant's principle, they can be considered to have local character. These considerations hold while the stresses in the cylindrical part are considerably smaller than the stresses in the zone of passing from the cylindrical part to the bottom. The greatest increase of stress in the tank and therefore the occurrence of cracks can be caused by hydraulic impact of fluid which arises at the impact of wagons.

This paper considers the impact between the tank-wagon (Fig. 1) and the tank-wagon Uah/Ra. Technical data for these wagon are given in [1].

By analyzing the results obtained by numeric simulation of the equations (20), it can be concluded that the tank of the tank-wagon loses its stability at the impact speed v=19,5 m/s (Fig. 6). This refers to the tank which does not have initial deflections. Diagrams of the change of amplitude f of the deflection w depending on whether there are initial deflections are given in Fig 7. It can be noticed from the diagram that if there are initial deflections, the tank of the tank-wagon will lose its stability earlier.

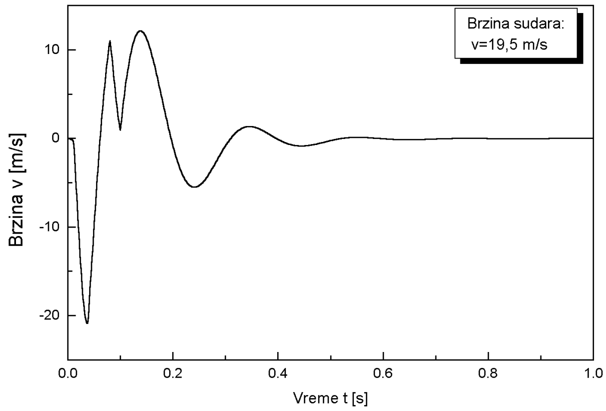

Fig. 6. *Change of speed of fluid at impact*

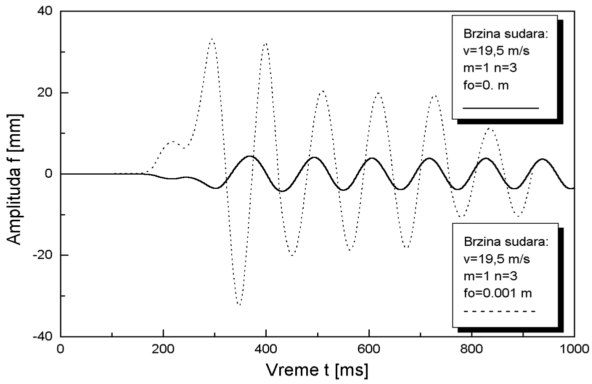

Fig. 7. *Change of amplitude of deflection of the tank*

At impact dynamic loading, the analysis of influences of shapes of oscillation on the occurrence of lost of stability has also been made, and the results are shown in Fig. 8. From the diagram it can be concluded that at dynamic action of load that arises at the impact of wagons at the speed of 19,5 m/s, critical load obtains at the number of semi-waves m=1 and n=4. In addition to that, the value of minimum critical load which the tank can stand is for higher at dynamic load in relation to static load.

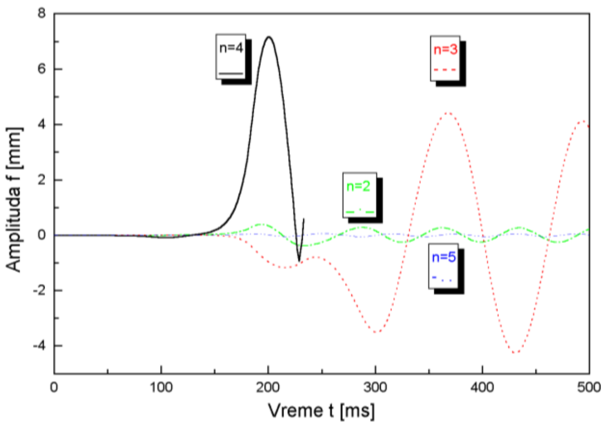

Fig. 8. *Influence of shapes of oscillation on amplitude of deflection of the tank for m=1*

By comparing the previous diagram with the diagram of change of speed of fluid (Fig. 6) and analyzing the time moment of lost of stability, it can be concluded that the abrupt increase of amplitude of deflection of the tank f arises only in the phase of unloading, that is when the load has considerably lower values in relation to its maximum values.

Fig. 9. *Deformed reservoir of tank-wagon*

Maximum load of tank occurs in the interval to 0.2 sec (Fig. 6), while loss of stability occurs in the interval after 0.2 sec (Fig. 8).

The appearance of deformed reservoir of tank-wagon at a loss of stability is given in Fig. 9.

3 CONCLUSION

By applying the principle of non-linear dynamics of plates and shells, a mathematical model of oscillation of the tank has been formed and, with appropriate initial and limiting conditions, it can be applied to all types of tank-wagons. The formed model of oscillation of the tank of the tank-wagon has been applied during calculation of the real structure, where critical loads at static and dynamic actions of forces have been determined as well as the influence of shapes of oscillation and initial deflections on the stability of the structure.

LITERATURE

[1] Volmir A. S., "Nelinearnaja dinamika plastinok i oboloček", Izdateljstvo «Nauka», Moskva, 1972.

[2] Petrović D., "Stabilnost noseće strukture vagona pri sudaru", Doktorska disertacija, Mašinski fakultet Kraljevo, 2000.

[3] Veršinski S. V., Danilov V. N., Čelnokov I., I., "Dinamika vagona", Izdateljstvo «Transport», Moskva, 1978.

[4] Dragan Petrovic, Milan Bizic, Mirko Djelosevic, "Determination of dynamic sizes during the process of impact of railway wagons", Archive of Applied Mechanics, 82:205–213, DOI: 10.1007/s00419-011-0549-5, 2011.

[5] Marković S., Lišanin R., Grajić J., "Proračun železničkih vozila – skripta", Mašinski fakultet Beograd, 1976.


ULT-160 CK
14. oktobar