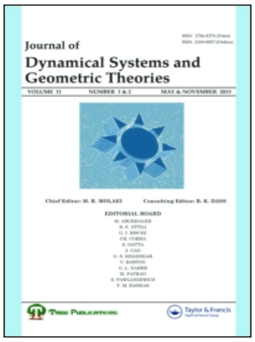

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ON T-SLANT, N-SLANT AND B-SLANT HELICES IN GALILEAN SPACE $\mathbb{G}_3$

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Abstract. In this paper, we define T-slant, N-slant and B-slant helices in Galilean space$\mathbb{G}_3$. In particular, we obtain the explicit parameter equations of the$T$-slant helices and prove that an admissible curve is aT-slant helix with a non-isotropic axis if and only if it has a non-zero constant conical curvature. We also prove that there are no N-slant, B-slant and Darboux helices in the same space.

AMS Classification: 53A20, 53A35, 53A40, 65D17
Keywords: Galilean space; general helix; slant helix; Darboux vector

1. Introduction

In Euclidean space $\mathbb{E}^3$, a regular curve whose tangent vector T and principal normal vector N make a constant angle with some fixed direction, is called the *general helix* (or curve of the constant slope) and the *slant helix*, respectively. It is well-known that a regular curve α in $\mathbb{E}^3$ with the curvature $\kappa \neq 0$ and the torsion τ in $\mathbb{E}^3$ is the general helix if and only if it has constant conical curvature τ/κ. In particular, the slant helices have constant geodesic curvature of the spherical image of their principal normal indicatrix ([7]). Some characterizations of the slant helices can be found in [1, 8, 9]. *Darboux helices* in $\mathbb{E}^3$ are defined in [16] as the curves whose Darboux vector makes a constant angle with some fixed direction. There is a simple relationship between the Darboux helices and the slant helices. Namely, every slant helix is the Darboux helix with respect to the same axis. Darboux helices whose Darboux vector has a constant speed, are known as the *curves of the constant precession* ([15]). In Minkowski space $\mathbb{E}^3_1$, the Darboux helices are studied in [10, 12].

In Galilean space $\mathbb{G}_3$, the general helices are defined in [14] as admissible curves which have a non-zero constant conical curvature τ/κ. In particular, the general helices in $\mathbb{G}_3$ with the natural equations $\tau(x) = b/ax$ and $\kappa(x) = 1/ax$, where $a, b = constant \neq 0$ lie on a cone and have a property that they are isogonal trajectories of the cone generators ([13]). In pseudo-Galilean space $\mathbb{G}^1_3$, the general helices are defined in [3] in terms of an angle between two isotropic vectors which lie in the pseudo-Euclidean plane $x = 0$. Some characterizations of the general helices in Galilean spaces can be found in [2, 4, 5, 6, 11].

In this paper, we introduce T-slant, N-slant and B-slant helices in $\mathbb{G}_3$ as admissible curves whose tangent, principal normal and binormal vector respectively makes a constant angle with some fixed straight line (an axis of the helix). We prove that an admissible curve is a T-slant helix with a non-isotropic axis if and only if it has a non-zero constant conical curvature. This means that the notion of T-slant helices corresponds to the notion of the general helices defined in [2, 14]. We obtain an explicit parameter equation of the T-slant helix and show that the obtained equation is more general than parameter equation of the general helix obtained in [2] (Lemma 5.8, page 207). We also prove that there are no N-slant, B-slant and Darboux helices in the same space.

2. Preliminaries

The Galilean geometry is one of the real Cayley-Klein geometries of projective signature $(0,0,+,+)$. The absolute of the pseudo-Galilean geometry is an ordered triple $\{w,f,I\}$, where w is the ideal (absolute) plane, f is the line in w and I is the fixed hyperbolic involution of the points of f ([4]). In the non-homogeneous affine coordinates, the similarity group H_8 of the Galilean space $\mathbb{G}_3$ has the following form

$$
\left\{
\begin{aligned}
\bar{x} &= a_{11} + a_{12}x,\\
\bar{y} &= a_{21} + a_{22}x + a_{23}\cos\varphi y + a_{23}\sin\varphi z,\\
\bar{z} &= a_{31} + a_{32}x - a_{23}\sin\varphi y + a_{23}z\cos\varphi z,
\end{aligned}
\right.
\tag{1}
$$

where a_{ij} and φ are real numbers. For $a_{12}=a_{23}=1$, the relation (1) defines the group $B_6 \subset H_8$ of isometries of $\mathbb{G}_3$.

The *scalar product* of two vectors $\mathbf{u}=(u_1,u_2,u_3)$ and $\mathbf{v}=(v_1,v_2,v_3)$ in $\mathbb{G}_3$ is given by

$$
\langle \mathbf{u},\mathbf{v}\rangle = \begin{cases} u_1v_1 & \text{, if } u_1\neq 0 \ \vee v_1\neq 0;\\ u_2v_2+u_3v_3 & \text{, if } u_1=0 \ \wedge \ v_1=0. \end{cases}
$$

This scalar product leaves invariant the Galilean norm of the vector $\mathbf{u}=(u_1,u_2,u_3)$ defined by

$$
\|\mathbf{u}\| = \begin{cases} |u_1| & \text{, if } u_1\neq 0;\\ \sqrt{u_2^2+u_3^2} & \text{, if } u_1=0. \end{cases}
$$

The *cross product* of two vectors $\mathbf{u}=(u_1,u_2,u_3)$ and $\mathbf{v}=(v_1,v_2,v_3)$ is given by

$$
\mathbf{u}\times\mathbf{v} = \begin{vmatrix} 0 & e_2 & e_3\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3 \end{vmatrix},
$$

where $e_2=(0,1,0)$ and $e_3=(0,0,1)$.

A plane of the form $x=$ *const.* in $\mathbb{G}_3$ is called *Euclidean plane*, since its induced geometry is Euclidean. Otherwise, it is called an *isotropic plane.*

The *angle measure* between two unit non-isotropic vectors is defined in [14] as the length of their difference.

Definition 2.1. *Let* $\mathbf{a}=(1,a_2,a_3)$ *and* $\mathbf{b}=(1,b_2,b_3)$ *be the unit non-isotropic vectors in general position in Galilean space* $\mathbb{G}_3^1$*. An angle* φ *between* $\mathbf{a}$ *and* $\mathbf{b}$ *is given by*

$$
\varphi = \sqrt{(a_2-b_2)^2+(a_3-b_3)^2}.
$$

The *angle measure* between two isotropic vectors, parallel to the Euclidean plane in $\mathbb{G}_3$, is defined in [14] as follows.

Definition 2.2. *An angle ω between two isotropic vectors $\mathbf{c} = (0, c_2, c_3)$ and $\mathbf{d} = (0, d_2, d_3)$, parallel to the Euclidean plane in $\mathbb{G}_3$, is equal the Euclidean angle between them. Namely,*

$$\cos\omega = \frac{c_2 d_2 + c_3 d_3}{\sqrt{c_2^2 + c_3^2}\sqrt{d_2^2 + d_3^2}}.$$

Definition 2.3. *The curve $\alpha(t) = (x(t), y(t), z(t))$ in the Galilean space $\mathbb{G}_3$ is said to be admissible if it has no inflection points $(\dot{\alpha}(t) \times \ddot{\alpha}(t) \neq 0)$ and no isotropic tangents $(\dot{x}(t) \neq 0)$.*

Each admissible curve can be written as

$$\alpha(x) = (x, y(x), z(x)). \tag{2}$$

The arc-length parameter of α is defined by $ds = |\dot{x}(t)dt| = |dx|$. For simplicity, we assume $ds = dx$ and $s = x$ as the arc-length parameter of α.

The curvature κ and the torsion τ of $\alpha(x)$ are given by

$$\kappa(x) = \sqrt{y''(x)^2 + z''(x)^2}, \tag{3}$$

$$\tau(x) = \frac{y''(x)z'''(x) - y'''(x)z''(x)}{\kappa^2(x)}. \tag{4}$$

The Frenet frame $\{T, N, B\}$ of an admissible curve $\alpha(x) = (x, y(x), z(x))$, has the form

$$\begin{aligned} T(x) &= (1, y'(x), z'(x)), \\ N(x) &= \frac{1}{\kappa(x)}(0, y''(x), z''(x)), \\ B(x) &= \frac{1}{\kappa(x)}(0, -z''(x), y''(x)), \end{aligned} \tag{5}$$

where T, N and B are called the *tangent*, the *principal normal* and the *binormal vector field* of α, respectively.

The Frenet equations of the curve $\alpha(x)$ are given by

$$\begin{bmatrix} T'(x) \\ N'(x) \\ B'(x) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(x) & 0 \\ 0 & 0 & \tau(x) \\ 0 & -\tau(x) & 0 \end{bmatrix} \begin{bmatrix} T(x) \\ N(x) \\ B(x) \end{bmatrix}. \tag{6}$$

Also, the Frenet's frame vectors of α satisfy the equations

$$T \times N = B, \quad N \times B = 0, \quad B \times T = N. \tag{7}$$

By using the relations $\alpha(x) = (x, y(x), z(x))$ and (5), we get

$$y''' = \frac{\kappa'}{\kappa}y'' - \tau z'', \quad z''' = \frac{\kappa'}{\kappa}z'' + \tau y''. \tag{8}$$

When the Frenet frame $\{T, N, B\}$ moves along an admissible curve α in $\mathbb{G}_3$, there exists an axis of the frame's rotation. The direction of such axis is given by *Darboux* vector (centrode), which has the equation

$$D(x) = \tau(x)T(x) + \kappa(x)B(x). \tag{9}$$

The Darboux vector satisfies *Darboux equations* given by

$$\begin{aligned} T'(x) &= D(x) \times T(x), \\ N'(x) &= D(x) \times N(x), \\ B'(x) &= D(x) \times B(x). \end{aligned}$$

Throughout the next sections, let $\mathbb{R}_0$ denote $\mathbb{R}\backslash\{0\}$.

3. T-slant, N-slant and B-slant helices in $\mathbb{G}_3$

In this section we define T-slant, N-slant and B-slant helices in the Galilean space $\mathbb{G}_3$ and obtain explicit parameter equations of the T-slant helices. We also prove that there are no N-slant and B-slant helices in $\mathbb{G}_3$.

Definition 3.1. *An admissible curve α in the Galilean space $\mathbb{G}_3$ is called a T-slant helix, if its tangent vector T makes a constant angle with some non-isotropic fixed direction.*

Definition 3.2. *An admissible curve α in the Galilean space $\mathbb{G}_3$ is called a N-slant and B-slant helix, if its principal normal and binormal vectors N and B respectively make a constant angle with some isotropic fixed direction.*

The fixed direction in the Definitions 3.1 and 3.2 is called an *axis* of the helix. We will exclude the case when the Frenet vectors T, N and B are constant, since they trivially make a constant angle with any fixed direction. Let us first characterize the T-slant helices.

Theorem 3.1. *Let α be an admissible curve with the curvature $\kappa \neq 0$ and the torsion τ in $\mathbb{G}_3$. Then α is T-slant helix if and only if it has a non-zero constant conical curvature τ/κ.*

Proof. Assume that an admissible curve $\alpha(x) = (x, y(x), z(x))$ is a T-slant helix with a non-isotropic axis spanned by the unit constant vector $U = (1, u_2, u_3)$. According to the Definition 3.1, its tangent vector $T = (1, y', z')$ makes the constant angle φ with U. By using the Definition 2.1, we have

$$\varphi^2 = (y' - u_2)^2 + (z' - u_3)^2 = c^2, \quad c \in \mathbb{R}_0^+.$$

From the previous equation we obtain

$$y' - u_2 = c \sin\psi, \quad z' - u_3 = c\cos\psi, \tag{10}$$

for some differentiable function $\psi = \psi(x)$. Differentiating the last two equations two times with respect to x, we get

$$y'' = c\psi' \cos\psi, \qquad z'' = -c\psi' \sin\psi, \tag{11}$$

$$y''' = -c\,(\psi')^2 \sin\psi + c\psi'' \cos\psi, \qquad z''' = -c\psi'' \sin\psi - c\,(\psi')^2 \cos\psi. \tag{12}$$

Substituting (11) and (12) in (8) and using linear independence of the trigonometric functions $\sin x$ and $\cos x$, we obtain

$$\psi'' = \frac{\kappa'}{\kappa}\psi', \quad \psi'^2 = -\tau\psi'. \tag{13}$$

If $\psi' = 0$, α is not an admissible curve, which is a contradiction. Hence $\psi' \neq 0$. From the relation (13), we find

$$\psi' = -\tau, \quad \frac{\kappa'}{\kappa} = \frac{\tau'}{\tau}. \tag{14}$$

The second equation of (14) gives

$$\frac{\tau}{\kappa} = constant \neq 0.$$

Conversely, assume that the admissible curve α has the constant conical curvature $\frac{\tau}{\kappa} = constant \neq 0$. Let us put $\frac{\tau}{\kappa} = -\frac{1}{c}$, $c \in \mathbb{R}_0$. Consider the unit non-isotropic vector U given by

$$U(x) = T\,(x) - cB\,(x)\,.$$

Differentiating the previous equation with respect to x and using the Frenet equations (6), we find $U' = 0$. Hence U is a constant vector. By using the Definition

2.1, it can be easily checked that an angle φ between the vectors T and U is given by $\varphi = |c| = constant$. According to the Definition 3.1, the curve α is a T-slant helix with an axis spanned by U. $\square$

Remark 3.1. *The notion of T-slant helices corresponds to the notion of the general helices defined in* [2, 14].

Corollary 3.1. *The non-isotropic axis of the T-slant helix α is spanned by*

$$U(x) = T(x) - cB(x).$$

where $c = -\frac{\kappa}{\tau} \in \mathbb{R}_0$.

In the next theorem, we obtain explicit parameter equations of the T-slant helices.

Theorem 3.2. *Let α be an admissible curve with the curvature κ and the torsion $\tau \neq 0$ in $\mathbb{G}_3$. Then α is a T-slant helix with an axis spanned by the unit non-isotropic fixed vector $U = (1, u_2, u_3)$ if and only if it has parameter equation given by*

(15)
$$\alpha(x) = \left(x, u_2 x + c\int \sin\left(\frac{1}{c}\int \kappa(x)dx\right)dx, u_3 x + c\int \cos\left(\frac{1}{c}\int \kappa(x)dx\right)dx\right),$$

where $u_2, u_3 \in \mathbb{R}$ and $c \in \mathbb{R}_0$.

Proof. Assume that an admissible curve α given by (2) is a T-slant helix with an axis spanned by the unit non-isotropic fixed vector $U = (1, u_2, u_3)$. By Theorem 3.1, α has a non-zero constant conical curvature $\frac{\tau}{\kappa} = -\frac{1}{c}$, $c \in \mathbb{R}_0$. By using relation (10), we obtain

$$y'(x) = u_2 + c\sin\psi(x), \quad z'(x) = u_3 + c\cos\psi(x).$$

Integrating the last two equations, we find

$$\begin{cases} y(x) = u_2 x + c\int \sin\psi(x)dx + c_1, & c_1 \in \mathbb{R}, \\ z(x) = u_3 x + c\int \cos\psi(x)dx + c_2, & c_2 \in \mathbb{R}. \end{cases} \tag{16}$$

Up to a translation, we may take $c_1 = c_2 = 0$. Substituting (16) in the relation (2), we obtain that α has parameter equation of the form

$$\alpha(x) = \left(x, u_2 x + c\int \sin\psi(x)dx, u_3 x + c\int \cos\psi(x)dx\right). \tag{17}$$

In particular, by using the first equation of (14), we get

$$\psi(x) = -\int \tau(x)dx + c_0, \quad c_0 \in \mathbb{R}. \tag{18}$$

Substituting $\tau = -\frac{\kappa}{c}$ in the last relation and putting $c_0 = 0$, we get

$$\psi(x) = \frac{1}{c}\int \kappa(x)dx.$$

Substituting this in (17), we get (15).

Conversely, assume that an admissible curve α has parameter equation given by (15). By using the Definition 2.1 it can be easily checked that an angle φ between the vectors T and U is constant. By Definition 3.1, α is a T-slant helix with an axis spanned by a non-isotropic fixed direction U. □

Remark 3.2. *In* [2], *the general helices are defined as the curves which have a constant conical curvature. It can be easily seen that parameter equation (14) of the T-slant helix (i.e. general helix) is more general than parameter equation of the general helix given in* [2] *(Lemma 5.8, page 207).*

Example 3.1. *Let us consider an admissible curve α in $\mathbb{G}_3$ with parameter equation (Figure 1)*

$$\alpha(x) = \left(x, -\frac{x}{2}\left[\cos(\ln x) - \sin(\ln x)\right], x + \frac{x}{2}\left[\cos(\ln x) + \sin(\ln x)\right]\right).$$

The tangent and the binormal vector of α are respectively given by

$$T(x) = (1, \sin(\ln x), 1 + \cos(\ln x)),$$

$$B(x) = (0, \sin(\ln x), \cos(\ln x)).$$

The curvature and torsion of α read $\kappa(x) = -\tau(x) = \frac{1}{x}$. Since $\frac{\tau}{\kappa} = -1 = -\frac{1}{c}$, according to the Theorem 3.1, the curve α is a T-slant helix. By Corollary 3.1, the non-isotropic axis of α is spanned by

$$U = T - cB = (1, 0, 1),$$

where $c = 1$ is the angle between T and U.

Example 3.2. *Let us consider an admissible curve β in $\mathbb{G}_3$ with parameter equation (Figure 2)*

$$\beta(x) = (x, 3x - \frac{1}{2}\sqrt{x}\cos(2\sqrt{x}) + \frac{1}{4}\sin(2\sqrt{x}), \frac{1}{2}\sqrt{x}\sin(2\sqrt{x}) + \frac{1}{4}\cos(2\sqrt{x})).$$

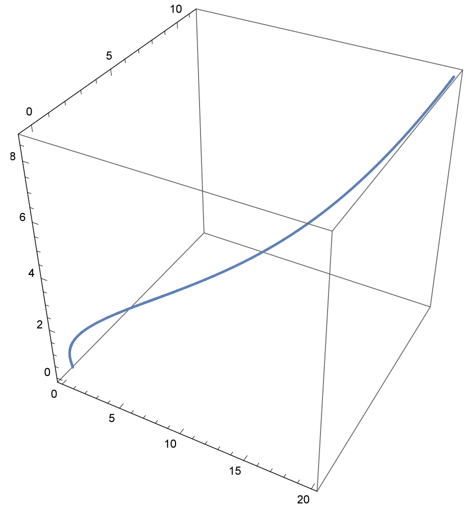

FIGURE 1. T-slant helix α

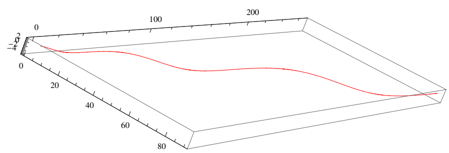

FIGURE 2. T-slant helix β

The tangent and the binormal vector of β have the form

$$T(x) = (1, 3 + \frac{1}{2}\sin(2\sqrt{x}), \frac{1}{2}\cos(2\sqrt{x})),$$

$$B(x) = (0, \sin(2\sqrt{x}), \cos(2\sqrt{x})).$$

The curvature and the torsion of β read $\kappa(x) = \frac{1}{2\sqrt{x}}$, $\tau(x) = -\frac{1}{\sqrt{x}}$. Since $\frac{\tau}{\kappa} = -2 = -\frac{1}{c}$, the Theorem 3.1 implies that β is a T-slant helix. According to the Corollary 3.1, the non-isotropic axis of β is spanned by

$$U = T - cB = (1, 3, 0),$$

where $c = \frac{1}{2}$ is the angle between T and U.

Next, let us consider N-slant helices. Let α be a N-slant helix whose principal normal vector $N(x)$ makes a constant angle ω with an isotropic axis determined by the unit isotropic fixed vector $U = (0, u_2, u_3)$. According to the Definition 2.2, we have

$$\omega = \angle(N, U) = \frac{1}{\kappa}(u_2 y'' + u_3 z'') = c_0, \quad c_0 \in \mathbb{R}_0.$$

The previous relation gives

$$y'' = \frac{1}{u_2}(c_0\kappa - u_3 z''). \tag{19}$$

Differentiating the previous equation with respect to x, we get

$$y''' = \frac{1}{u_2}(c_0\kappa' - u_3 z'''). \tag{20}$$

Substituting (19) and (20) in the first equation of (8), we get

$$z''' = \frac{\kappa'}{\kappa} z'' + \tau \frac{u_2}{u_3} z''.$$

By using the last equation and the second equation of (8), we find

$$\tau(y'' - \frac{u_2}{u_3} z'') = 0.$$

If $\tau = 0$, then the Frenet equations (6) imply $N = constant$, which we have excluded as the possibility. Thus

$$y'' = \frac{u_2}{u_3} z''. \tag{21}$$

Differentiating the last relation with respect to x, we obtain

$$y''' = \frac{u_2}{u_3} z'''. \tag{22}$$

From the relations (4), (21) and (22) we get $\tau = 0$, which gives a contradiction again.

The above results can analogously be proved for the B-slant helices. Thus, we can state the following theorem.

Theorem 3.3. *There are no N-slant and B-slant helices in $\mathbb{G}_3$ with non-constant Frenet vectors.*

4. Darboux helices in $\mathbb{G}_3$

In Euclidean and Minkowski 3-space, Darboux helices are defined as the curves whose Darboux vector makes a constant angle with some fixed axis. In this section, we show that there are no Darboux helices in Galilean space $\mathbb{G}_3$. First we give the definition of such helices.

Definition 4.1. *An admissible curve α in Galilean space $\mathbb{G}_3$ is called a Darboux helix, if its Darboux vector makes a constant angle with some fixed direction.*

The fixed direction in the Definition 4.1 is called an *axis* of the helix. We will exclude the case when the Darboux vector is constant, since it trivially makes constant angle with any fixed direction. By using the relations (5) and (9), we find that the Darboux vector of an admissible curve α is given by

$$D(x) = (\tau, \tau y' - z'', \tau z' + y''). \tag{23}$$

Theorem 4.1. *There are no Darboux helices in $\mathbb{G}_3$ with non-constant Darboux vector.*

Proof. Assume that there exists Darboux helix α in $\mathbb{G}_3$ with non-constant Darboux vector. Depending on the torsion τ of α, we may consider two cases:
(A) Assume that $\tau = 0$. Substituting $\tau = 0$ in the relation (9) it follows that the Darboux vector of the Darboux helix is given by $D = \kappa B$. Moreover, from the Frenet equations (6) it follows that the binormal vector B is constant. This implies that the Darboux vector D always has a fixed direction, which is a contradiction.
(B) Assume that $\tau \neq 0$. According to the Definition 4.1 and the relation (23), the unit Darboux vector D_0 of α given by

$$D_0 = \frac{D}{||D||} = \left(1, y' - \frac{z''}{\tau}, z' + \frac{y''}{\tau}\right) \tag{24}$$

makes a constant angle with some fixed axis spanned by the unit non-isotropic constant vector $U = (1, u_2, u_3)$. By Definition 2.1, it holds

$$\varphi^2 = \left(y' - \frac{z''}{\tau} - u_2\right)^2 + \left(z' + \frac{y''}{\tau} - u_3\right)^2 = c^2, \quad c \in \mathbb{R}_0^+.$$

From the previous equation we obtain

$$y' - \frac{z''}{\tau} - u_2 = c \sin\psi, \quad z' + \frac{y''}{\tau} - u_3 = c\cos\psi, \tag{25}$$

for some differentiable function $\psi = \psi(x)$. The last two equations imply

$$z'' = \tau y' - \tau u_2 - c\tau \sin\psi, \quad y'' = -\tau z' + \tau u_3 + c\tau \cos\psi. \tag{26}$$

Differentiating the second equation of (26) with respect to x, we get

$$y''' = -\tau' z' - \tau z'' + \tau' u_3 + c\tau' \cos\psi - c\tau\psi' \sin\psi. \tag{27}$$

Substituting the second equation of (26) and (27) in the first equation of (8), we find

$$-\tau' z' + \tau' u_3 + c\tau' \cos\psi - c\tau\psi' \sin\psi = \frac{\kappa'}{\kappa}(-\tau z' + \tau u_3 + c\tau \cos\psi).$$

The last equation is satisfied if and only if $\psi' = 0$ and $\tau = c_1\kappa$, $c_1 \in \mathbb{R}_0^+$. Substituting $\psi(x) = constant$ in (25) and using (24), we get $D_0 = constant$ which is a contradiction. $\square$

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