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COMPARATIVE BENDING ANALYSIS OF COMPOSITE LAMINATE AND FUNCTIONALLY GRADED PLATES BASED ON THE NEW SHAPE FUNCTION

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Abstract:

The paper analyzes the bending of composite laminate and functionally graded plates based on a higher order shear deformation theory. A new shape function has been implemented in the known theoretical formulations and the possibility of applying such a developed shape function has been shown through comparative analysis. The essential difference of the constitutive model of the functionally graded material in relation to the composite laminate is described. Equilibrium equations are defined using Hamilton's principle. Analytical methods were used to solve partial differential equations of equilibrium. In the bending plane, both plates are loaded with a force given as a trigonometric function. Numerical examples of simply supported square plates were done. The displacement of the central point of the plate as well as the distribution of stresses components through the thickness of the plate were analyzed. Comparative results are given in tables and graphs on the basis of which the newly developed shape functions are verified. The analysis of the obtained results shows the basic advantages of functionally graded materials in relation to conventional composite laminates.

Key words: composite laminate, functionally graded material, higher order shear deformation theory, shape function, bending.

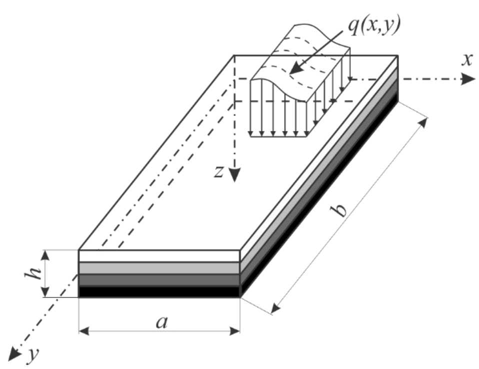
1. Introduction

In recent years, there has been a tendency to replace conventional materials with new materials. In that sense, there is an increasing use of composite materials which consist of two or more constituents. The use of these materials is increasingly pronounced in various branches of engineering. Composite materials have recently been used in the automotive industry, the aerospace industry, the electrical industry, civil and mechanical engineering, etc. Therefore, the use of multilayer composite materials-laminates is important. The advantages of these materials are high strength and toughness in certain directions and significantly lower weight compared to conventional materials. The main problems of composite laminates are different mechanical properties at the interlayers due to the merging of two discrete materials. As a result, a stress concentration on the interlayer usually occurs which can lead to damage to the laminate such as delamination of layer or matrix fracture. In order to overcome these problems, functionally graded (FG) materials have been developed in recent times. FG material is a composite material made of two or more constituents with a continuous change in material properties. Practically, it eliminates the stress concentration that occurs with conventional laminate composites.

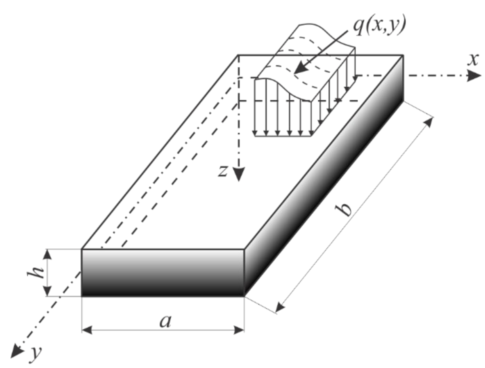
The behavior of laminate and functionally graded plates under the mechanical or thermal load can be analyzed using 3D elasticity theory or equivalent single-layer (ESL) theories. ESL theories have been developed from 3D elasticity theory with appropriate assumptions about strains or stress state through plate thickness. They represent approximate theories that have found wide application. The historical development of these theories begins with the classical plate theory, also known as Kirchhoff's theory [1]. Theories related to laminate plates based on Kirchhoff's hypothesis were further developed by many authors. Mindlin and Reissner dealt with the further development of plate theories. They developed a new plate theory in parallel, so the first order shear deformation theory (FSDT) is usually called the Mindlin-Reissner plate theory. There are essential differences between Mindlin and Reissner theories that can be found in [2]. Improvements of FSDT theory have been achieved with higher order shear deformation theories (HSDT). Different HSDT theories are based on different forms of assumed displacement fields. From the aspect of this paper, theories are significant in which the authors introduced the shape function into the assumed displacement fields. The shape functions that can be found in the papers belong to the group of polynomial, hyperbolic, exponential, parabolic, logarithmic, etc [3]. It should be emphasized that these shape functions are very often equally applicable to composite laminates as well as functionally graded materials [4-6]. In this paper, a new shape function is applied to the bending analysis of composite laminate and functionally graded plates.

2. Theoretical formulation

A rectangular composite laminate plate (Fig. 1a) as well as functionally graded plate are analyzed (Fig. 1b) under a sinusoidal load.



a) composite laminate plate



b) functionally graded plate

Fig. 1. Rectangular plate under sinusoidal load

The plate is analyzed under the sinusoidal transverse load:

$$q(x,y) = q_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right), \tag{1}$$

where q_0 is load amplitude.

In order to solve the problem of plate bending by applying higher order shear deformation theories based on shape functions, it is necessary to assume a displacement field in the form:

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w}{\partial x}(x,y,t) + f(z)\theta_x, \\
v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w}{\partial y}(x,y,t) + f(z)\theta_y, \\
w(x,y,z,t) &= w_0(x,y,t).
\end{aligned} \tag{2}
$$

where: u_0, v_0, w_0 - displacement of the middle plane of the laminate, $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$ - rotation angles of the normal of plate, θ_x, θ_y - displacement due to transversal shear and $f(z)$ - shape function.

For such an assumed field of displacement, the deformation of the normal perpendicular to the middle plane of the plate can be shown as in Fig. 2.

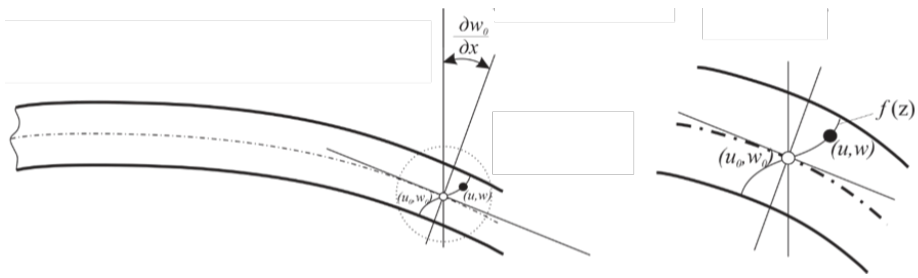


Fig. 2. Deformation of the normal perpendicular to the middle plane of the plate according HSDT

The strains components can be determined in a simple way by well-known strain-displacement relations in the area of linear elasticity.

Using the generalized Hooke's law in the form of:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad (i,\ j,\ k,l = 1,\ 2,\ 3), \tag{3}$$

the stress components are defined on the basis of which the unit loads of the plate are obtained in the form:

$$
\begin{aligned}
\mathbf{N} &= \int_{h^-}^{h^+} \boldsymbol{\sigma} dz = \sum_{l=1}^{n}\left(\int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k_0} dz + \int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k}_1 z dz + \int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k}_2 f(z) dz \right), \\
\mathbf{M} &= \int_{h^-}^{h^+} \boldsymbol{\sigma} z dz = \sum_{l=1}^{n}\left(\int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k_0} z dz + \int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k}_1 z^2 dz + \int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k}_2 z f(z) dz \right), \\
\mathbf{P} &= \int_{h^-}^{h^+} \boldsymbol{\sigma} f(z) dz = \sum_{l=1}^{n}\left(\int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k_0} f(z) dz + \int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k_1} z f(z) dz + \int_{h_l^-}^{h_l^+} C^{(l)}\mathbf{k_2} \left(f(z)\right)^2 dz \right), \\
\mathbf{R} &= \int_{h^-}^{h^+} \boldsymbol{\tau} f'(z) dz = \sum_{l=1}^{n} \int_{h_l^-}^{h_l^+} C_{\mathbf{s}}^{(l)}\mathbf{k_S} \left(f'(z)\right)^2 dz,
\end{aligned} \tag{4}
$$

where is:

$$\mathbf{C}^{(l)}=\begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix},\ C_S^{(l)}=\begin{bmatrix} C_{44} & C_{45} \\ C_{45} & C_{55} \end{bmatrix}$$ – constitutive elasticity tensor that depends on the class of symmetry

$$\boldsymbol{\sigma}=\left\{\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{xy}\right\}^T, \qquad \boldsymbol{\tau}=\left\{\tau_{xz} \quad \tau_{yz}\right\}^T$$ – stress components

$$\mathbf{N}=\left\{N_{xx} \quad N_{yy} \quad N_{xy}\right\}^T, \mathbf{M}=\left\{M_{xx} \quad M_{yy} \quad M_{xy}\right\}^T,$$

$$\mathbf{P}=\left\{P_{xx} \quad P_{yy} \quad P_{xy}\right\}^T, \mathbf{R}=\left\{R_y \quad R_x\right\}^T$$ – unit loads

$$\mathbf{k_0}=\left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right\}^T, \mathbf{k_1}=\left\{-\frac{\partial^2 w_0}{\partial x^2} \quad -\frac{\partial^2 w_0}{\partial y^2} \quad -2\frac{\partial^2 w_0}{\partial x \partial y}\right\}^T,$$

$$\mathbf{k_2}=\left\{\frac{\partial \theta_x}{\partial x} \quad \frac{\partial \theta_y}{\partial y} \quad \frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}\right\}^T, \mathbf{k_S}=\left\{\theta_x \quad \theta_y\right\}^T$$ – strain components

In the case of a functionally graded plate, the coefficients of the constitutive elasticity tensor are not constant values. Due to the gradient change of the material characteristics in the direction of the *z* coordinate, the constitutive elasticity tensor is defined in the form:

$$\begin{bmatrix} C_{11}(z) & C_{12}(z) & 0 & 0 & 0 \\ C_{12}(z) & C_{22}(z) & 0 & 0 & 0 \\ 0 & 0 & C_{44}(z) & 0 & 0 \\ 0 & 0 & 0 & C_{55}(z) & 0 \\ 0 & 0 & 0 & 0 & C_{66}(z) \end{bmatrix}, \tag{5}$$

where the coefficients of the constitutive elasticity tensor can be defined by engineering constants as:

$$\begin{gathered} C_{11}(z)=C_{22}(z)=\frac{E(z)}{1-\nu^2}, \qquad C_{44}(z)=C_{55}(z)=C_{66}(z)=\frac{E(z)}{2(1+\nu)}, \\ C_{12}(z)=\frac{\nu E(z)}{1-\nu^2}. \end{gathered} \tag{6}$$

Due to the gradient change of the material characteristics in the direction of the *z* coordinate, the modulus of elasticity can be defined according to the power law function [7]:

$$E(z)=E_\mathrm{b}+E_\mathrm{tb}\left(\frac{1}{2}+\frac{z}{h}\right)^p, \qquad E_\mathrm{tb}=E_\mathrm{t}-E_\mathrm{b}, \tag{7}$$

where $E(z)$, E_t, E_b are material characteristics in arbitrary cross-section *z*, at the top and bottom of the plate, respectively.

The Poisson's ratio can be considered constant $\nu = const$ due to the small variation of the value in the direction of the plate thickness.

As can be seen, the coefficients of the constitutive elasticity tensor are functionally dependent on the *z* coordinate, which practically means that in case of $p \neq 0$ there is a finite number of planes parallel to the middle plane that each of these planes having different values of the tensor coefficients C_{ij}.

For previously obtained unit load and the given plate load, it is possible to define equilibrium equations using Hamilton's principle:

$$\begin{aligned}
&\delta u_0:\quad N_{xx,x}+N_{xy,y}=0, \qquad\qquad \delta v_0:\quad N_{yy,y}+N_{xy,x}=0,\\
&\delta w_0:\quad M_{xx,xx}+2M_{xy,xy}+M_{yy,yy}+N_{xx}w_{0,xx}+2N_{xy}w_{0,xy}+N_{yy}w_{0,yy}+q=0,\\
&\delta\theta_x:\quad P_{xx,x}+P_{xy,y}-R_x=0, \qquad\qquad \delta\theta_y:\quad P_{xy,x}+P_{yy,y}-R_y=0.
\end{aligned} \tag{8}$$

The boundary conditions along the edges of the simply supported rectangular plate are:

$$\begin{aligned}
&v_0=w_0=\theta_y=N_x=M_x=P_x=0,\ \text{along the edge}\ x=0,\ x=a,\\
&u_0=w_0=\theta_x=N_y=M_y=P_b=0,\ \text{along the edge y}=0,\ y=b.
\end{aligned} \tag{9}$$

Taking into account the previously defined boundary conditions it is possible to assume Navier's form of the solution as [8]:

$$\begin{aligned}
&u_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b};\ v_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b},\\
&w_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},\\
&\theta_x(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}T_{xmn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b};\ \theta_y(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}T_{ymn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b},
\end{aligned} \tag{10}$$

where $U_{mn}, V_{mn}, W_{mn}, T_{xmn}, T_{ymn}$ are arbitrary parameters to be determined.

3. Numerical examples and analysis of results

In order to apply the previously obtained theoretical results for solving examples, the advantages of the symbolic and numerical part of programming in the MATLAB program are used. The results of bending analysis of composite laminate and FG plates are presented. Bearing in mind that a new shape function was implemented in the previously described theoretical formulations, its verification was first performed through the given tabular results. After that, based on a comparative diagram of the stress distribution, the advantages of FG material in relation to composite laminates were analysed. To solve the numerical example, the material characteristics from Table 1 were used.

Table 1. Material characteristics of composite laminate and FGM constituents

Material	Material characteristics		
Composite laminate	Lamina	$E_1/E_2=25,\ G_{12}/E_2=0.5,\ G_{13}/E_2=0.5,\ G_{23}/E_2=0.2,$ $\nu_{12}=\nu_{13}=\nu_{23}=0.25$	
Functionally graded	Aluminum (Al)	$E_m=70$	$\nu=0.3$
	Alumina (Al_2O_3)	$E_c=380$	$\nu=0.3$

Table 2. Shear deformation shape function

No. of function	Source	Shape function, *f(z)*
F1	Present study	$z\left(\cosh\left(\frac{z}{h}\right)-1.388\right)$
F2	[9]	$\sin\left(\frac{\pi}{h}z\right)e^{\frac{1}{2}\cos\left(\frac{\pi}{h}z\right)}+\frac{\pi}{2h}z$
F3	[10]	$ze^{-2\left(\frac{z}{h}\right)^2}$, $\left(ze^{\frac{-2\left(\frac{z}{h}\right)^2}{\ln\alpha}}\right)$, $\forall\alpha>0$
F4	[11]	$\xi\left[\frac{h}{\pi}\sin\left(\frac{\pi}{h}z\right)-z\right],\ \xi=\left\{1,1/\cosh(\frac{\pi}{2})-1\right\}$
F5	[12]	$h\sinh\left(z/h\right)-z\cosh\left(1/2\right)$
F6	[13]	$z\sec h\left(\frac{z^2}{h^2}\right)-z\sec h\left(\frac{\pi}{4}\right)\left[1-\frac{\pi}{2}\tanh\left(\frac{\pi}{4}\right)\right]$

Verification of the results obtained by applying the newly developed shape function was performed by comparison with the results from the literature obtained by applying the different shape functions given in Table 2.

Table 3. Normalized values of displacement and stress of symmetrical cross laminate [0°/90°/90°/0°] for ratio *a/b*=1 and *a/h* =4

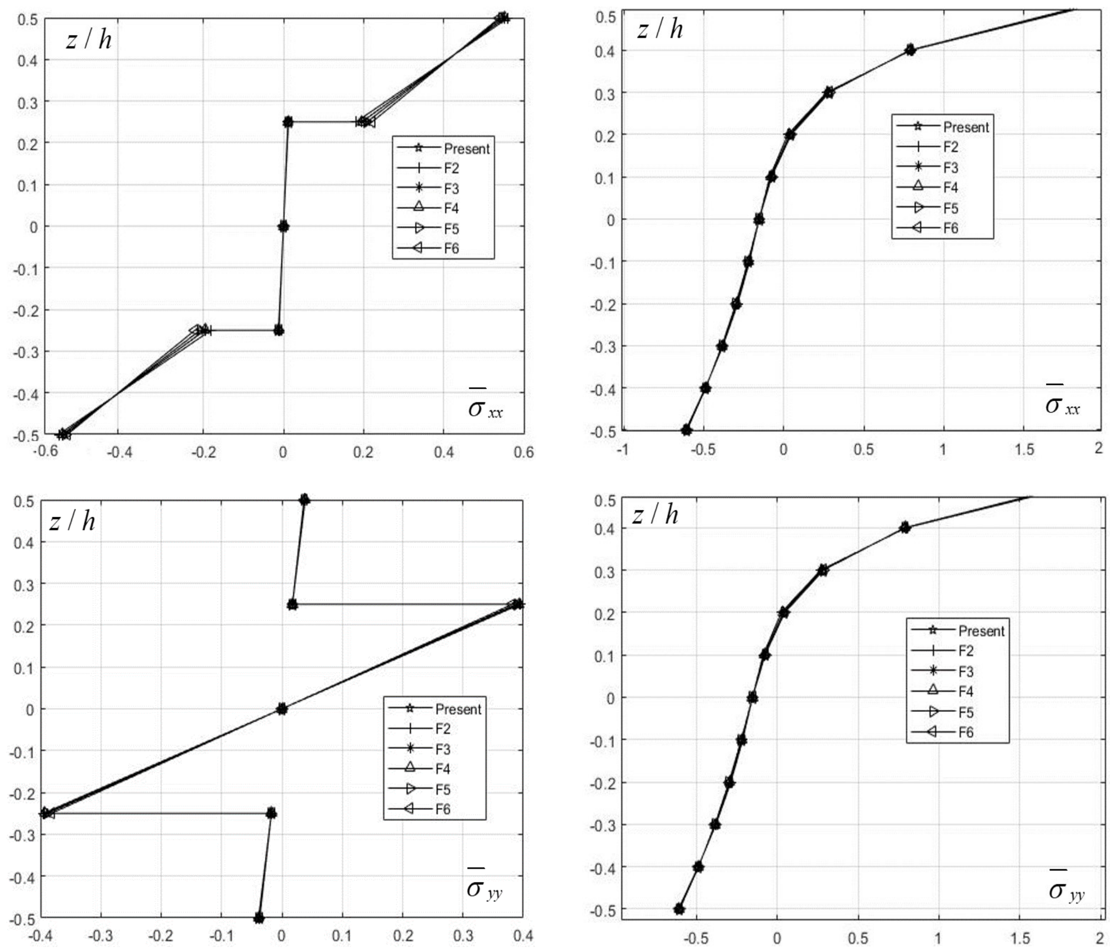
Theory	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\tau}_{xy}$	$\bar{\tau}_{yz}$	$\bar{\tau}_{xz}$
Present study	1.892	0.663	0.632	0.440	0.238	0.227
F2	1.920	0.741	0.636	0.048	0.270	0.253
F3	1.920	0.700	0.637	0.046	0.253	0.226
F4	1.920	0.702	0.637	0.046	0.254	0.227
F5	1.892	0.663	0.632	0.440	0.238	0.227
F6	1.905	0.677	0.634	0.045	0.244	0.214

Table 4. Normalized values of displacement and stress of functionally graded plate (*a/b*=1, *a/h*=4) for power law index *p*=2 and *p*=5.

p	Theory	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\tau}_{xy}$	$\bar{\tau}_{yz}$	$\bar{\tau}_{xz}$
2	Present study	0.949	0.538	0.538	0.214	0.272	0.272
	F2	0.942	0.533	0.533	0.213	0.271	0.271
	F3	0.948	0.535	0.535	0.213	0.276	0.276
	F4	0.948	0.535	0.535	0.213	0.276	0.276
	F5	0.949	0.538	0.538	0.214	0.272	0.272
	F6	0.945	0.540	0.540	0.214	0.266	0.266
5	Present study	1.220	0.413	0.413	0.224	0.236	0.236
	F2	1.219	0.403	0.403	0.222	0.244	0.244
	F3	1.224	0.408	0.408	0.223	0.244	0.244
	F4	1.224	0.407	0.407	0.223	0.244	0.244
	F5	1.220	0.413	0.413	0.224	0.236	0.236
	F6	1.209	0.418	0.418	0.225	0.227	0.227

In order to analysis and compare results of vertical displacement $\overline{w}$ (deflection), normal stresses $\overline{\sigma}_{xx}$ and $\overline{\sigma}_{yy}$, shear stress $\overline{\tau}_{xy}$ as well as transversal shear stresses $\overline{\tau}_{xz}$ and $\overline{\tau}_{yz}$, it is necessary to normalize the obtained values according to [14]:

Tables 3 and 4 show comparative results of normalized values of vertical displacement and stress of laminate and FG plates. On the basis of the obtained displacement and stress results by applying the newly developed shape function and the shape function given in Table 2, a good match can be observed.



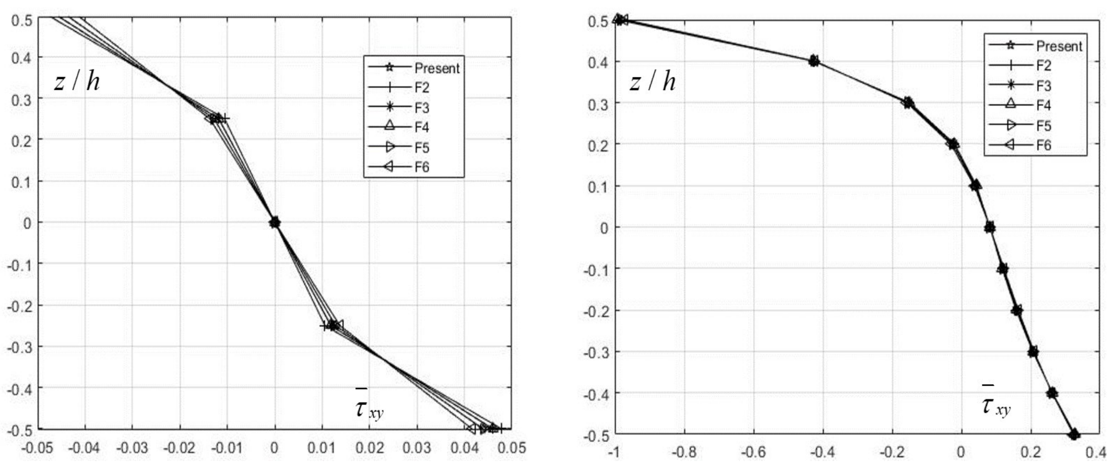
a) laminate plate, *a/h*=4, [0°/90°/90°/0°] б) functionally graded plate, *a/h*=4, *p*=5

Fig. 3. Distribution of the normal stresses $\overline{\sigma}_{xx}$ and $\overline{\sigma}_{yy}$ through the thickness of the square plate (*a/b*=1) for different shape functions

Figure 3 shows the distribution of normal stresses $\overline{\sigma}_{xx}$ and $\overline{\sigma}_{yy}$ through the thickness of the laminate and functionally graded square plate for different shape functions. By analyzing the diagrams, it can be seen that in the case of FG plates, the distribution curves are almost identical for all shape functions as for the newly introduced shape function. On the other hand, in the case of symmetrical cross laminate [0°/90°/90°/0°], minor deviations are observed in the layers with orientation angle of 0°. In general, by comparing the normal stress distribution diagrams of symmetrical cross laminate (Fig. 3a) and FG material (Fig. 3b), the following differences can be observed:

- In the case of composite laminate, the presence of symmetry of the normal stress distribution in relation to the coordinate origin is noticeable while in the case of FG material this symmetry does not exist.
- In the case of composite laminates, the maximum values of normal stress in the zones of pressure and tension are equal in absolute value. In the case of FG plate, these values are not the same due to different materials on the lower and upper edge of the plate.
- There is a stress discontinuity on the interlayers of the composite laminate while with the FG material we have a continuous stress distribution.

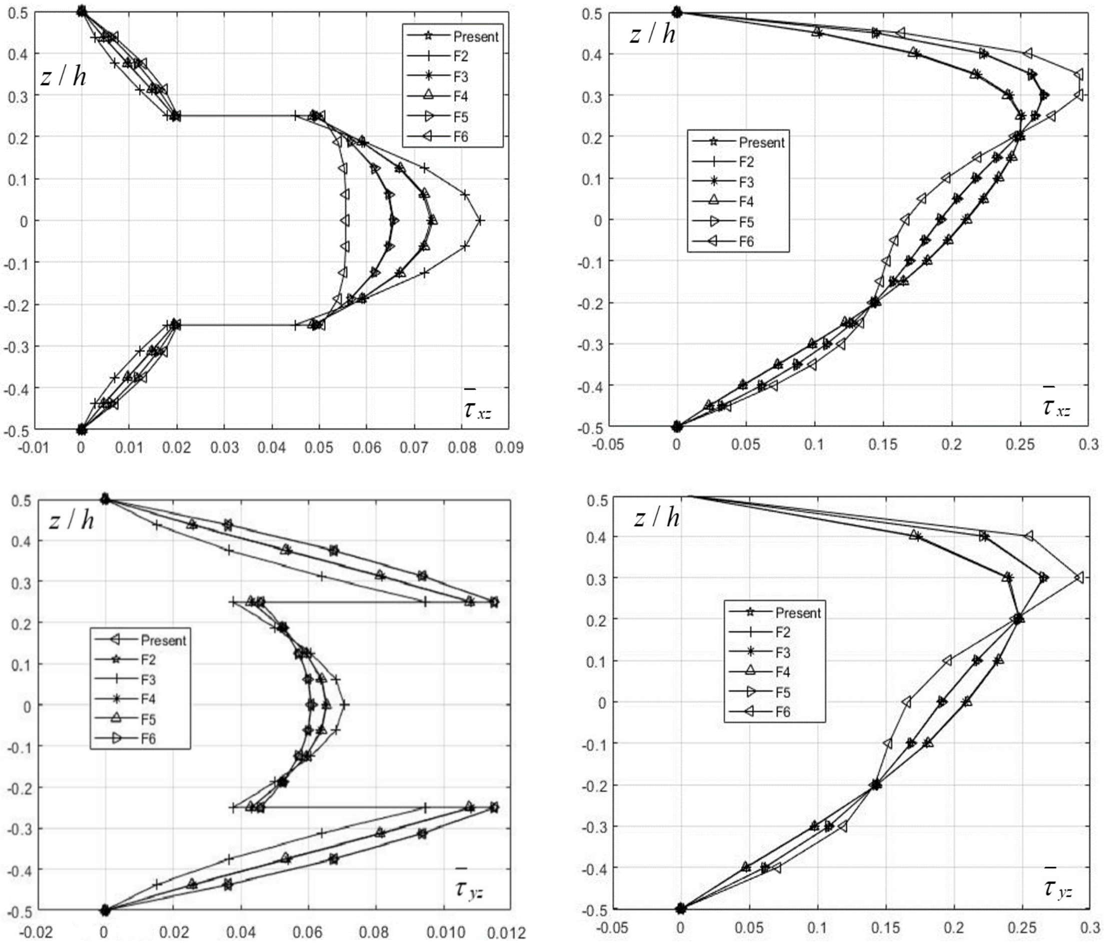
Figure 4 shows distribution of the shear stress $\overline{\tau}_{xy}$ through the thickness of the laminate and functionally graded square plate for different shape functions. Comparative analysis in Figure 4b showed that all the above shape functions give identical results as the newly developed shape function. This means that in the case of FG plates the type of shape function does not affect the shear stress $\overline{\tau}_{xy}$ results. It can be seen in Figure 4a that the stress distribution curves differ slightly depending on the applied shape function.



a) laminate plate, *a/h*=4, [0°/90°/90°/0°] б) functionally graded plate, *a/h*=4, *p*=5

Fig. 4. Distribution of the shear stress $\overline{\tau}_{xy}$ through the thickness of the square plate (*a/b*=1) for different shape functions

Figure 4 shows distribution of the transversal shear stress $\overline{\tau}_{xz}$ and $\overline{\tau}_{yz}$ through the thickness of the laminate and functionally graded square plate for different shape functions. In figure 5a, unlike all previous stress diagrams, the differences between the different shape functions are more pronounced. The largest deviations exist in the shape functions denoted by F2 and F6. The proposed new shape function gives the results that most closely match the results obtained by applying the F5 function. In FG plates (Fig. 5b), in contrast to shear stress $\overline{\tau}_{xy}$, the results for transversal shear stresses $\overline{\tau}_{xz}$ and $\overline{\tau}_{yz}$ do not match for all shape functions. The largest deviation is observed in the results for shape functions numbered as F6. Also, there is a slight deviation at shape functions F3 and F4 where the maximum stress value occurs at *z/h*=0.25, while the results for other shape functions are almost identical, reaching the maximum stress value in the plane *z/h*=0.3. Analysis of transverse shear stresses in figure 5a and 5b shows one of the basic differences between laminate and FG plates. In the case of laminate plates, it can be noticed that both transversal shear stresses reach the maximum value in the plane at the height *z/h*=0. On the other hand, in the case of FG plates asymmetry in relation to the plane *z/h*=0 is observed. The stresses reach the maximum value at plane *z/h*=0.3 for the observed volume fraction of constituents in the FG material (p=5). Finally, in the case of laminate, the occurrence of stress discontinuity at the interlayers is clearly expressed while in the case of FG plate there is a continuous stress distribution.



a) laminate plate, a/h=4, [0°/90°/90°/0°] б) functionally graded plate, a/h=4, p=5

Fig. 5. Distribution of the transversal shear stress $\overline{\tau}_{xz}$ and $\overline{\tau}_{yz}$ through the thickness of the square plate (a/b=1) for different shape functions

3. Conclusions

The paper points out the importance and topicality of the research of composite materials, with special reference to laminate and functionally graded materials. A brief overview of deformation theories applied in the field of composite plate's research is given. Kinematic displacement-strain relations based on a higher order shear deformation theory involving shape functions are defined. A new shape function has been implemented in the known theoretical formulations. The essential difference in the constitutive model of the functionally graded material in relation to the composite laminate is described. Using Hamilton's principle partial differential equations of equilibrium are defined which are solved by applying analytical methods. Numerical examples of bending of square simply supported laminated and FG plates have been made. Based on the obtained results, the newly developed shape function was verified and the basic advantages of FG material in relation to conventional composite laminates were shown:

- in the case of composite laminate, the presence of symmetry of the normal stress distribution in relation to the coordinate origin or symmetry of the transversal shear stresses distrubution in relation to the plane *z/h*=0 is noticeable while in the case of functionally graded material this symmetry does not exist,

- there is a stress discontinuity on the interlayers of the composite laminate while with the functionally graded material we have a continuous stress distribution.

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