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# Application of Strain Gauges in Experimental Testing of Mechanical Structures

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Abstract— The paper deals with application of strain gauges in the field of experimental testing of mechanical structures. The basic construction of strain gauge and its principle of work from the aspect of measurement of strain of tested object are analysed. The way of connection in Wheatstone bridges is considered and the advantages and disadvantages of certain solutions are discussed. The methods of indirect determination of other mechanical quantities, such as forces, moments, etc., are analysed. Special attention is paid to the construction and operation principle of converters which are based on the strain gauges. In the end, practical examples realized within the Laboratory of Railway Vehicles Centre at the Faculty of Mechanical and Civil Engineering in Kraljevo, are exposed. The known fact that strain gauges will for a long time represent the leading sensors in the field of measuring technique, is confirmed.

*Keywords*— Strain gauges, Testing of Mechanical Structures, Wheatstone bridge, Converters.

#### I. INTRODUCTION

The modern way of development of mechanical structures is based on the application of theoretical and experimental methods [1, 2]. Theoretical methods are mainly based on the application of analytical or numerical calculations which in the most cases enable complete definition of product being developed. Software packages for modeling and calculation are so much developed today and provide fantastic opportunities. However, engineers are very often (even in the product development phase) faced with the need for practical verification of the results obtained using analytical or numerical methods. For this purpose, it is necessary to carry out certain experimental tests on a real object, most often on a prototype of the product being developed. In any case, experimental tests are most often inevitable in the certification stage. For the most complex mechanical structures those tests are prescribed and defined by international standards. For example, in the case of railway vehicles in the certification phase experimental tests of the carrying structure, impact tests, brake tests, running quality and safety tests, etc., should be carried out [3]. In this sense, one of the simplest sensors that provides enormous possibilities in experimental testing of mechanical structures is the strain gauge [4-6]. In 1936, these sensors were found and in 1942 they were patented by American engineer Edward Simons [7]. Their primary purpose is to measure strains, but they can indirectly be used to measure mechanical stresses, forces, moments, as well as

other sizes that can be associated with the strains. For these reasons, strain gauges are today the most used sensors in experimental tests of the structures. In this paper, the basic postulates related to strain gauges are presented, and some implemented solutions of their application in tests carried out by the Laboratory of Railway Vehicle Center at the Faculty of Mechanical and Civil Engineering in Kraljevo, are presented.

### II. BASIC PRINCIPLES

The strain gauge is a conductor (sensor) with defined resistance that is fixed (glued) to the surface of the object being tested. Each deformation of the tested object due to the load causes an appropriate deformation of the strain gauge, which leads to a change in the electrical resistance in it.



Fig. 1 Deformations of the structure at the action of tensile force

At the action of tensile force on the structure shown in Fig. 1, the relative longitudinal strain is:

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$$S = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}$$
(1)

In addition, the ratio of the transverse and longitudinal strain (Poisson's ratio) is:

$$\mu = -\frac{\varepsilon_{tr}}{\varepsilon} \tag{2}$$

It is known that for steel the Poisson's ratio is approximately 0.3. The transverse deformation is:

$$\varepsilon_{tr} = \frac{\Delta d}{d_0} = \frac{d - d_0}{d_0} \tag{3}$$

If measured strain of the structure is marked with  $\varepsilon$  (as indicated above), resistance of the strain gauge glued to the structure with *R*, and the change in the resistance due to the load (force *F*) with  $\Delta R$ , there is the following correlation between these sizes:

$$\frac{\Delta R}{R} = k \cdot \varepsilon \tag{4}$$

In the previous expression k represents the so-called "k factor" or factor of the strain gauge. Under normal conditions of testing of mechanical elements and structures this factor has a value  $k = 1.8 \div 2.2$ . In analyzed case, there is a uni-axial stress state when the strain gauge is always oriented in the direction of the maximum stress  $\sigma_{max}$ , or in the direction of the action of tensile or pressure load. On the basis of measured strain, the stress is determined by its multiplication with the modulus of elasticity *E*, that is, from the Hooke's law is:

$$\sigma = E \cdot \varepsilon \tag{5}$$

If it's a plane stress state, when the directions of major stresses  $\sigma_1$  and  $\sigma_2$  are known (they occur in two normal directions), the strain gauges should be glued along these directions. On the basis of measured strains, the stresses are determined from generalized Hooke's law as following:

$$\sigma_1 = \frac{E}{\left(1 - \mu^2\right)} (\varepsilon_1 + \mu \varepsilon_2) \tag{6}$$

$$\sigma_2 = \frac{E}{\left(1 - \mu^2\right)} (\varepsilon_2 + \mu \varepsilon_1) \tag{7}$$

Based on the obtained stresses, it is possible to calculate the comparative stress on the basis of which the conclusion on the condition of the tested structure is made:

$$\sigma_i = \sqrt{\sigma_1^2 - \sigma_1 \cdot \sigma_2 + \sigma_2^2} \tag{8}$$

If the directions of major stresses are unknown (which is the most common case in the testing of mechanical structures), the measurement of strains is done in three different directions using independent strain gauges, or fish-bone strain gauges (so called rosettes), are used. On the basis of the measured strains in three different directions (Fig. 2), it is possible to determine the major stresses and the comparative or ideal stress according to the methods which are known from the resistance of materials and the theory of elasticity [2]. Accordingly, it is known that the strains in the given directions are [2]:

$$\varepsilon_a = \varepsilon_x \cdot \cos^2 \varphi_a + \varepsilon_y \cdot \sin^2 \varphi_a + \gamma_{xy} \cdot \sin \varphi_a \cdot \cos \varphi_a \quad (9)$$

$$\varepsilon_b = \varepsilon_x \cdot \cos^2 \varphi_b + \varepsilon_y \cdot \sin^2 \varphi_b + \gamma_{xy} \cdot \sin \varphi_b \cdot \cos \varphi_b \quad (10)$$

$$\varepsilon_c = \varepsilon_x \cdot \cos^2 \varphi_c + \varepsilon_y \cdot \sin^2 \varphi_c + \gamma_{xy} \cdot \sin \varphi_c \cdot \cos \varphi_c \quad (11)$$



Fig. 2 Example of measurement of strains in three different directions

It is very important to emphasize that the angles of the chosen directions  $\varphi_a$ ,  $\varphi_b$  and  $\varphi_c$  can be arbitrary and they are measured in a positive mathematical direction with respect to the X axis (Fig. 2). Therefore, if at some point of the tested structure there are strain gauges in certain directions, dilatations  $\varepsilon_a$ ,  $\varepsilon_b$  and  $\varepsilon_c$  can be measured, and, from the equations (9–11) the unknown sizes of strains  $\varepsilon_x$  and  $\varepsilon_y$  and slipping  $\gamma_{xy}$  can be determined. On the basis of this, the values of the major strains can be determined:

$$\varepsilon_{1,2} = \frac{1}{2} \left( \varepsilon_x + \varepsilon_y \right) \pm \frac{1}{2} \sqrt{\left( \varepsilon_x - \varepsilon_y \right)^2 + \gamma_{xz}^2}$$
(12)

The direction of the first major strain (stress) is determined by the following angle measured from X axis in the positive mathematical direction:

$$\theta = \frac{1}{2} \operatorname{arctg} \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \tag{13}$$

The angle of inclination of the second major strain is  $\theta + \pi/2$  and is also measured from X axis in a positive mathematical direction. Based on determined major strains, the values of major normal stresses  $\sigma_1$  and  $\sigma_2$  can be determined from expressions (6) and (7). The value of the tangential stress is determined by the product of the slip modulus *G* and sliding  $\gamma_{xy}$ :

$$\tau_{\max} = G \cdot \gamma_{xy} = \frac{E}{2 \cdot (1 + \mu)} \cdot \gamma_{xy} \tag{14}$$

## III. PRINCIPLE OF CONNECTING OF STRAIN GAUGES

The relative changes in the resistance of the strain gauge are very small sizes as well as the strains being measured. That is why no direct measurements of resistance change are applied in practice, but Wheatstone bridges are used. Wheatstone bridge is an electric circuit intended for accurate measurement of the electric resistance, in this case changes in the resistance of strain gauges. It consists of: 4 resistors connected to form a rectangle, power supply (voltage  $U_E$ ), and a measuring instrument that measures the output voltage of bridge  $U_A$ (Figs. 3–5). The principle of applying of balanced Wheatstone bridges is based on the fact that the output voltage of the bridge  $U_A=0$  when the products of resistance of opposite branches of the bridge are the same:

$$R_1 \cdot R_3 = R_2 \cdot R_4 \tag{15}$$

In the unbalanced Wheatstone bridges, for certain values of  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$  (R is resistance for all four identical strain gauges) and voltage  $U_E$ , the output voltage

of the bridge  $U_A$  is calculated according to a specific expression [2]:

$$U_{A} = U_{E} \cdot \frac{\frac{\Delta R_{1}}{R} + \frac{\Delta R_{3}}{R} - \frac{\Delta R_{2}}{R} - \frac{\Delta R_{4}}{R}}{4}$$
(16)

At measurements with unbalanced Wheatstone bridges, output voltage  $U_A$  is affected by the changes in the resistance of all strain gauges in the bridge (increase of resistance of the resistors  $R_1$  and  $R_3$  increases the output voltage  $U_A$ , while the increase of resistance of the resistors  $R_2$  and  $R_4$  reduces the output voltage  $U_A$ . Consequently, the output voltage  $U_A$  increases if the strain gauges  $R_1$  and  $R_3$  are positioned in the direction of tension, and the strain gauges  $R_2$  and  $R_4$  in the direction of compression. In measurement of strain by an unbalanced bridge, an error is up to 3%, and is caused by various parameters such as non-linearity of the voltage, cable resistance, humidity, etc. It is important to emphasize that connecting the strain gauges in Wheatstone bridge allows compensation of temperature or other impacts that represent disturbances in measurement.





By combining active strain gauges, passive strain gauges and resistors, it is possible to realize the three types of configurations of Wheatstone bridges. At full bridge (Fig. 3), all four strain gauges are active - strain gauges with resistances  $R_1$  and  $R_3$  are positioned in the direction of measured strain, and gauges  $R_2$  and  $R_4$  are placed in the direction normal to measured strain. This configuration has the highest sensitivity and stability. It has the highest price due to the using of 4 strain gauges, enable temperature compensation and is used for dynamic measurements. At half-bridge configuration (Fig. 4) there are two active strain gauges  $-R_1$  in the direction of measured strain and  $R_2$  in the direction normal to measured strain. It has a double smaller sensitivity and the price is twice as low relative to the full bridge, and allows temperature compensation. The quarter-bridge configuration (Fig. 5) has one active strain gauge placed in the direction of measured strain. It has about 30% less sensitivity and the price is lower relative to the half-bridge, and doesn't allow temperature compensation without an additional passive strain gauge.

All mentioned characteristics enable the wide application of strain gauge in the field of testing of mechanical structures. On the one hand, this application is related to the testing of stress-strain states at certain points of structures, and on the other hand for the development of measuring converters. The next chapter shows few performed solutions of testing of mechanical structures using the strain gauges that have been realized within the Laboratory of Railway Vehicles Centre at the Faculty of Mechanical and Civil Engineering in Kraljevo.



Fig. 5 Quarter-bridge configuration

#### IV. EXAMPLES OF PERFORMED SOLUTIONS

## A. Testing of stress-strain state of supporting structure of railway vehicle

These tests were carried out in cooperation with the Test Center of the Wagon Factory Kraljevo at a special test stand in this factory (Fig. 6). The data acquisition system UPM 100 is used and strains are measured in about 100 measuring points. Each measuring point is composed of one half-bridge configuration with one active strain gauge in direction of maximal stress and one passive strain gauge for compensation.



Fig. 6 Testing of supporting structure of railway vehicle [2]

## B. Testing of stress-strain state of box girder

These tests are carried out with the aim of verification of analytical and numerical model for identification of stress-strain state of box girder (Fig. 7) [8]. The data acquisition system HBM QUANTUM MX 840A is used. Measurements were carried out at 4 measuring points with one half-bridge configuration in order to temperature compensation.



Fig. 7 Testing of box girder [8]

#### C. RB converter for measurement of forces on buffers

This converter, intended for measurement of forces on the buffers at testing of wagons impact, has been developed in cooperation with the Test Center of the Wagon Factory Kraljevo. Its design, layout and way of connection of strain gauges are shown in Fig. 8 [2].

#### D. Instrumented wheelset

This converter has been developed in cooperation with company OSS and serves for measurement of wheel-rail contact forces in process of certification of railway vehicles. Its design with layout and way of connection of strain gauges are shown in Fig. 9.



Fig. 8 Design of RB converter with layout and way of connection of strain gauges [2]



Fig. 9 Design of instrumented wheelset with layout and way of connection of strain gauges [9]

## E. H converter

This converter (Fig. 10) has also been developed in cooperation with company OSS and serves for measurement of forces at the height of axle-box bearings, also in process of certification of railway vehicles.



Fig. 10 Design of H converter

#### V. CONCLUSION

The paper deals with application of strain gauges in experimental testing of mechanical structures. The basic principles of work and connection of strain gauges are analyzed. A few practical examples of application of strain gauges are shown. It can be concluded that strain gauges will be for a long time leading sensors in experimental testing of mechanical structures. Especially is important to emphasize their importance in development and production of special converters where they provide a huge possibilities.

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