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ANALYSIS OF METHODS FOR DETERMINING OF IMPACT FORCES AT CROSSING OF WHEEL OVER RAIL IRREGULARITIES

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Abstract *– During the exploitation, the rail is exposed to intensive forces caused by the trains passing. These forces lead to the wear and non-uniform damages of the rail head. When the wheels pass over these places, sudden change of rail geometry causes very intensive impact forces which are transmitted to the track and railway vehicle. The consequences of action of these dynamic forces are very often fatigue and failure of elements of vehicles and tracks which in many cases cause derailments with enormous consequences. From this reason, these problem is very actual in the field of research of dynamic behavior of railway vehicles. Hence, the aim of this paper is to analyze some of the existing methods for determination of impact forces at crossing of railway wheel over the rail irregularities. This initial research of authors should draw attention of wide scientific and professional auditorium on this problem and to give base for further research and improvements in this area.*

Keywords – Impact forces, wheel-rail contact, track irregularities.

1. INTRODUCTION

During the running of railway vehicles there are intensive impact forces due to the various irregularities of the track. When the wheels pass over these places, sudden change of rail geometry causes very intensive impact forces which are transmitted to the track and railway vehicle. Due to the significant influence on the safety on railway, this problem is very actual in the field of dynamic of railway vehicles [1, 2]. The subject of the state of the art papers in this field is usually concerned to the determination of impact forces at dipped rail joints and modeling of discontinuities in the wheel-rail contact [3, 4]. The very actual issue is also research of the damages of the rails under fatigue of material, improper maintenance, etc [5]. The intensity of impact forces is proportional to the increase of geometric abnormalities of the track, the stiffness of the rails, the running speed, uneven distribution of the cargo on the vehicle, etc. The next chapters show analysis of some existing ways for determination of mentioned impact forces and for studying of this phenomena in general.

2. MODELING OF WHEEL-RAIL CONTACT

The dynamic model of moving the wheel along the rail is shown in Fig. 1. This model is usually used for describing the single point wheel-rail contact. The differential equation which describes this motion has the following form [3]:

$$m_w \dot{u}(t) + k_h \cdot u(t) = k_h \cdot z(t) \tag{1}$$

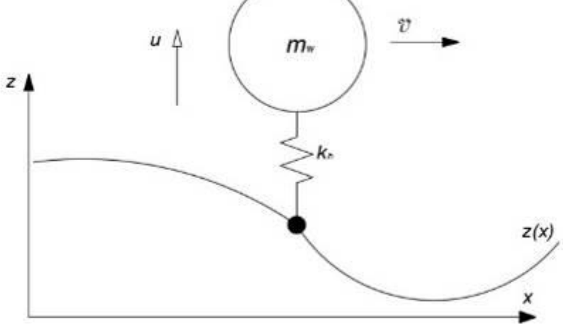

Fig.1. The modelling of continuous single point wheel-rail contact [3]

In equation (1) are: m_w – mass per wheel, u – the

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degree of freedom in the vertical direction, k_h – Hertz's rigidity on contact, z – vertical movement along the apripriate axis.

However, given model is not applicable in the analysis of crossing of wheel over rail irregularities. For these analysis, the wheel-rail contact in two points must be used. In this aim, we will observe the rail and the wheel as two rigid bodies. The wheel of radius R is moving with certain velocity ϑ and crossing over the step with height u_0 (Fig. 2). The distance a represents the minimum length at which the wheel is in contact with the rail at two points. Also, we assume that $u_0 << R$.

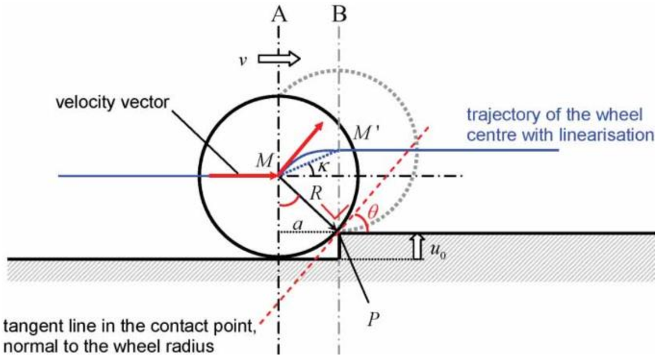

Fig.2. The kinematical wheel displacement for a vertical step in the rail surface [3]

The intensity of vertical velocity at wheel moves from point A to point B is [3]:

$$\dot{u}(0) = \vartheta \cdot \sin\theta = \vartheta \cdot \sqrt{\frac{2u_0}{R}} \tag{2}$$

The time required for the wheel transition from position A to position B is:

$$t_B = \frac{\sqrt{2Ru_0}}{\vartheta} \tag{3}$$

The equation which represents the movement of the wheel in vertical direction is [3]:

$$z(t) = \sqrt{\frac{u_0}{2R}}\vartheta t \cdot H(t_B - t) + u_0 \cdot H(t - t_B) \tag{4}$$

The previous equation (where H is Heaviside function) establishes the relation between the height of the step, radius of the wheel and running speed.

3. DETERMINATION OF IMPACT FORCES

3.1. Standard approach

For analysis and analytical determination of impact forces, we will consider situation when the wheel with a high velocity passes over the damaged rail (Fig. 3). In the moment of transition of the wheel from the rail 1 to the rail 2, intensive shock and high frequency vibrations occur. This situation can be modeled with the system with two degrees of freedom, as shown in Fig. 4. The wheel with mass m_w moves at a constant speed ϑ as it passes on the rail at an angle 2α. Here k_t represents rail stiffness which is in range 50÷200 MN/m, while k_h is Hertz's contact stiffness which is usually increasing and amounts about 1000 MN/m [1]. The impact force P_1 occurs at the moment when the wheel hits the bottom of the irregularity. This force has very short duration and it is caused by existing of some elasticity in the wheel-rail contact, which is taken into account with the Hertz's contact stiffness.

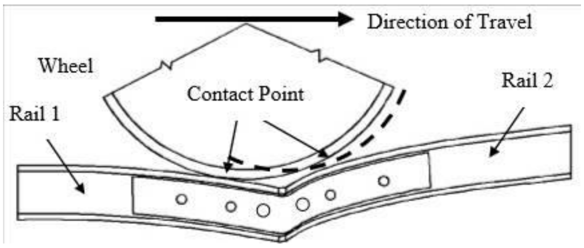

Fig.3. The wheel that passes over damaged rail [3]

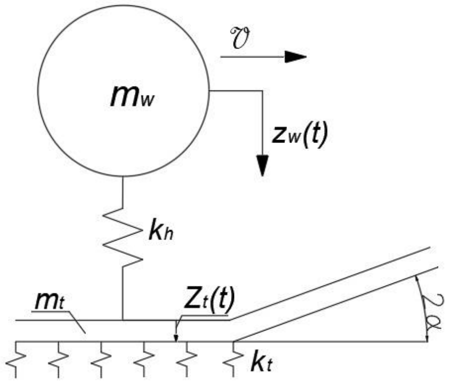

Fig.4. The model of wheel that passes over damaged rail [1]

The impact force P_2 occurs at the moment when the wheel come across to the curve with an sudden change of the rail geometry. Its duration is for a few milliseconds longer regard to P_1 (Fig. 5).

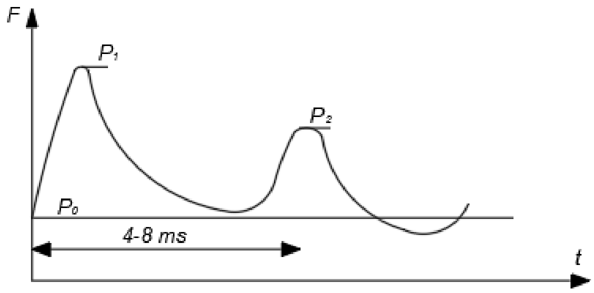

Fig.5. The impact forces P_1 and P_2

It should be noted that the force P_1 is caused by inertia of the rail and connection elements, while the force P_2 is caused by the rejection from the rail (P_0

is static load which wheel generates on the rail). The force P_1 cause a frequency in range 200÷1000 Hz, while the force P_2 cause a somewhat lower frequency in range 50÷200 Hz [3].

With assumption that $k_t << k_h$, the eigenfrequency of the rail and the wheel is [1]:

$$\omega_{P1} = \sqrt{\frac{k_h(m_w + m_t)}{m_w \cdot m_t}} \tag{5}$$

The impact force is calculated with equation [1]:

$$\Delta P_1 = k_h(z_w + z_t) \approx k_h z_w \tag{6}$$

If the initial vertical movement is equal to zero, then we can calculate the vertical velocity with the following formula:

$$\dot{z}_w(t = t_1) = \vartheta \cdot tg(2\alpha) \approx 2\alpha\vartheta \tag{7}$$

By using equations (5) and (7) we can write equation for vertical movement of the wheel [1]:

$$z_w(t) = \frac{\dot{z}_w(t_1)}{\omega_{P1}} \sin(\omega_{P1} t) \tag{8}$$

The change of equation (8) in equation (6) gives the equation for maximum impact force P_1:

$$P_1 = P_0 + k_h z_{w,\max} = P_0 + 2\alpha\vartheta \sqrt{\frac{k_h m_t}{1 + \frac{m_w}{m_t}}} \tag{9}$$

With assumption that $k_h = \infty$, the expression for eigenfrequency is:

$$\omega_{P1} = \sqrt{\frac{k_t}{m_w + m_t}} \tag{10}$$

The equation for the second force is [1]:

$$\Delta P_2 = -m_w \cdot \ddot{z}_w(t) \tag{11}$$

The impact force P_2 is:

$$P_2 = P_0 + 2\alpha\vartheta \sqrt{\frac{k_t m_w}{1 + \frac{m_t}{m_w}}} \tag{12}$$

The equations (9) and (12) are very significant for further considerations and research of influences of stiffness, mass, velocity, etc.

Despite the simplicity, this model gives clear identification of key problems in crossing of the wheel over rail irregularities.

3.2. Steenbergen's approach

This approach implies that during the wheel crossing over the rail irregularity the change of the speed and the resulted impact are happening only in some finite interval. Also, plasticity plays an important role.

For the case from Fig. 2, with the assumption of vertical linear movement and the wheel-rail contact in two points, the differential equation of system is [4]:

$$m_w \ddot{u}(t) + k_h \cdot u(t) = m_w \vartheta_0 \delta(t) \tag{13}$$

In the equation (13), δ represents the Dirac delta function while ϑ_0 can be calculated from formula (2). From equation (13), expression for calculation the amplitude can be obtained [4]:

$$\hat{u} = \vartheta \sqrt{\frac{2 m_w u_0}{k_h R}} \tag{14}$$

The equation for determination of impact force is [4]:

$$\hat{F} = \vartheta \sqrt{\frac{2 u_0 k_h m_w}{R}} \tag{15}$$

3.3. Simplified approach

Lately there are methods with a new simplified equations for determination of impact forces at crossing of wheel over the rail irregularities. One of the most important is method of Mandals et al. The dynamic of cargo semi-wagons that passes over the track irregularities of different depths with different speeds is analyzed (Fig. 6) [3].

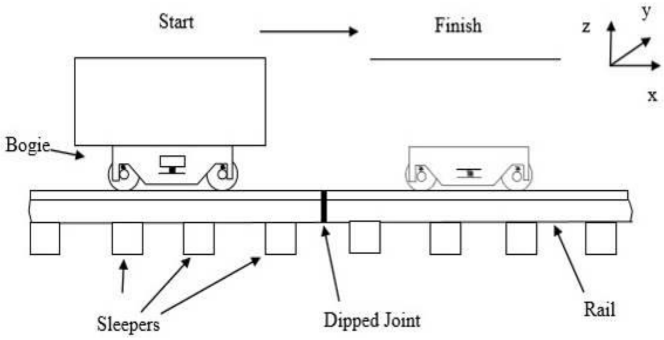

Fig.6. The dynamic wheel-rail interaction of a half-wagon model on the symmetrical dipped joint [3]

For this analysis, 1719 degrees of freedom is used. Research is conducted for speeds of 25 km/h, 50 km/h, 75 km/h and 100 km/h, as well as for irregularities with deep of 0.4 mm, 0.8 mm and 1.2 mm. The approach is based on the fact that forces P_1 and P_2 have linear relation which depends from the coeficient of corelation X.

The equations for determination of impact forces are [3]:

$$P_1 = 13X + 1,05 \quad (16)$$

$$P_2 = 6X + 1,05 \quad (17)$$

The correlation between the dynamic impact forces and non-dimensional coefficient X is shown in diagram in Fig. 7.

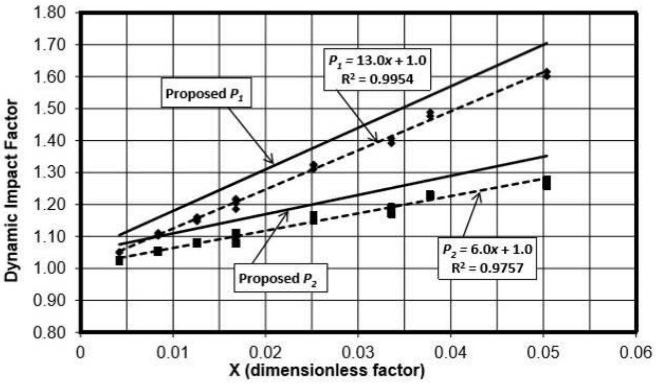

Fig.7. The non-dimensional expresions for P_1 and P_2 [3]

4. IMPORTANCE OF FATIGUE OF RAIL JOINTS

A lot of railway disasters in the period between 1996 and 2002 are caused necessity for research of influence of fatigue of rail joints. These research are usually concerned to analyzing of phisical influences of the railway wheels on the rail joints during the train passing. The estimation of bending stresses on the place of rail joints under the wheel load is usually based on the FEM (finite element method) analysis.

For rail type 132RE, bending moment can be calculated from following formula [5]:

$$M_R(x) = \frac{P}{4\lambda}\left[\exp(-\lambda x)(\cos(\lambda x) - \sin(\lambda x)\right] \quad (18)$$

In the previous expression, P represents the dynamic load of the wheel, while λ (constant) is defined with the following relation:

$$\lambda = \sqrt[4]{\frac{k_v}{4EI_R}} \quad (19)$$

In the expression (19) are: k_v – rail stiffness, E– modulus of elasticity, I_R – moment of inertia.

Accordingly, the maximum bending moment on the rail joints can be calculated with the following formula [5]:

$$M_J = \beta \frac{P_2}{4\lambda} \quad (20)$$

It can be noticed that in this expression the factor of joint efficiency β plays important role. The laboratory and field tests indicate that this factor depends on the condition of the joint. For a good joint it needs to be in the interval 0.6÷0.8 [5].

The factor β can be calculated from the following expression:

$$\beta_{\max} = \left(\frac{M_J}{M_0}\right)_{\max} = \sqrt[4]{\frac{I_J}{I_R}} \quad (21)$$

In the previous expression, I_J and I_R are the moments of inertia of the rail joint and rail.

The maximum bending stress of the upper part of the rail in case when wheel passes directly over the rail joint and when it is at a certain distance, can be determined by using the following equations [5]:

$$S_{J-} = \frac{M_J c_J}{I_J} \quad (22)$$

$$S_{J+} = \frac{\beta M_R(x_{rb}) c_J}{I_J} \quad (23)$$

5. CONCLUSION

This paper analyzes some of the existing methods for determination of impact forces at crossing of railway wheel over the rail irregularities. The impact forces which occur when wheel passes over the rail irregularities can be even 3 times larger than static load. In many cases these forces are caused catastrophic scenarios and derailments on the railway.

This initial research of authors in this field should draw attention of wide scientific and professional auditorium on this problem. Besides, it should provide the proper base and motivation for further research and improvements in this area with the main aim of enhancing of safety on the railway.

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