

## Numerical Simulation of Wagons Impact

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*Impact of wagons and change in moving tempo cause very intensive longitudinal forces and accelerations that affect the stress-strain state of the supporting structures of wagons. The magnitudes of impact loads can be such to constitute a risk to the support structure, and therefore for passengers and cargo. This paper is focused on an accurate analysis of the impact of two wagons. The analytical formulation of wagons impact is established and equations are solved numerically. The obtained results are compared with results of experimental testing of wagons impact. Comparative analysis of numerical and experimental results has shown satisfactory agreement. In that way, the formed analytical model can be used for determining the dynamic parameters of the impact in the design of new wagons.*

**Keywords:** Numerical simulation, Impact, Railway vehicle, Wagon

### 0. INTRODUCTION

Generally, the construction of wagon consists of body, underframe, bogies, buffers, braking system, and various additional parts and equipment. Impact causes the dynamic parameters such as forces and accelerations, whose changes and maximum values depend on the speed of the impact and the characteristics of the mentioned sub-assemblies, as well as the type of cargo and its fixing for the supporting structure of wagon.

Such a complex system with many degrees of freedom is replaced with the simpler theoretical models with a limited number of degrees of freedom. In the formation of mathematical models the following restrictions are usually introduced: the construction of wagons and bogies, as well as cargo, are considered to be absolutely rigid bodies, the railway track is horizontal, centres of masses of cargo and wagons move in parallel, i.e. there is no relative movement between the cargo and wagons.

Experimental testing has shown that these simplifications lead to the insufficiently accurate results in analysis of wagons impact. The most frequently used models of wagons impact do not take into account the influence of cargo movement during the impact, gaps in assemblies of wagons, influence of the loss of energy during the impact, etc [1, 2].

In order to determine the above-mentioned influences on the values of forces on the buffers during the impact of two wagons, more accurately and somewhat more complex model has been formed [3].

### 1. THEORETICAL ANALYSIS OF IMPACT OF TWO WAGONS

Consider the case of two wagons impact with buffers stiffness  $c_1$  and  $c_2$ , and stiffness of underframes  $c_{ns1}$  and  $c_{ns2}$ . Wagon whose mass is  $m_1$  is moving with the speed  $\vec{v}_1$  and collides with wagon of mass  $m_2$  which moves with the speed  $\vec{v}_2$ . Wagons are loaded with cargoes whose masses are  $m_3$  and  $m_4$ . This is resulted with the relative movement of masses over the wagons for values  $\vec{x}_3$  and  $\vec{x}_4$ . In addition, between the wagons and cargoes there are elastic connections with stiffness  $c_3$  and  $c_4$ .

Apart from that, the relative movements of cargoes with masses  $m_3$  and  $m_4$  are opposed by forces of dry friction and forces of viscous friction. These forces are proportional to the first degree of speeds of relative movements of cargoes  $\vec{x}_3$  and  $\vec{x}_4$ . Also, the movements of the first and the second wagon are opposed by forces of rolling friction  $\mu_1 \cdot g \cdot (m_1 + m_3)$  and  $\mu_2 \cdot g \cdot (m_2 + m_4)$ .

The process of impact is observed from the moment when buffers of two wagons touch each other. All buffers have the same elastic properties, so that the total buffers movement is  $2\Delta\ell$ .

Since the impact of wagons can be considered as event in an isolated system where the forces, speeds, accelerations or other dynamic sizes of the wagons and cargoes are in the same direction before and after the impact, they can be written without vector label.

Equations for kinetic energy  $E_k$ , potential energy  $E_p$  and dissipation function  $\Phi_r$  of the system on Fig.1 are

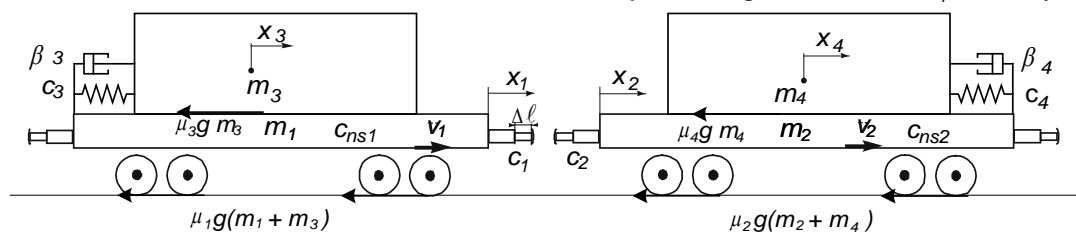


Figure 1: The impact of two wagons when the movement of cargo is present

[4]:

$$\begin{aligned}
 E_k &= \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 (\dot{x}_3 + \dot{x}_1)^2 + \frac{1}{2} m_4 (\dot{x}_4 + \dot{x}_2)^2 \\
 E_p &= \frac{1}{2} c_s (x_2 - x_1)^2 + \frac{1}{2} c_3 x_3^2 + \frac{1}{2} c_4 x_4^2 \\
 \Phi_r &= \mu_1 (m_1 + m_3) g |\dot{x}_1| + \mu_2 (m_2 + m_4) g |\dot{x}_2| + \\
 &\quad + \mu_3 m_3 g |\dot{x}_3| + \mu_4 m_4 g |\dot{x}_4| + \frac{1}{2} \beta_3 \dot{x}_3^2 + \frac{1}{2} \beta_4 \dot{x}_4^2
 \end{aligned} \quad (1)$$

where:

- $c_s$  – equivalent stiffness of the system at the moment of impact,  
 $\mu_1, \mu_2, \mu_3$  and  $\mu_4$  – coefficients of dry friction,  
 $\beta_3$  and  $\beta_4$  – dynamic viscosities that define the environment resistance,  
 $g$  – acceleration due to gravity.

As the system is influenced by the conservative and dissipative forces, the Lagrange equations of second kind have the following form:

$$\frac{d}{dt} \frac{\partial E_k}{\partial \dot{q}_i} + \frac{\partial \Phi_r}{\partial \dot{q}_i} + \frac{\partial E_p}{\partial q_i} = 0 \quad (2)$$

By changing the equations for kinetic and potential energy into the previous equations, as well as changing the equation for dissipation function, we get the following differential equations:

$$\begin{aligned}
 (m_1 + m_3) \ddot{x}_1 + m_3 \ddot{x}_3 + c_s x_1 - c_s x_2 + \mu_1 (m_1 + m_3) g \cdot \text{sign} \dot{x}_1 &= 0 \\
 (m_2 + m_4) \ddot{x}_2 + m_4 \ddot{x}_4 + c_s x_2 - c_s x_1 + \mu_2 (m_2 + m_4) g \cdot \text{sign} \dot{x}_2 &= 0 \\
 m_3 \ddot{x}_3 + m_3 \ddot{x}_1 + \beta_3 \dot{x}_3 + c_3 x_3 + \mu_3 m_3 g \cdot \text{sign} \dot{x}_3 &= 0 \\
 m_4 \ddot{x}_4 + m_4 \ddot{x}_2 + \beta_4 \dot{x}_4 + c_4 x_4 + \mu_4 m_4 g \cdot \text{sign} \dot{x}_4 &= 0
 \end{aligned} \quad (3)$$

The system of differential equations (3) defined in this way takes into account both the movement of cargoes and the influence of friction that occurs between the individual sub-assemblies when two wagons collide, and it is suitable for numerical solution.

Also, the function signum (sign) allows the determination of the sign of friction forces that depends on the speeds of the movement of wagons and cargoes.

In order to preparation for numerical solution of system of differential equations (3), the changing of the third equation in the first one, and the fourth equation in the second one, is performed:

$$\begin{aligned}
 \ddot{x}_1 &= a_1 (x_2 - x_1) + a_2 x_3 + a_9 \dot{x}_3 - a_{10} \text{sign} \dot{x}_1 + a_{13} \text{sign} \dot{x}_3 \\
 \ddot{x}_2 &= a_3 (x_1 - x_2) + a_4 x_4 + a_5 \dot{x}_4 - a_{11} \text{sign} \dot{x}_2 + a_{14} \text{sign} \dot{x}_4
 \end{aligned} \quad (4)$$

By changing the first equation (4) in the third equation (3) and second equation (4) in the fourth equation (3) we obtain the following equations:

$$\begin{aligned}
 \ddot{x}_3 &= a_1 (x_1 - x_2) - (a_2 + a_{12}) x_3 - (a_9 + a_7) \dot{x}_3 + \\
 &\quad + a_{10} \text{sign} \dot{x}_1 - (a_{13} + a_{15}) \text{sign} \dot{x}_3 \\
 \ddot{x}_4 &= a_3 (x_2 - x_1) - (a_4 + a_8) x_4 - (a_5 + a_6) \dot{x}_4 + \\
 &\quad + a_{11} \text{sign} \dot{x}_2 - (a_{14} + a_{16}) \text{sign} \dot{x}_4
 \end{aligned} \quad (5)$$

The constants  $a_1 \div a_{16}$  are given in the following Table 1.

Table 1: The values of the constants  $a_1 \div a_{16}$

$a_1$	$a_2$	$a_3$	$a_4$
$c/m_1$	$c_3/m_1$	$c/m_2$	$c_4/m_2$
$a_5$	$a_6$	$a_7$	$a_8$
$\beta_4/m_2$	$\beta_4/m_4$	$\beta_3/m_3$	$c_4/m_4$
$a_9$	$a_{10}$	$a_{11}$	$a_{12}$
$\beta_3/m_1$	$\mu_1 \frac{m_1 + m_3}{m_1} g$	$\mu_2 \frac{m_2 + m_4}{m_2} g$	$c_3/m_3$
$a_{13}$	$a_{14}$	$a_{15}$	$a_{16}$
$\mu_3 \frac{m_3}{m_1} g$	$\mu_4 \frac{m_4}{m_2} g$	$\mu_3 g$	$\mu_4 g$

This system, however, does not take into account the reduction of impact forces due to the loss of energy in oscillating structures of wagons during the impact. This effect can be taken into account by using Newton's coefficient of restitution. However, when it comes to the railway vehicles, the coefficient of restitution at impact can be determined only by the experimental testing [3, 5].

By introducing the following sizes:

$$\begin{aligned}
 y_1 &= x_1, & y_2 &= \dot{x}_1, & y_3 &= x_2, & y_4 &= \dot{x}_2, \\
 y_5 &= x_3, & y_6 &= \dot{x}_3, & y_7 &= x_4, & y_8 &= \dot{x}_4
 \end{aligned} \quad (6)$$

the final form of differential equations of two wagons impact is obtained:

$$\begin{aligned}
 \ddot{x}_1 &= a_1 (y_3 - y_1) + a_2 y_5 + a_9 y_6 - a_{10} \text{sign} y_2 + a_{13} \text{sign} y_6 \\
 \ddot{x}_2 &= a_3 (y_1 - y_3) + a_4 y_7 + a_5 y_8 - a_{11} \text{sign} y_4 + a_{14} \text{sign} y_8 \\
 \ddot{x}_3 &= a_1 (y_1 - y_3) - (a_2 + a_{12}) y_5 - (a_7 + a_9) y_6 + \\
 &\quad + a_{10} \text{sign} y_2 - (a_{13} + a_{15}) \text{sign} y_6 \\
 \ddot{x}_4 &= a_3 (y_3 - y_1) - (a_4 + a_8) y_7 - (a_5 + a_6) y_8 + \\
 &\quad + a_{11} \text{sign} y_4 - (a_{14} + a_{16}) \text{sign} y_8
 \end{aligned} \quad (7)$$

Such defined system of differential equations takes into account the movement of cargoes during the wagons impact, and is suitable for solution in numerical way.

## 2. NUMERICAL SIMULATION OF IMPACT OF TWO WAGONS

Based on the formed system of differential equations which describe the dynamic process of the impact of two wagons, a program for their solution is made. It is important to note that method of Runge-Kutta of IV level and programming language Fortran 77 were used [6, 7]. Based on the underlying methodology, the process of two wagons impact is simulated whereby model enables calculations with and without movement of cargo during the impact.

Dynamic parameters that were followed during the impact are:

- $t$  – time,  
 $x_1$  – movement of the wagon that hits into the other one,  
 $\dot{x}_1$  – speed of the wagon that hits into the other one,  
 $\ddot{x}_1$  – acceleration of the wagon that hits into the other one,  
 $x_2$  – movement of the wagon that is being hit,  
 $\dot{x}_2$  – speed of the wagon that is being hit,

- $\ddot{x}_2$  – acceleration of the wagon that is being hit,  
 $x_3$  – movement of the cargo on the wagon that hits into the other one,  
 $\dot{x}_3$  – speed of the cargo on the wagon that hits into the other one,  
 $\ddot{x}_3$  – acceleration of the cargo on the wagon that hits into the other one,  
 $x_4$  – movement of the cargo of the wagon that is being hit,  
 $\dot{x}_4$  – speed of the cargo of the wagon that is being hit,  
 $\ddot{x}_4$  – acceleration of the cargo of the wagon that is being hit,  
 $F_u$  – the total force on the buffers.

Total force on the buffers in period until the complete compaction of buffers is:

$$x_1(t) - x_2(t) \leq 2\Delta l; \quad F_u(t) = F^o \quad (8)$$

where  $F^o$  is the force which occurs when buffers are completely compacted.

Total force on the buffers in period of solid impact occurs when buffers are completely compacted:

$$x_1(t) - x_2(t) > 2\Delta l; \quad F_u(t) = F_u \quad (9)$$

Consequently, the mathematical model of change of force during the impact is defined by the following equation:

$$F_u(t) = \begin{cases} c_I [x_1(t) - x_2(t)]^{n_1} & ; x_1(t) - x_2(t) \leq 2\Delta l \\ c_I 2\Delta l^{n_1} + c_{II} [x_1(t) - x_2(t) - 2\Delta l]^{n_2} & ; x_1(t) - x_2(t) > 2\Delta l \end{cases} \quad (10)$$

In the above expressions, non-linearity between the force and the displacement is taken into account with coefficients  $n_1$  and  $n_2$ . In the case of linear dependence between the force and displacement, these coefficients are equal 1.

### 3. ANALYSIS OF RESULTS OF NUMERICAL SIMULATION

The numerical calculation of dynamic parameters of impact is carried out for three types of wagons: Tadmss-z (Fig. 2), Uacns-z (Fig. 3), and Hccrrs-z (Fig. 4). These wagons are previously experimentally tested.



Figure 2: The wagon Tadmss-z for transportation of grain materials



Figure 3: The wagon Uacns-z for transportation of powdered materials



Figure 4: The wagon Hccrrs-z for transportation of cars

During the first period of impact, the overall stiffness of the system is  $c_s = c_I$ , while in the second period of rigid impact, the stiffness of the system is  $c_s = c_{II}$ . Here, a certain approximation is introduced, namely, the characteristics of stiffness can be considered linear only in static and quasi-static loads. However, at the dynamic load, the parameters of stiffness are nonlinear.

The experimental determination of the dynamic stiffness  $c_I$  and  $c_{II}$  could contribute to obtaining more accurate values of the requested parameters of wagons impact. In this paper the results of quasi-static tests [8–11] are used and according to them specific stiffness of buffers  $c_I$  and supporting structures of wagon which participating in the impact  $c_{II}$ , are determined.

Although in the calculation we get change more dynamic parameters of wagons impact, in the following analysis comparison of numerical results of parameters obtained by the experimental tests (Fig. 5) is performed.



Figure 5: The experimental testing of wagons impact

Table 2: The results of impact for wagon Uacns-z

Size	Unit	Exp. results	Num. results
$F^o$	[kN]	1280	1240
$v^o$	[m/s]	-	2.1
$F_u^{max}$	[kN]	2980	3020
$a_2^{max}$	[m/s <sup>2</sup> ]	+56/-55	+66/-52
$t$	[ms]	245	225

Table 3: The results of impact for wagon Tadnss-z

Size	Unit	Exp. results	Num. results
$F^o$	[kN]	1502	1480
$v^o$	[m/s]	-	2,36
$F_u^{max}$	[kN]	3540	3220
$a_2^{max}$	[m/s <sup>2</sup> ]	+55/-45	+58/-43
$t$	[ms]	243	223

Table 4: The results of impact for wagon Hccrrss-z

Size	Unit	Exp. results		Num. results	
		Empty	Full	Empty	Full
$F_u^{max}$	[kN]	1142	748	1161	761
$t$	[ms]	279	298	271	294

In previous Tables 2–3  $v^o$  is the speed which occurs when buffers are completely compacted.

When wagon Uah/Ra hits into an empty wagon Hccrrss-z, it should be taken into account that there is a movement of the roof and platform of this wagon. This movement is measured during the testing (Fig. 6) and was 11.8 mm [9]. By numerical calculation this value is 10.9 mm (Fig. 7.).



Figure 6: The testing of roof movement (wagon Hccrrss-z)

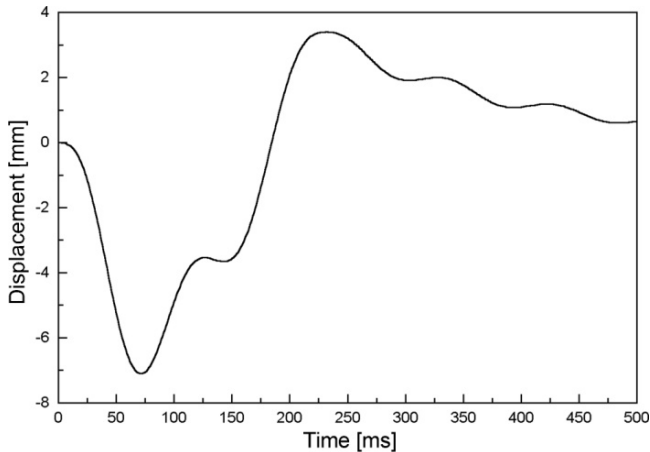


Figure 7: The diagram of roof movement obtained by the numerical simulation (wagon Hccrrss-z)

#### 4. COMPARATIVE ANALYSIS OF EXPERIMENTAL AND NUMERICAL RESULTS

By summarizing the presented research and obtained results, it can be said that proposed methodology enables determination of behaviour and stability of the wagons supporting structure at the action of impulse loads. The way of determination of dynamic parameters (forces, displacements, accelerations, etc.) and the stress-strain state of main parts of the wagon supporting structure at impact is presented, by using the principles of non-linear dynamics and the theory of elasticity.

By analysing the duration time of impact at all three cases of tested wagons, the large similarity of the numerical and experimental results is noticed (Tables 2–4). The other dynamic parameters are also very similar. This is primarily related to the values of acceleration, maximal values of total forces, as well as values of forces at completely compacted buffers.

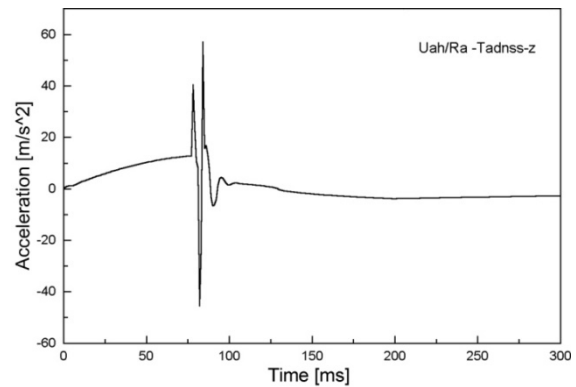


Figure 8: The diagram of acceleration change - obtained by numerical simulation (wagon Tadnss-z)

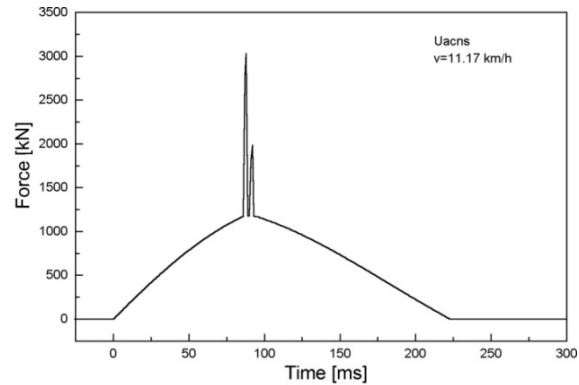


Figure 9: The diagram of total force on buffers of Uacns-z wagon - obtained by numerical simulation

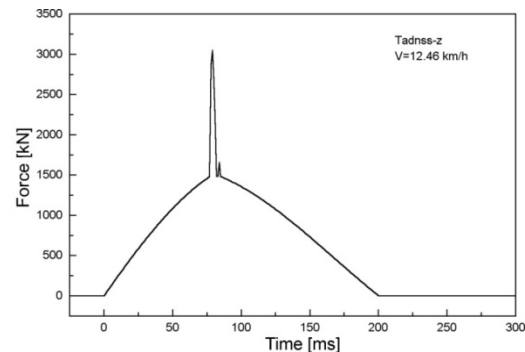


Figure 10: The diagram of total force on buffers of Tadnss-z wagon - obtained by numerical simulation

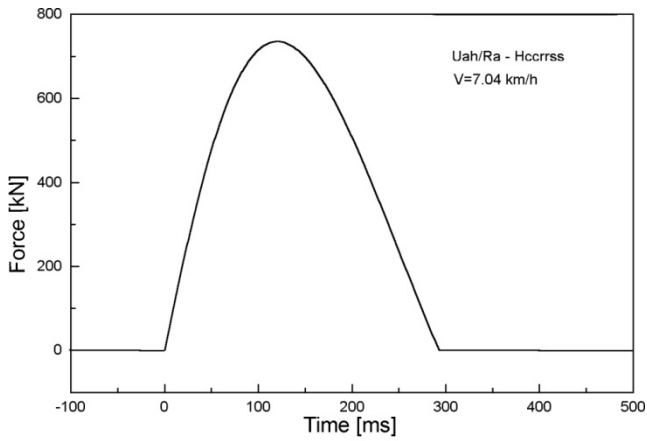


Figure 11: The diagram of total force on buffers of Hccrrss-z wagon obtained by numerical simulation (while there is cargo movement)

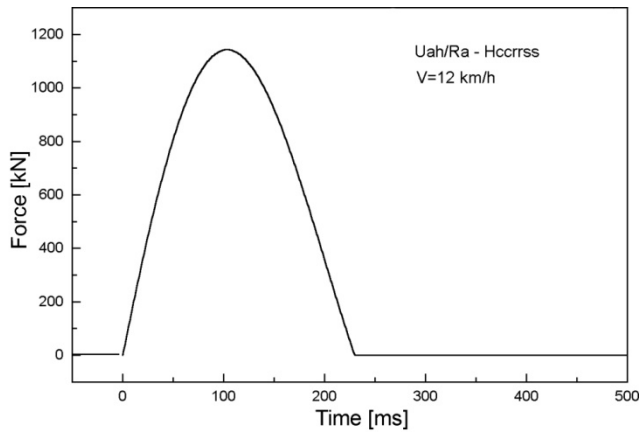


Figure 12: The diagram of total force on buffers of empty Hccrrss-z wagon - obtained by numerical simulation

The diagrams of experimentally obtained results are given in the following figures.

The experimental tests are performed on the special polygon for testing the railway vehicles impact in Wagon Factory Kraljevo, Serbia (Fig. 5).

In contrast to the diagrams obtained by the numerical simulation which show total force on buffers, the diagrams obtained by the experimental tests show the individual force at each buffer.

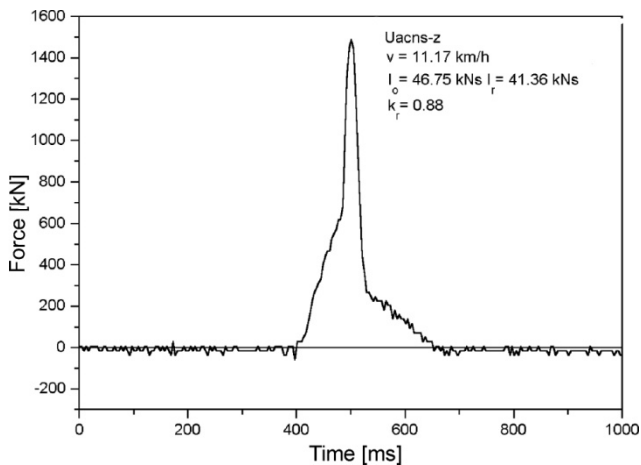


Figure 13: The diagram of force change on the first buffer of Uacns-z wagon - obtained experimentally

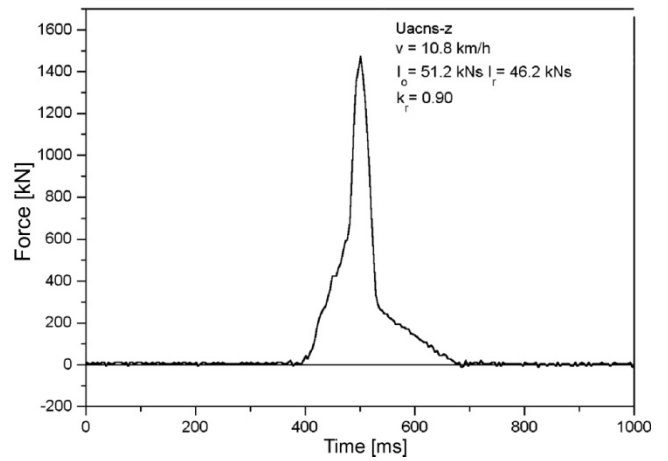


Figure 14: The diagram of force change on the second buffer of Uacns-z wagon obtained experimentally

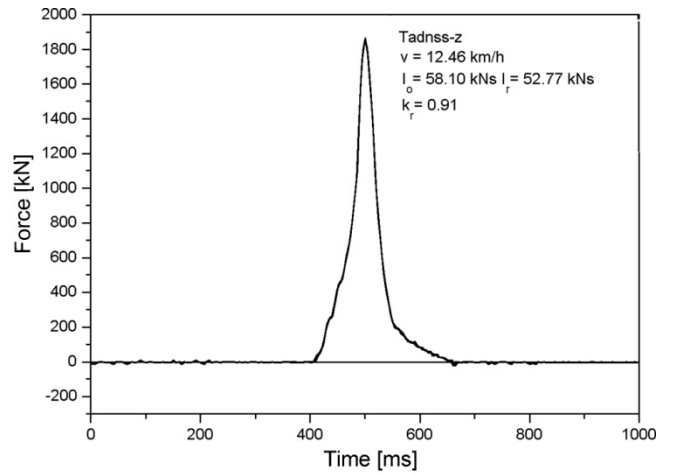


Figure 15: The diagram of force change on the first buffer of Tadnss-z wagon obtained experimentally

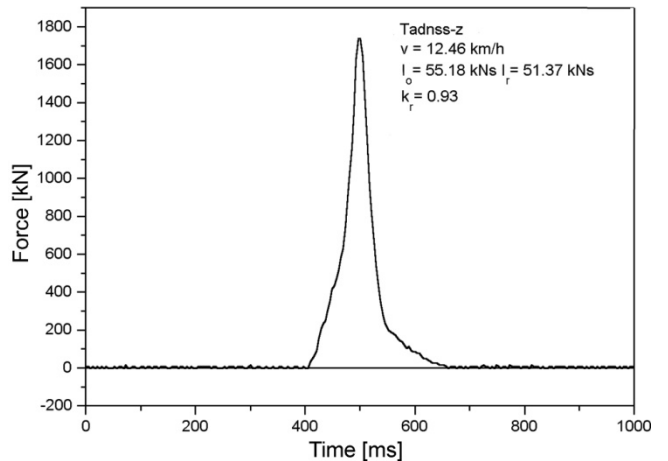


Figure 16: The diagram of force change on the second buffer of Tadnss-z wagon obtained experimentally



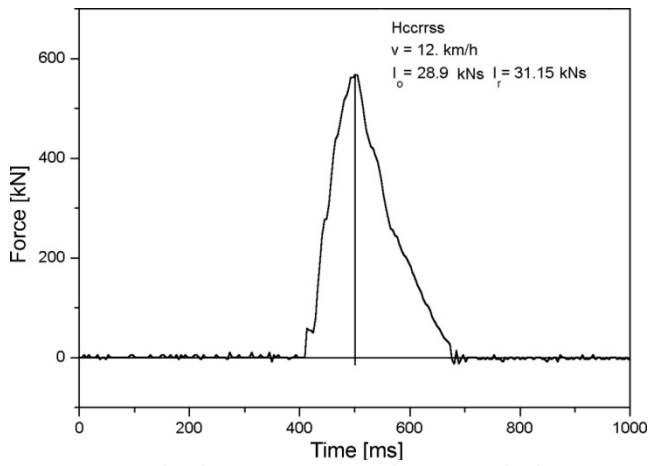


Figure 17: The diagram of force change on the buffer of empty Hccrrss-z wagon obtained experimentally

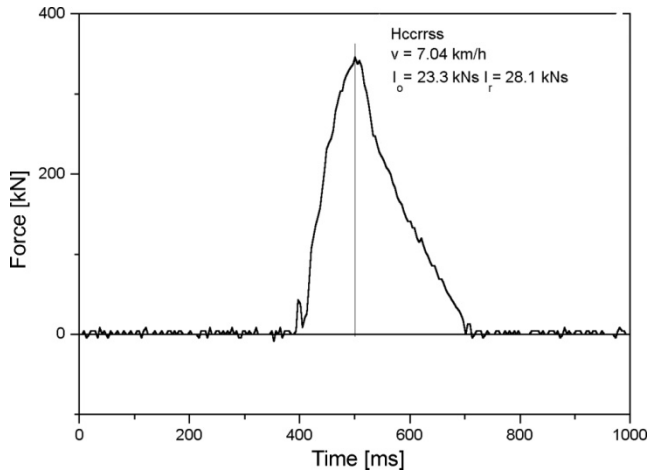


Figure 18: The diagram of force change on the buffer of laden Hccrrss-z wagon obtained experimentally (while there is cargo movement)

4.1. Results of numerical simulation which are not experimentally measured

In addition to the results obtained by numerical simulation which we could verify by experimental tests, the values of other dynamic parameters which are not experimentally measured (speeds of wagons during impact, movements and accelerations of cargoes and wagons, etc.) are obtained. These results of numerical simulation can be checked in future experimental tests. These results are given in the following figures.

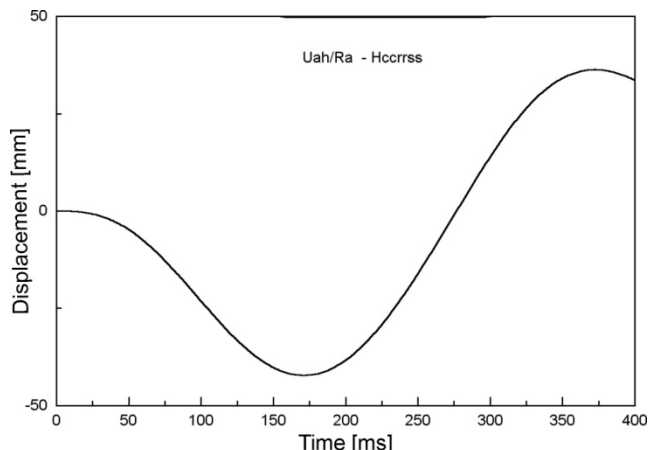


Figure 19: The diagram of cargo movement during the impact of Hccrrss-z wagon

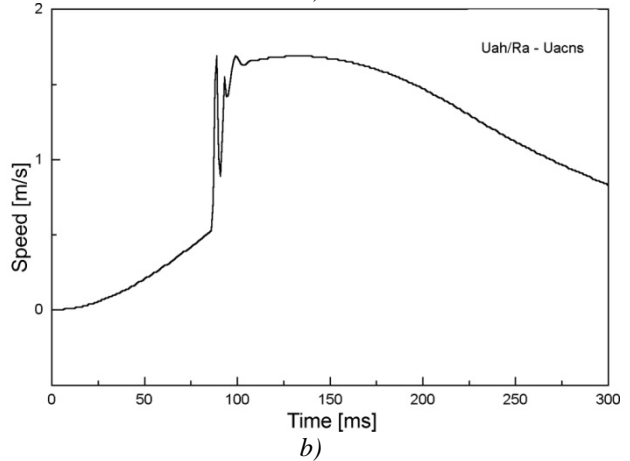
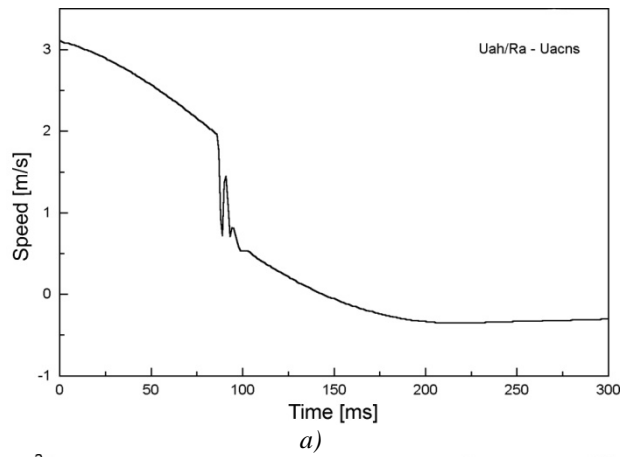


Figure 20: The diagrams of speed change of first (a) and second (b) wagon Uacns-z during the impact

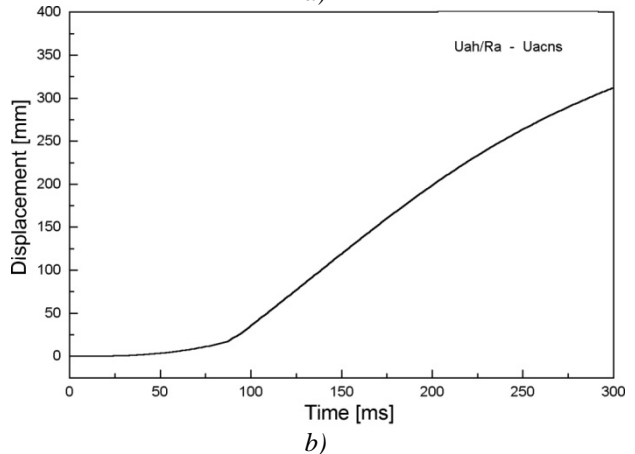
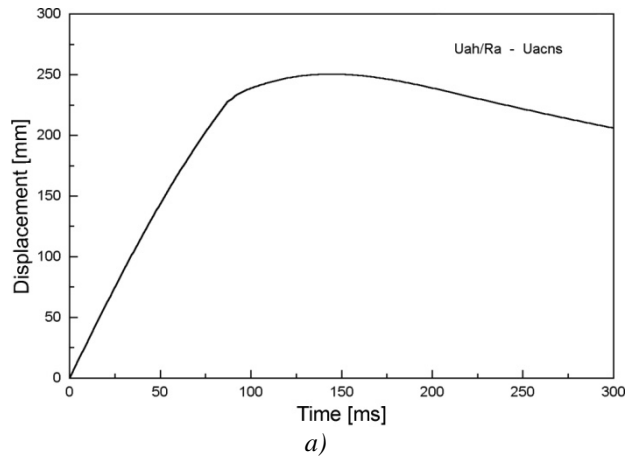
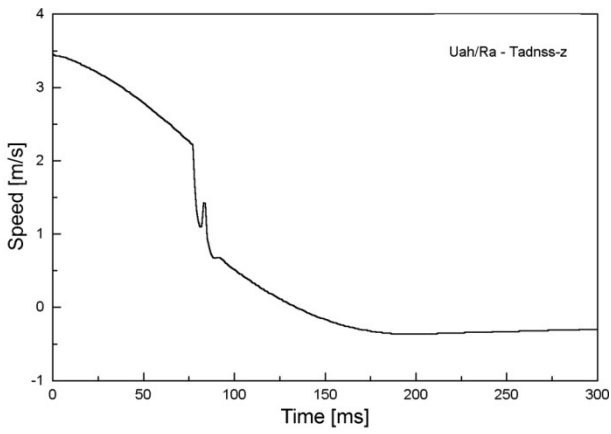
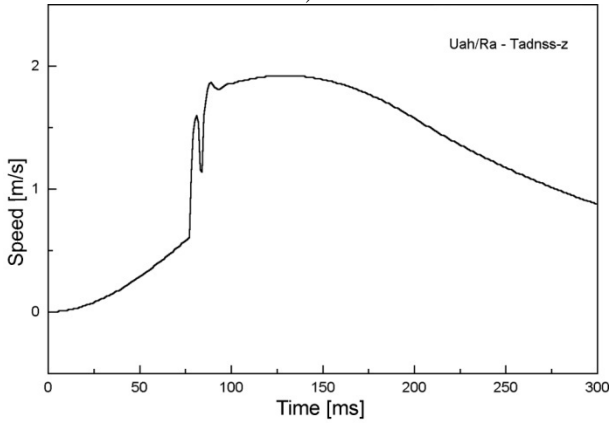


Figure 21: The diagrams of movement change of first (a) and second (b) wagon Uacns-z during the impact

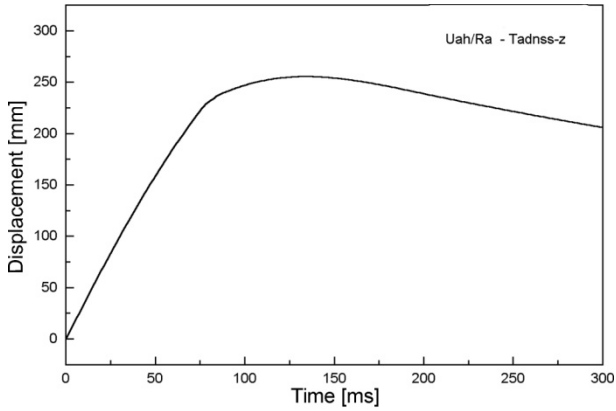


a)

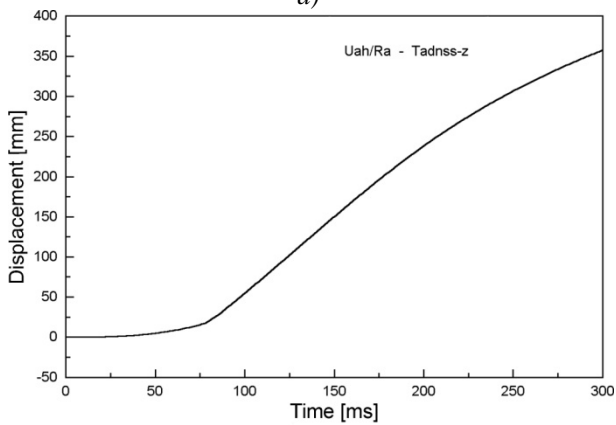


b)

Figure 22: The diagrams of speed change of first (a) and second (b) wagon Tadnss-z during the impact

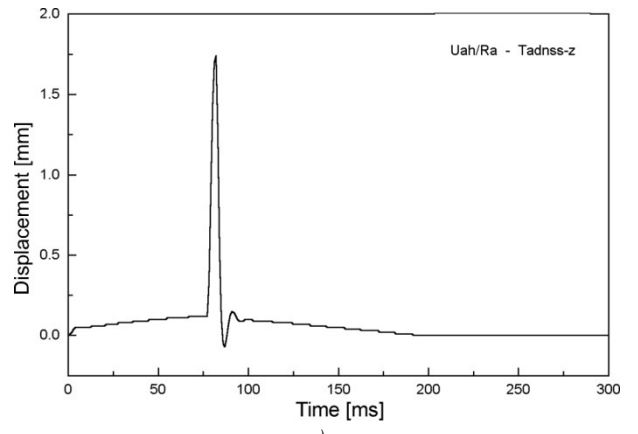


a)

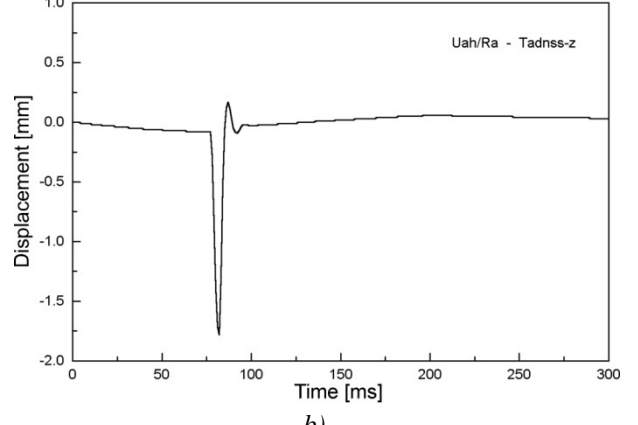


b)

Figure 23: The diagrams of movement change of first (a) and second (b) wagon Tadnss-z during the impact

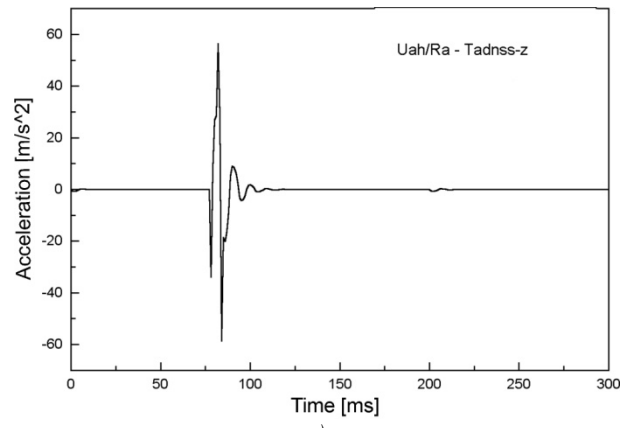


a)

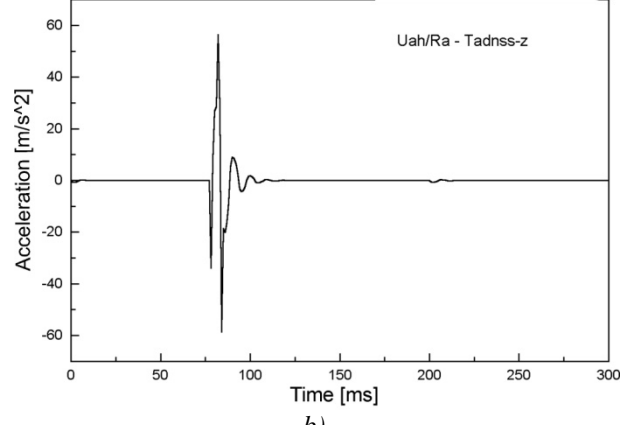


b)

Figure 24: The diagrams of cargo movement change of first (a) and second (b) wagon Tadnss-z during the impact



a)



b)

Figure 25: The diagrams of cargo acceleration change of first (a) and second (b) wagon Tadnss-z during the impact

## 5. CONCLUSION

In the research of vehicle dynamics so far, significant results have been achieved, before all, by using experimental methods. Examinations of values of forces at the impact of wagons as well as those of the stress-strain state of the supporting structure have been carried out. In this paper, experimental research was done at three different wagons: the wagon for transportation of powdered materials Uacns-z, the wagon for transportation of grain material Tadmss-z, and the wagon Hccrrss-z which is used for transportation of cars.

Beside the results of experimental testing of mentioned wagons, the analytical model of impact of two wagons is developed. The model takes into account the movement of cargo during the impact. The established differential equations are solved by numerical way.

By analysing the obtained results of numerical simulation of impact of wagons presented in this paper, it can be concluded that they are consistent with the results obtained experimentally. In that way, the developed analytical model can be used for determination of the dynamic parameters of the impact in the design phases of new wagons.

The improvements of the proposed model can be achieved by further exploring of dynamic stiffness of the wagon supporting structure and its sub-assemblies, as well as determination of the energy that is lost on the oscillation of the wagon during the impact.

## 6. ACKNOWLEDGEMENT

*The authors wish to express their gratitude to Serbian Ministry of Education, Science and Technology for supporting this paper through project TR35038.*

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