THE 7TH INTERNATIONAL CONFERENCE RESEARCH AND DEVELOPMENT OF MECHANICAL ELEMENTS AND SYSTEMS

ANALYSIS OF THE INFLUENCE OF LOCAL STRESS ON THE CARRYING CAPACITY OF BOX BEAMS

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Abstract: *This paper presents the identification of influence of local stress on the carrying capacity of box beams with quadrilateral cross section using the method of decomposing the cross section of the beam into its structural elements (plates). A mathematical model of the box beam was created on the basis of this methodology from the aspect of local stress. Deformation and stress parameters (deflection, moment and stress) of the constituent plates were defined. A comparative analysis of the results, i.e. deformation and stress values for the rectangular and trapezoidal cross sections was carried out. It was done under the same conditions of global carrying capacity and for the same ratio between the area of cross section and the resistance moment of the beam area. It was shown that the local stress can have the intensity which is several times higher than that of the global stress and that the selection of an appropriate cross section of the beam can significantly reduce the local effects. The results of research into the influence of local stress on the total carrying capacity of box beams were verified by means of the finite element method (FEM) using the software package ANSYS 12.*

Key words*: local stress, strain, carrying capacity, box beams*

1. INTRODUCTION

Structural elements of many carrying structures are derived by using box beams. Typical representative of these beams is traditional rectangular cross section. Modern research [1–6] has shown that other shapes are significant too, such as trapezoidal shapes. A specific field of research is the research of multi-polygonal shapes. The fact that is worth noting is that the above mentioned research is the result of analysis in the field of global stress structures. Namely, monitoring of the exploitation process in construction of box type, designed only according to the criteria of global capacity, has shown that there are extensive plastic deformations and even damages that lead to destruction of the material [7]. The same conclusion is reached in the process of controlled experimental testing [8], where the dominant effect of the local stresses was considered in the case of telescopic boom of auto crane.
The above mentioned phenomenon, characterized as local stress, is the case in this paper, with a tendency to mathematically reformulate deformation and stress state, in order to define effective parameters on the carrying capacity of box beams. Basic research of development of box beams is reflected in the optimization of its cross section.
The papers [1–3] and [5–6] has shown optimization of the defined dimensions for the given shapes of cross section of beam area, by using the objective function with more variables. The results of these optimizations are limited, since in the process of minimization of mass of beam structure the possibility of shape variability of cross section was not taken into account. From this stems the need to develop methodologies of cross-sectional shape optimization within the global stress. For comprehensive analysis of optimization problems it is necessary to recognize the influence of local stress state. Regarding this, special emphasis is given to the influence of geometric parameters of cross section on optimization of beam structures in the field of stress of the local character.

2. DEFINITION OF PROBLEM

This paper is primarily about the analysis of local stress. It focuses on the following topics:
1. Stress and deformation analysis of rectangular cross section from the aspect of local stress by applying analytical methods and FEM,
2. Identification of influential parameters of cross section on the beam structure from the aspect of optimal design of beam structure.
For realization of the above mentioned topic it is necessary to use a suitable physical or mathematical model. The analysis of local stress is defined by the model of local stability [9], which is established on the principle of decomposing the cross section of the beam into its structural elements (plates). Consideration of local stability and stress is made on the applicable beam segment whose length is L (Fig. 1 and 2). Research on the

effects of local stress is important in the domain of design, especially of high-responsible constructions, as constructive solutions of shape of cross-section ''relieve'' the most burdened parts and also create more suitable stress state for beam structure.

3. MATHEMATICAL FORMULATION

The above considered segment of beam structure with rectangular cross section (after decomposing into plates and after applying reaction moments) is shown on Fig.3. The model is based on the following assumptions:
1. Plates are considered to be freely leant,
2. Transverse forces are neglected in comparison to the effect of external stress and reaction moments,
3. Moving of the supporting structure is negligible compared to the plate deflection.

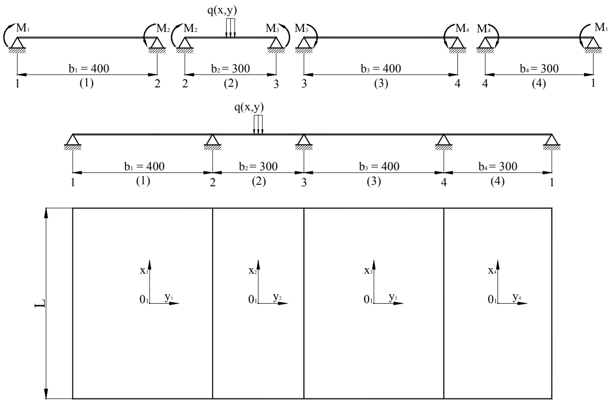

Fig.1. Physical model of box beam (unvaried-continual effect)

Deflection of plate "2" (Fig.2) from unvaried-continual effect q(x,y), is:

$$w_2(q)=$$
$$=\frac{16\cdot q_0}{\pi^6 D}\cdot\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi\xi}{a}\cdot\sin\frac{n\pi\eta}{b}\cdot\sin\frac{m\pi u}{2a}\cdot\sin\frac{n\pi v}{2b}}{mn\cdot\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}\tag{1}$$

Inclined plate "2" caused by the stress q(x,y), is defined by the equation:

$$\left(\frac{\partial w_2(q)}{\partial y_2}\right)=$$
$$=\frac{16\cdot q_0}{\pi^5 Db}\cdot\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi\xi}{a}\cdot\sin\frac{n\pi\eta}{b}\cdot\sin\frac{m\pi u}{2a}\cdot\sin\frac{n\pi v}{2b}}{m\cdot\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\cdot\sin\frac{m\pi x}{a}\cdot\cos\frac{n\pi y}{b}\tag{2}$$

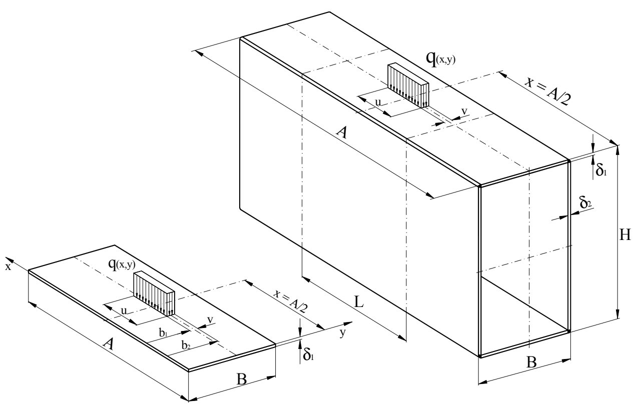

Fig.2. Plate of box beam structure exposed to unvaried-continual stress

Plate deflection "i" from the moment "M_i" i "M_{i+1}", is:

$$w_i(M_i,M_{i+1})=$$
$$=\frac{a_i^2}{4\pi^2 D_i}\cdot\sum_{m=1}^{\infty}\frac{\sin\frac{m\pi x}{a_i}}{m^2}\cdot\left[\begin{array}{l}\frac{K_{i,m}+K_{(i+1),m}}{\cosh\alpha_{i,m}}\cdot\left(\alpha_{i,m}tgh\alpha_{i,m}\cosh\frac{m\pi y}{a_i}-\frac{m\pi y}{a_i}\sinh\frac{m\pi y}{a_i}\right)+\\ \frac{K_{(i+1),m}-K_{i,m}}{\sinh\alpha_{i,m}}\cdot\left(\alpha_{i,m}ctgh\alpha_{i,m}\sinh\frac{m\pi y}{a_i}-\frac{m\pi y}{a_i}\cosh\frac{m\pi y}{a_i}\right)\end{array}\right]\tag{3}$$

Plate inclination "i" can be defined from the equation:

$$\left(\frac{\partial w_i(M_i,M_{i+1})}{\partial y_i}\right)_{y_2=-\frac{b_2}{2}}=$$
$$=\frac{a_i}{4\pi D_i}\cdot\sum_{m=1}^{\infty}\frac{\sin\frac{m\pi x}{a_i}}{m}\cdot\left[\begin{array}{l}\left(K_{i,m}+K_{(i+1),m}\right)\left(\frac{\alpha_{i,m}}{\cosh^2\alpha_{i,m}}+tgh\alpha_{i,m}\right)+\\ \left(K_{i,m}-K_{(i+1),m}\right)\left(ctgh\alpha_{i,m}-\frac{\alpha_{i,m}}{\sinh^2\alpha_{i,m}}\right)\end{array}\right]\tag{4}$$

$$\left(\frac{\partial w_i(M_i,M_{i+1})}{\partial y_i}\right)_{y_2=-\frac{b_2}{2}}=$$
$$=-\frac{a_i}{4\pi D_i}\cdot\sum_{m=1}^{\infty}\frac{\sin\frac{m\pi x}{a_i}}{m}\cdot\left[\begin{array}{l}\left(K_{i,m}+K_{(i+1),m}\right)\left(\frac{\alpha_{i,m}}{\cosh^2\alpha_{i,m}}+tgh\alpha_{i,m}\right)+\\ \left(K_{(i+1),m}-K_{i,m}\right)\left(ctgh\alpha_{i,m}-\frac{\alpha_{i,m}}{\sinh^2\alpha_{i,m}}\right)\end{array}\right]\tag{5}$$

Moments M_i and M_{i+1} are defined according to the following order equations:

$$M_i=\sum_{m=1}^{\infty}K_{i,m}\cdot\sin\frac{m\pi x}{a}\tag{6}$$

$$M_{i+1}=\sum_{m=1}^{\infty}K_{(i+1),m}\cdot\sin\frac{m\pi x}{a}\tag{7}$$

$K_{i,m}$ and $K_{(i+1),m}$ are unidentified coefficients in "m" function

Supporting structure number 1:

$$\left(\frac{\partial w_4(M_4,M_1)}{\partial y_4}\right)_{y_3=\frac{b_3}{2}}=\left(\frac{\partial w_1(M_1,M_2)}{\partial y_1}\right)_{y_1=-\frac{b_1}{2}}\tag{8}$$

Supporting structure number 2:

$$\left(\frac{\partial w_2(q_0)}{\partial y_2}\right)_{y_2=-\frac{b_2}{2}}+\left(\frac{\partial w_2(M_2,M_3)}{\partial y_2}\right)_{y_2=-\frac{b_2}{2}}=\left(\frac{\partial w_1(M_2,M_3)}{\partial y_2}\right)_{y_1=\frac{b_1}{2}}\tag{9}$$

Supporting structure number 3:

$$\left(\frac{\partial w_2(q_0)}{\partial y_2}\right)_{y_2=\frac{b_2}{2}}+\left(\frac{\partial w_2(M_2,M_3)}{\partial y_2}\right)_{y_2=\frac{b_2}{2}}=\left(\frac{\partial w_3(M_3,M_4)}{\partial y_3}\right)_{y_3=-\frac{b_3}{2}}\tag{10}$$

Supporting structure number 4:

$$\left(\frac{\partial w_3(M_3,M_4)}{\partial y_3}\right)_{y_3=\frac{b_3}{2}} = \left(\frac{\partial w_4(M_4,M_5)}{\partial y_4}\right)_{y_4=-\frac{b_4}{2}} \tag{11}$$

Coefficients "$B_{i,m}$" and "$C_{i,m}$", are derived from the following equations:

$$B_{i,m} = -\left(\frac{\alpha_{i,m}}{\cosh^2 \alpha_{i,m}} + tgh\alpha_{i,m}\right) \tag{12}$$

$$C_{i,m} = \left(\frac{\alpha_{i,m}}{\sinh^2 \alpha_{i,m}} - ctgh\alpha_{i,m}\right) \tag{13}$$

Coeficient $\alpha_{i,m}$ is:

$$\alpha_{i,m} = \left(\frac{m\pi b_i}{2a}\right), \quad i = (1,2,3,4) \tag{14}$$

4. ANALYSIS BY FEM

Calculation requirements of FEM models in terms of loads, geometry and bonds are identical with the theoretical model. To generate FEM model, the finite elements of tetrahedron shape (size 10 mm) were applied. Readings from the FEM model are shown in comparative diagrams (Fig. 5–8). According to it, high compatibility of results in terms of value and trend distribution can be noted.

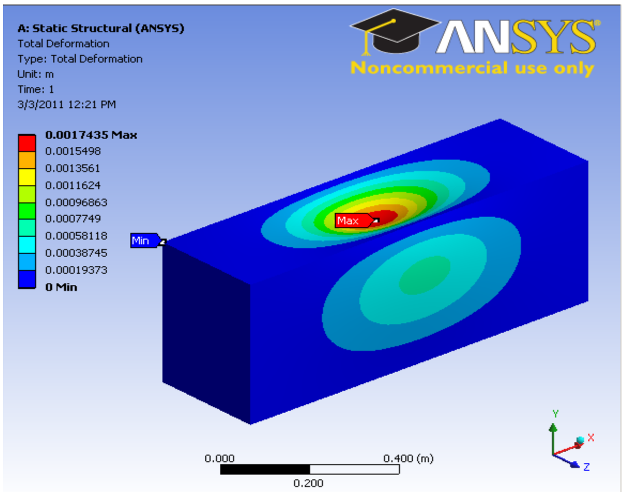

Fig.3. Overall deflection of beam structure segments with rectangular shaped cross section

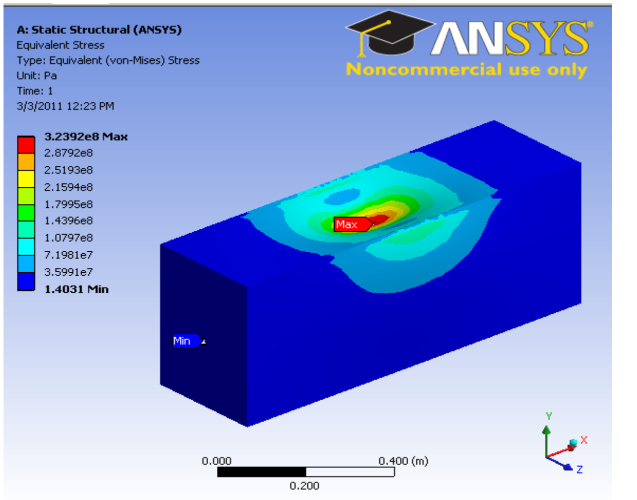

Fig.4. Equivalent stress of beam structure segments with rectangular shaped cross section

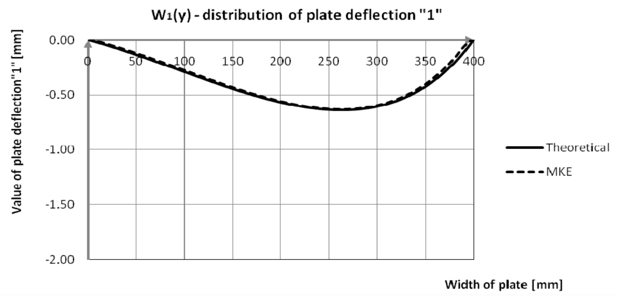

Fig.5. Comparative diagram of plate deflection "1" of beam structure

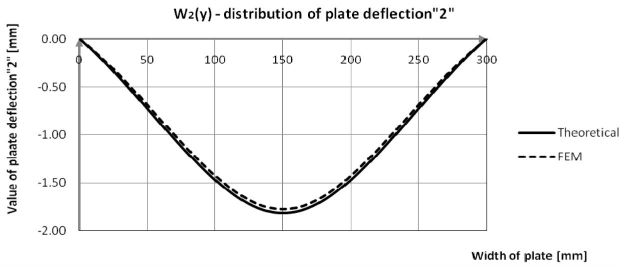

Fig.6. Comparative diagram of plate deflection "2" of beam structure

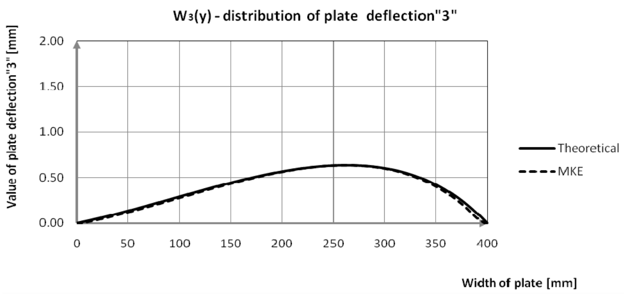

Fig.7. Comparative diagram of plate deflection "3" of beam structure

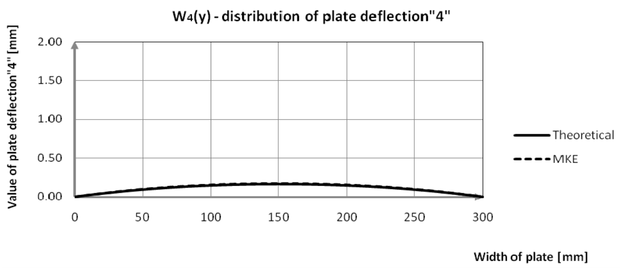

Fig.8. Comparative diagram of plate deflection "4" of beam structure

5. COMPARATIVE ANALYSIS OF RECITANGULAR AND TRAPEZOIDAL SHAPED CROSS SECTIONS

Use of analytical method and FEM has shown that upper belt plate, which is the plate that is exposed to direct stress q(x,y). According to the well-developed mathematical model, width (B_2) and thickness (δ_2) are identified as the most influential parameters of local stress. The mentioned is also proven for FEM (Fig. 3–4, 9–12), for the same conditions of global capacity and for the same slenderness of vertical tins of beam structure. The results are shown in comparative diagrams (Fig. 13–20).

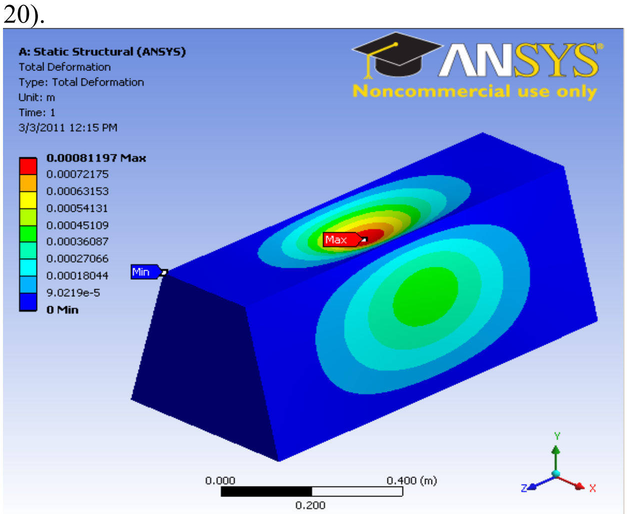

Fig.9. Overall deflection of beam structure segments with trapezoidal cross section (variant 1)

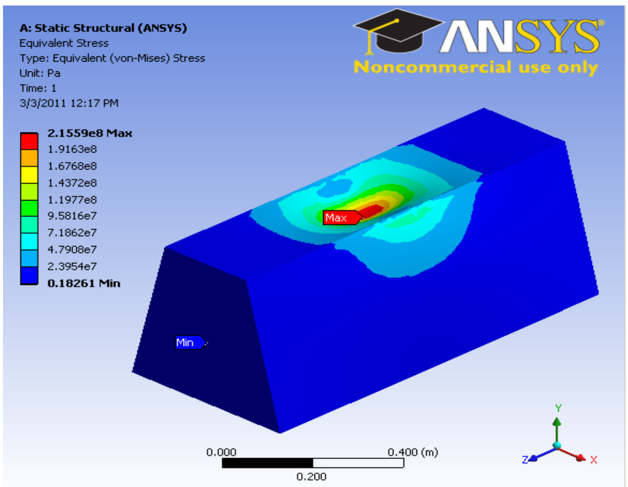

Fig.10. Comparative stress of beam structure segments with trapezoidal cross section (variant 1)

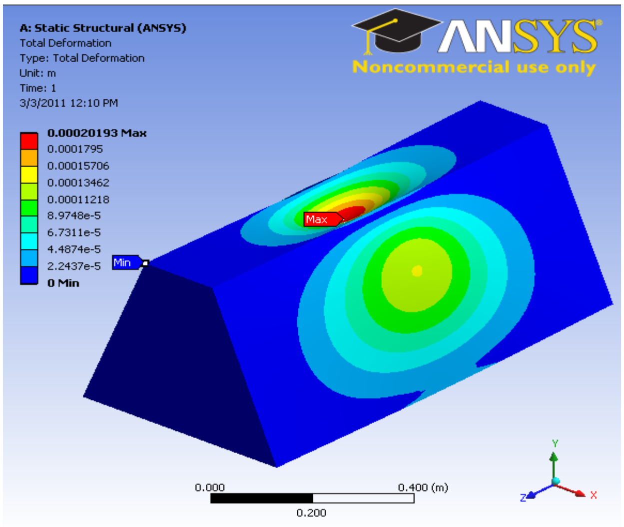

Fig.11. Comparative deflection of carrying structure segments with trapezoidal cross section (variant 2)

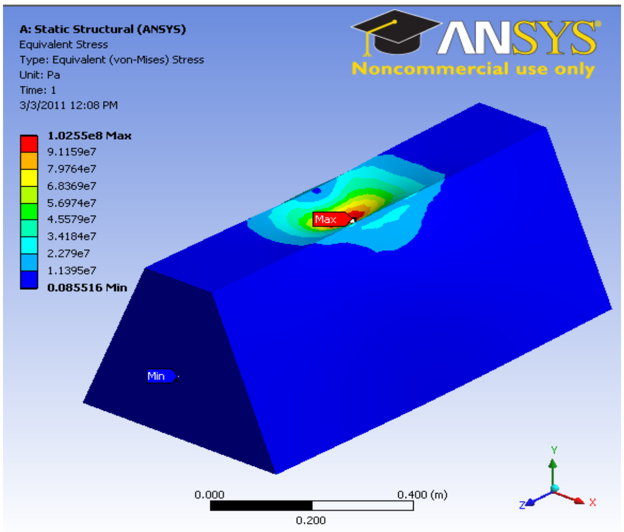

Fig.12. Comparative stress of beam structure with trapezoidal cross section (variant 2)

Table 1. Comparative table of rectangular and trapezoidal cross secton

item	shape cross section			
	rectangular	trapezoidal		
		var. 1	var. 2	var. 3
δ_1 [mm]	10	10	10	10
δ_2 [mm]	10	12	14	16
δ_3 [mm]	10	10	10	10
δ_4 [mm]	10	6	6	5
B_1 [mm]	300	250	205	175
B_2 [mm]	300	455	455	506
H [mm]	400	400	400	400
w_1 [mm]	**0.633**	**0.420**	**0.234**	**0.135**
w_2 [mm]	**1.744**	**0.812**	**0.373**	**0.202**
w_3 [mm]	**0.633**	**0.420**	**0.234**	**0.135**
w_4 [mm]	**0.141**	**0.186**	**0.091**	**0.050**
σ_u[kN/cm^2]	**32.39**	**21.56**	**14.76**	**10.26**

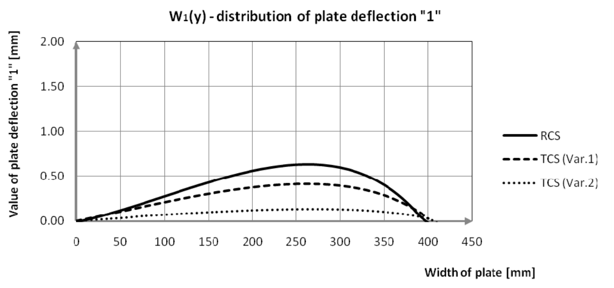

Fig.13. Diagram of comparative analysis of plate deflection "1" that has rectangular and trapezoidal shaped cross section (lateral direction)

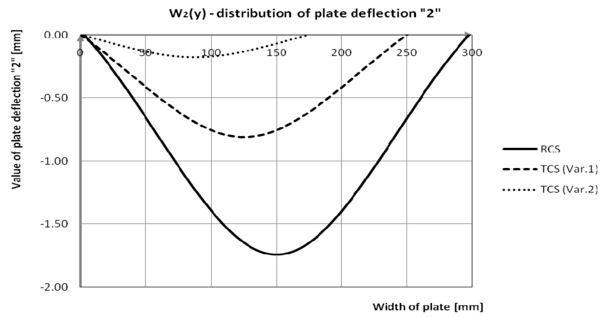

Fig.14. Diagram of comparative analysis of plate deflection "2" that has rectangular and trapezoidal shaped cross section (lateral direction)

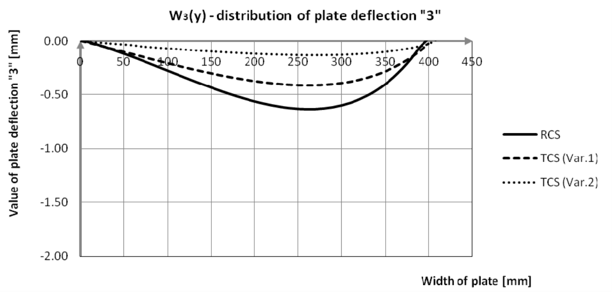

Fig.15. Diagram of comparative analysis of plate deflection "3" that has rectangular and trapezoidal shaped cross section (lateral direction)

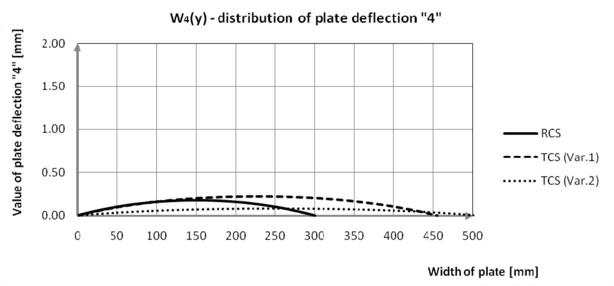

Fig.16. Diagram of comparative analysis of plate deflection "4" that has rectangular and trapezoidal shaped cross section (lateral direction)

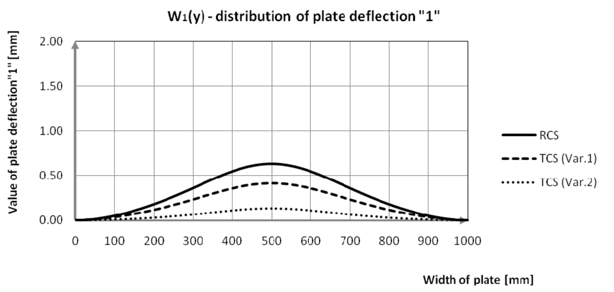

Fig.17. Diagram of comparative analysis of plate deflection "1" that has rectangular and trapezoidal shaped cross section (longitudinal direction)

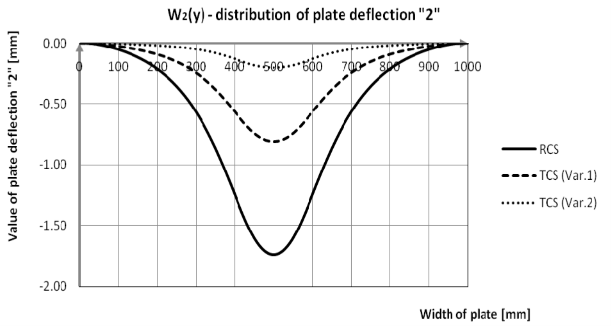

Fig.18. Diagram of comparative analysis of plate deflection "2" that has rectangular and trapezoidal shaped cross section (longitudinal direction)

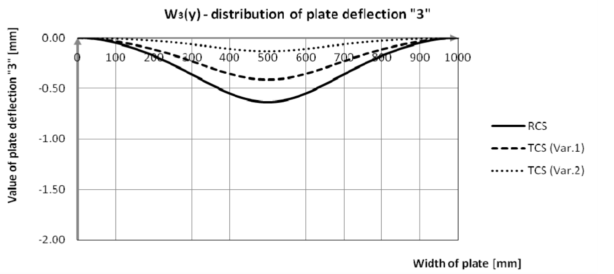

Fig.19. Diagram of comparative analysis of plate deflection "3" that has rectangular and trapezoidal shaped cross section (longitudinal direction)

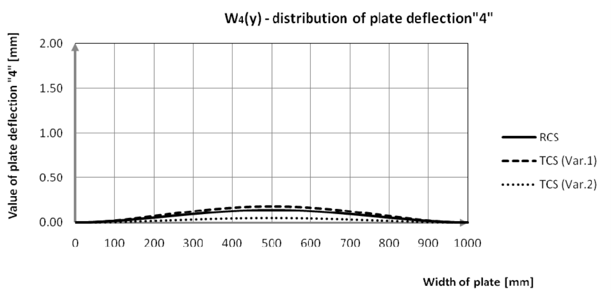

Fig.20. Diagram of comparative analysis of plate deflection "4" that has rectangular and trapezoidal shaped cross section (longitudinal direction)

6. CONCLUSION

According to the carried out research that was presented in this paper, the methodology of stress and deformational identification of local character of box beams that have rectangular cross-section is presented. Also, according to the formed physical model of local stability, its mathematical formulation was carried out, thus enabling definition of local stress state by analytic procedure. By verification of the calculated sizes, and by implemented methodologies, FEM was carried out by using the software package ANSYS 12. By implying comparative analysis of deflection of the plates of beam structure with rectangular cross section, it was noticed that the results are compatible as far as trends of distribution and values are concerned. Maximal variations do not do over 5%, which only proves that this method is reliable as far as both analysis of local stress state and defining influential geometrical parameters for increasing the capacity of the considered beam structure are concerned. Using this fact, the comparative analysis of a rectangular cross-section of defined geometrical characteristics of the corresponding trapezoidal shapes was carried out under the same conditions of global capacity, while maintaining a constant level of vertical beams and thick tin plates. The aim of these conditions is to identify the most influential parameters on the local strain carriers at a constant ratio between the area of cross section and the resistance moment of the beam area (*A/W = const.*), i.e. the same global carrying capacity, and for the same slenderness of vertical tin plates by variation of the geometric sizes of the elements of cross section. The analysis has shown that the most burdened is the upper belt plate or panel that is exposed to the immediate effect of external stress, while the lower belt plates (as opposed to above) is the least loaded.

It was found that the width and thickness of upper plate belt predominantly affect the value of local stress and strain, which is not the case with the lower band plate. By using FEM it was proved that the trapezoidal cross section has a more favorable stress state of rectangular, since the upper tin is less burdened, and consequently the other elements of the carrying structure. Research [10] has shown the generality of the mathematical model, given in Chapter 3, when applied to complex problems such as stiffening drawn from the system of plates. Generally, exposure identification indicates that the application of multipolygon forms significantly contributes to reducing the local impact by eliminating the sudden increase of stress in the critical zone, which is important for further development in this area.

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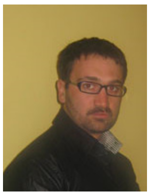

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