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Identification of wave phenomena at wagons impact

Dragan Petrović[1,*] – Milan Bižić[1] – Mirko Đelošević[1] – Ranko Rakanović[1]
[1] University of Kragujevac, Faculty of Mechanical Engineering Kraljevo

At longitudinal impacts, when the structure members are very quickly deformed, complex physical phenomena occur, such as: changes of rheological properties of the material, temperature and chemical changes, etc. During these phenomena, the behaviour of the structure can be completely different from its behaviour at static loading. The structure fails in getting displacements which correspond to fast changes of loads. Such delay can cause abrupt deformation of the structure. This paper presents theoretical and experimental analysis of wave phenomena at impact of railway wagons. Theoretical considerations have been realized on an idealized beam model, and experimental results refer to test of wagon type Zagkks for transport of liquid petroleum gas.
Keywords: Railway vehicle, wagon, wave, identification, impact

0 INTRODUCTION

Theoretical and experimental analysis of behaviour of body at wagon impact cannot be precisely determined without considering wave processes. In railway vehicles, where the geometry of the carrying structure is complex, and speeds of impact are not so great, a model of elastic body neglecting some phenomena can be formed. In that way, local effects which refer to the three-axis stress state is avoided. This postulation defines impact by a certain speed of a cross-section of the member or the shell and the ratio between masses of the observed elements and load. Consideration of impact phenomena is, in this way, different from the case where the change of several physical factors is present and where changes of the structure of the material are dominant. Most real structures subjected to impact action can be treated in this way. In that case, equations of motion [1, 2] have the form:

$$
\begin{aligned}
&(\lambda+G)\frac{\partial\varepsilon_v}{\partial x}+G\nabla^2 u+F_x-\rho\frac{\partial^2 u}{\partial t^2}=0\\
&(\lambda+G)\frac{\partial\varepsilon_v}{\partial y}+G\nabla^2 v+F_y-\rho\frac{\partial^2 v}{\partial t^2}=0 \qquad (1)\\
&(\lambda+G)\frac{\partial\varepsilon_v}{\partial z}+G\nabla^2 w+F_z-\rho\frac{\partial^2 w}{\partial t^2}=0
\end{aligned}
$$

Where:
λ, G, ρ – constants of material
ε_v – volume deformation
u, v, w – displacements in x, y, z directions
F_x, F_y, F_z – external volume forces,
t – time, and ∇^2 – Laplace operator

1 BEHAVIOUR OF ELASTIC BODIES AT IMPULS LOADS

Behavior of an elastic body loaded with forces which do not change in time belongs to the field of statics. These problems can include the case where the change of load in time is slow, i.e. quasi-static. If changes of load in time are faster, as in the case of impact loads, then the problems are transferred to the field of dynamics. In this case, action of dynamic (impact) load is not immediately transmitted to all points of the body. Waves of stresses and strains start to propagate from the loaded surface and they have finite speed of propagation.

1.1 Longitudinal and transversal waves in isotropic elastic continuum

If a certain point of the elastic continuum is incited, waves will start to propagate from that point to all sides. At a distance from the centre of incitation, all particles will move in parallel with the direction of propagation of waves (longitudinal waves) or normally to that direction (transversal waves) (Fig. 1).

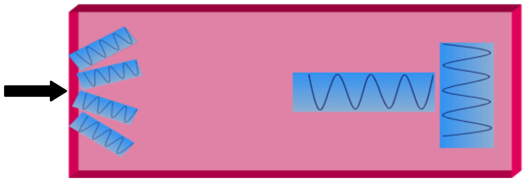

Fig. 1. *Distribution of waves in elastic continum*

Under the assumption that, in the existence of waves, the volume deformation is equal to zero, i.e. that deformation consists of sliding and rotating only, equations (1) obtain the form [1, 2]:

*Corr. Author's Address: Faculty of Mechanical Engineering Kraljevo, Dositejeva 19, 36000 Kraljevo, Serbia, petrovic.d@mfkv.kg.ac.rs

$$\frac{\partial^2 u}{\partial t^2} = c_i^2 \frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial^2 v}{\partial t^2} = c_i^2 \frac{\partial^2 v}{\partial y^2} \tag{2}$$
$$\frac{\partial^2 w}{\partial t^2} = c_i^2 \frac{\partial^2 w}{\partial z^2},$$

Where i=1, 2

Previously obtained equations have shown that the waves in elastic environment can be distributed in two different speeds. When *i=1* these are longitudinal waves, and when *i=2* these are transversal waves.

If the axis *x* is in the direction of propagation of waves (Fig. 2), then *v=w=0*, so that the displacement *u* is a function of the coordinate *x*. Every function $f(x+c_1t)$ can be a solution to the previous equation. Also, every function $f_1(x-c_1t)$ is a solution to that equation, so that it is possible to write the general solution in the form [3]:

$$u = f\left(x + c_1 t\right) + f_1\left(x - c_1 t\right) \tag{3}$$

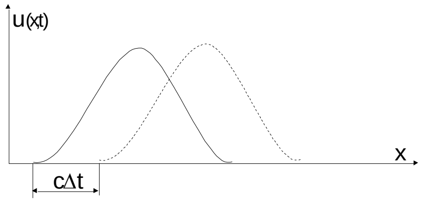

Fig. 2. *Propagation of waves in the elastic continuum*

The general solution to the equation (3) can be represented by two waves moving along the axis *x* in two opposite directions at the constant speed c_1 (Fig. 3).

Fig 3. *Moving of waves over the elastic body*

1.2 Beam at longitudinal impact

At the beginning of impact (Fig. 4), the beam is compressed, so that the initial speed v_1 of the mass m_1 is momentary changed until the speed of displacement of the beam end which undergoes the impact $\dot{u} = \partial u/\partial t$. This leads to fast occurrence of deformations $\varepsilon = \partial u/\partial x$, that is the stresses:

$$\sigma_x = E \cdot \varepsilon \tag{4}$$

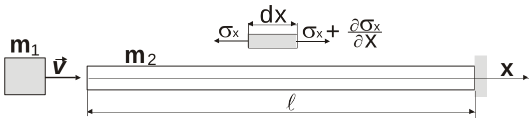

Fig. 4. *Beam at longitudinal impact*

On the basis of the expression (2), the differential equation of displacement of the beam along the axis *x* has the form:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{5}$$

Where the speed of propagation of waves in the beam is [4]:

$$c = \sqrt{\frac{E \cdot g}{\gamma}} = \sqrt{\frac{E}{\rho}} \tag{6}$$

At the moment of reaching the maximum displacement in the beam u_{max}, the mass m_1 will be in the state of rest. If the kinetic energy before the impact is $E_{k,o}$ and the maximum potential energy of the system is $E_{p,max}$, then, on the basis of the law of conservation of energy, it can be written:

$$E_{k,o} = E_{p,max} = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} EA\ell \left(\frac{u_{max}}{\ell}\right)^2 \tag{7}$$

$$\varepsilon_{max} = -\frac{u_{max}}{\ell} = -v_1\sqrt{\frac{m_1}{EA\ell}} = -\frac{v_1}{c}\sqrt{\frac{m_1}{m_2}} = -\frac{v_1}{c}\sqrt{\kappa} \tag{8}$$

Where κ is the ratio between the mass of load and the mass of the beam.

In the case of propagation of waves in the beam whose end $x = \ell$ is stationary, the solution must satisfy the following boundary conditions:

$$u(x,0) = 0\,; \quad \frac{\partial u}{\partial t}(x,0) = v_1\,; \text{ when } x=0$$
$$\frac{\partial u}{\partial t}(x,0) = 0\,; \text{ when } x>0 \tag{9}$$
$$\frac{\partial^2 u}{\partial t^2}(0,t) = \frac{c^2}{\kappa\ell}\frac{\partial u}{\partial x}(0,t)\,; \quad u(\ell,t) = 0$$

The local speed of the beam particles $\dot{u}$ and deformations ε is determined by appropriate derivations:

$$\left.\begin{aligned}\dot{u}=\frac{\partial u}{\partial t}=cf_1^{'}(ct-x)+cf^{'}(ct+x)\\ \varepsilon=\frac{\partial u}{\partial x}=-f_1^{'}(ct-x)+f^{'}(ct+x)\end{aligned}\right\}\qquad(10)$$

Let us consider the initial period of deformations $0\le t\le \ell/c$. If $f=0$ and $x=0$, the equation for determination of displacement of the loaded end is obtained:

$$f_1^{''}(t^*)+\frac{1}{\kappa\ell}f_1^{'}(t^*)=0\qquad(11)$$

Where $t^*=ct$

By using the limiting conditions, the following expressions for the speed of displacement of the movable end of the beam and for the corresponding deformation are obtained:

$$\left.\begin{aligned}\dot{u}(0,t)=\frac{\partial u(0,t)}{\partial t}=v_1\cdot e^{-\frac{t^*}{\kappa\ell}}\\ \varepsilon(0,t)=-\frac{v_1}{c}\cdot e^{-\frac{t^*}{\kappa\ell}}\end{aligned}\right\}\qquad(12)$$

Hence, it follows that at the moment of impact the members of the beginning of the beam, which are subjected to impact, obtain the deformation equal to the ratio of the local speed of the initial point of the beam and the speed of sound in the beam. The displacement of the end point of the beam is determined by the expression:

$$u(0,t)=v_1\frac{\kappa\ell}{c}(1-e^{-\frac{t^*}{\kappa\ell}})\qquad(13)$$

If the mass of the body which performs impact is considerably greater than the mass of the beam, it can be considered that $\kappa=\infty$, and at the speed $v_1=const.$ from the equation (12), it follows $u(0,t)=v_1t$.

For the analysis of the time period $\ell\le ct\le 2\ell$, it is necessary to determine the function *f* and limiting conditions at the stationary end. In that way, direct integration of the equation (5) results in functions whose form is changed upon running out of the period which is equal to the period of passing of the elastic wave along the beam. In the time period $t=2\ell/c$, the pressure wave returns to the beam beginning, which is in contact with the body. The speed of the body cannot be abruptly changed, so that the wave will reflect as if from the fixed end, and thus be doubled.

The characteristic curve of the beam deformation at longitudinal impact has the exponential form which decreases in time and after the period $2\ell/c$ has a rise. The value of the exponent is determined by the ratio between the masses of the body and the beam κ. The length of duration of the contact depends on the speed of members at impact v_1 and the ratio of masses κ. The contact stops at the moment when deformation of the beam beginning is equal to zero, which corresponds to passing through the equilibrium state.

2 SPEEDS OF WAGONS AT IMPACT

The impact of two wagons can be observed as the impact of two beams (Fig. 5) moving at the speeds v_1 and v_2 ($v_1 > v_2$).

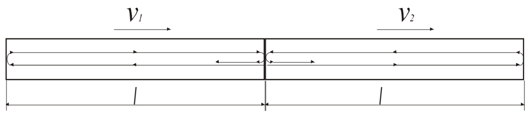

Fig. 5. *Impact of two beams*

At the moment of impact, two identical pressure waves start moving along both beams. In order to obtain equal absolute speeds of particles of both beams over the continuous surface, the values of those speeds must be equal to $(v_1-v_2)/2$. After the time interval ℓ/c, pressure waves reach free ends of the beams. At this moment, both beams are in the state of uniform pressure and the absolute speeds of all particles of the beams are:

$$v_1-\frac{v_1-v_2}{2}=v_2+\frac{v_1-v_2}{2}=\frac{v_1+v_2}{2}\qquad(14)$$

Pressure waves will then reflect from the free end, and at the moment $2\ell/c$, when these waves reach the contiguous surface of both beams, their speeds become:

$$\frac{v_1+v_2}{2}-\frac{v_1-v_2}{2}=v_2\qquad(15)$$

$$\frac{v_1+v_2}{2}+\frac{v_1-v_2}{2}=v_1\qquad(16)$$

The previously exposed theory of impact is based on several assumptions, such as, it is the impact of two homogeneous beams, the contact occurs over the whole surface of the beam, at the same moment, etc. In practice, such a case is rare and that is why the results of theoretical and experimental research do not agree. However, the knowledge of principles of occurrence and propagation of waves can help us in the analysis of experimentally obtained results of tests of real structures.

3 EXPERIMENTAL RESULTS OF IMPACT OF WAGONS

Experimentally obtained results (Fig. 6.) show effects of wavy motion, i.e. the time necessary for the wave to pass from the buffer to the end of the wagon and back. The experimentally determined time for this is between *21* and *24 ms* and it is somewhat longer than in the case when two homogeneous members of the same length would be at impact. The cause of this "delay" of wave is explained by the non-homogeneous structure which is interweaved with elements of different characteristics, then by the shape of the contiguous surfaces participating in the impact, etc. It can be indirectly concluded that the transducers, which record the impact force, have a satisfactory dynamic characteristic because they are able to record a phenomena which lasts more than ten times less than the time of impact duration. In the transducers which do not have a satisfactory dynamic characteristic, the curve would have a continual increase (without rises), and in that case there would appear an error in recording the maximum impact force.

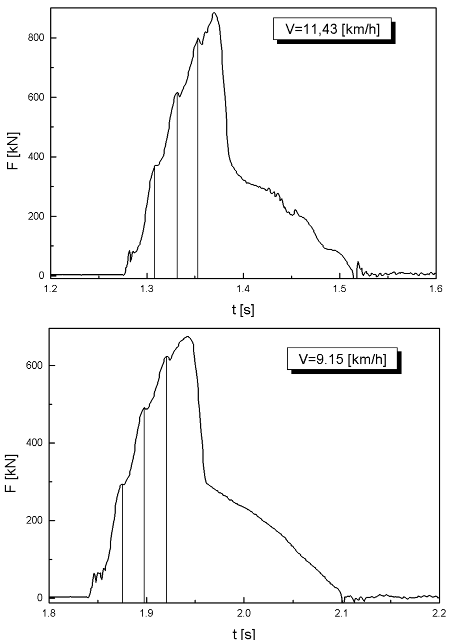

Fig. 6. *Change of force on the buffers at impact of wagons*

4 CONCLUSION

The aim of this paper is to draw attention to the phenomena that occur in impulse loading. This is particularly important for experimental investigation of wagons impact when it is necessary to determine the data acquisition frequency. As shown in Fig. 7, if the speed of acquisition is insufficient it is possible that the measurement does not register the maximum value of force at a collision.

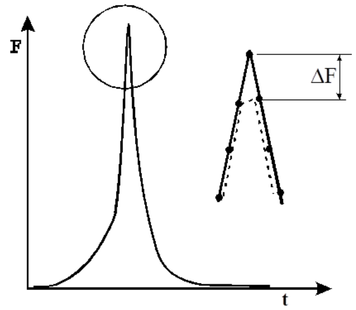

Fig. 7. *Measurement of force in a collision with insufficient a frequency of acquisition*

For this reason it is necessary to carry out the more theoretical study of impulse phenomena, and based on that performing the preparation for experimental investigation.

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