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Distribution of Bending Moments on the Plates of Carrier with Trapezoidal Cross Section

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This paper presents analysis of carrying capacity of the elements of cross-section (plates) with trapezoidal shape in carriers of box type. According to the model of linear character, some exact equations are formed. These equations are used for determination of moments that are transferred by flanges and webs of box carrier of trapezoidal cross-section. The criteria for application of the simplified expressions depending on the slenderness of the plate and the required accuracy of calculation are defined. This identification enabled the exact definition of plane compressive forces in order to analyze the buckling of plate carriers. Application of the results of this paper is a contribution to the process of optimal design of supporting structures, especially those that are used for construction of transport equipment, where the effect of reducing the weight affects on the efficiency of transport in supply chains.

Keywords: bending moment, plate, carrier, trapezoidal cross section

0 INTRODUCTION

Design of supporting structures is carried out through several phases, including a special procedure that is used to define the shape and dimensions based on the analysis of stress state, for whose definition it is necessary to identify the value of attack load. Carriers of steel structures are usually designed as thin-walled open profiles or box profiles (Fig.1,2) which are formed of plate elements and they are predominantly exposed to bending moments. In this paper, the analysis is limited to the linear distribution of normal stresses and bending moments (Fig. 3). The aim of this paper is primarily to correct expressions for the distribution of moments of attack (M_{P1}, M_{P2}, M_r) at the plate carrier of complex cross section (such as a trapezoidal), and then to point to the expediency of application of simplified expression, in certain cases.

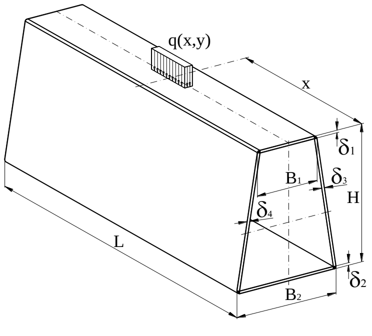

Fig. 1. *Nosač trapeznog poprečnog preseka*

Recent studies have paid special attention to the importance of box girder with a trapezoidal cross-section [1-3]. Researchers [1] have pinpointed that trapezoidal cross section has much more favourable stress state for the same requirements of global capacity and slender vertical plate, in comparison to the traditional rectangular shape (Fig. 2). Research [2] refers to the optimization of trapezoidal cross section in terms of global capacity indicating the usefulness of practical application. Trapezoidal shapes for carriers are particularly important to reduce the effect of local stress [1, 3].

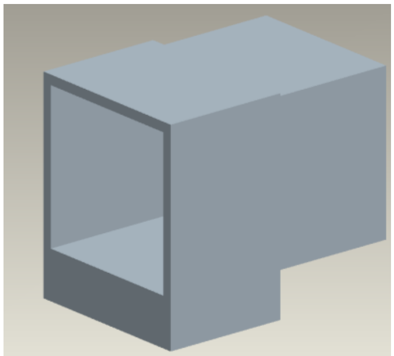

Fig. 2. *Segment carrier of boom track crane*

1 PROBLEM ANALYSIS

The total carrying capacity of any box beam corresponds to the sum capacity of its segments

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or plates (flanges and ribs). Under the influence of external loads, each segment of the cross section is subjected to the appropriate load according to their rigidity and resistance to deformation.

In order to make equations for attack sizes, it is necessary to know the following sizes:

$$\frac{N_r(y=H_{C1})}{H_{C1}}=\frac{N_r(y)}{y}\text{, for } y>0 \tag{1}$$

$$N_r(y)=\frac{N_r(y=H_{C1})}{H_{C1}}\cdot y \tag{2}$$

That is:

$$\frac{N_r(y=H_{C2})}{H_{C2}}=\frac{N_r(y)}{y}\text{, for } y<0 \tag{3}$$

$$N_r(y)=\frac{N_r(y=H_{C2})}{H_{C2}}\cdot y \tag{4}$$

Taking this into consideration:

$$\frac{N_r(y=H_{C1})}{H_{C1}}=-\frac{N_r(y=H_{C2})}{H_{C2}} \tag{5}$$

We get:

$$N_r(y=H_{C2})=-\frac{H_{C2}}{H_{C1}}\cdot N_r(y=H_{C1}) \tag{6}$$

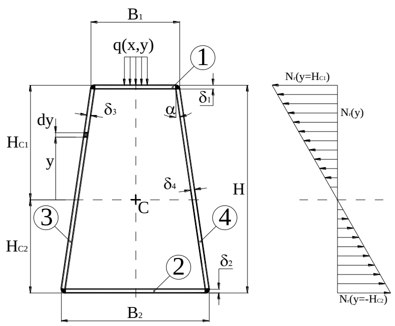


Fig. 3. *Distribution of force $N_r(y)$*

$N_r(y)$ - force per unit area, the function of "y"

H_{C1} - Distance between the centre of gravity C and the flange "1"

H_{C2} - Distance between the centre of gravity C and the flange "1

q(x,y) - arbitrary continuous load

2 MATHEMATICAL FORMULATIONS

The mathematical model is related to the bending moments of linear distribution. Areas of materials away from the neutral axis convey intense moments. Maximum acceptance of moments is achieved through flanges, while the remaining difference from the total moment is taken by web beams.

Moment of flange "1" (M_{p1}) is:

$$\begin{aligned}M_{p1}&=\int_{H_{C1}-\delta_1}^{H_{C1}}N_r(y)\cdot(B_1dy)\cdot y=\int_{H_{C1}-\delta_1}^{H_{C1}}\frac{N_r(y=H_{C1})}{H_{C1}}\cdot B_1\cdot y^2\cdot dy\\ M_{p1}&=\frac{N_r(y=H_{C1})}{H_{C1}}\cdot B_1H_{C1}^2\delta_1\cdot\left(1-\frac{\delta_1}{H_{C1}}+\frac{\delta_1^2}{3H_{C1}^2}\right)\end{aligned} \tag{7}$$

where:

B_1 - flange width "1"

δ_1 - flange thickness "1"

Moment of flange "2" (M_{p2}) is:

$$\begin{aligned}M_{p2}&=\int_{-(H_{C2}-\delta_2)}^{-H_{C2}}N_r(y)\cdot(B_2dy)\cdot y=\int_{-(H_{C1}-\delta_1)}^{-H_{C1}}\frac{N_r(y=-H_{C2})}{H_{C2}}\cdot B_2\cdot y^2\cdot dy\\ M_{p2}&=\frac{N_r(y=H_{C1})}{H_{C1}}\cdot B_2H_{C2}^2\delta_2\cdot\left(1-\frac{\delta_2}{H_{C2}}+\frac{\delta_2^2}{3H_{C2}^2}\right)\end{aligned} \tag{8}$$

where:

B_2 - flange width "2"

δ_2 - flange thickness "2"

Resulting flange moment (M_p) is:

$$M_p = M_{p1} + M_{p2} \tag{9}$$

By replacing (7) and (8) into (9), we get:

$$M_p=\frac{N_r(y=H_{C1})}{H_{C1}}\cdot\left[\begin{aligned}&B_1H_{C1}^2\delta_1\cdot\left(1-\frac{\delta_1}{H_{C1}}+\frac{\delta_1^2}{3H_{C1}^2}\right)+\\&+B_2H_{C2}^2\delta_2\cdot\left(1-\frac{\delta_2}{H_{C2}}+\frac{\delta_2^2}{3H_{C2}^2}\right)\end{aligned}\right] \tag{10}$$

Web moment above the heavy axis (M_{r1}) is:

$$\begin{aligned}M_{r1}&=2\int_0^{H_{C1}-\delta_1}N_r(y)\cdot\left(\frac{\delta_3}{\cos\alpha}dy\right)\cdot y=2\int_0^{H_{C1}-\delta_1}\frac{N_r(y=H_{C1})}{H_{C1}}\cdot\frac{\delta_3}{\cos\alpha}\cdot y^2\cdot dy\\ M_{r1}&=\frac{2}{3}\cdot\frac{N_r(y=H_{C1})}{H_{C1}}\cdot\frac{\delta_3}{\cos\alpha}\cdot H_{C1}^3\cdot\left(1-3\frac{\delta_1}{H_{C1}}+3\frac{\delta_1^2}{H_{C1}^2}-\frac{\delta_1^3}{H_{C1}^3}\right)\end{aligned} \tag{11}$$

where:

δ_3 - web thickness "3"

α - web slope angle "3" and "4" to the vertical axis

Moment of web under the heavy axis (M_{r2}) is:

$$\begin{aligned}M_{r2}&=2\int_0^{-(H_{C2}-\delta_2)}N_r(y)\cdot\left(\frac{\delta_4}{\cos\alpha}dy\right)\cdot y=2\int_0^{-(H_{C2}-\delta_2)}\frac{N_r(y=-H_{C2})}{H_{C2}}\cdot\frac{\delta_4}{\cos\alpha}\cdot y^2\cdot dy\\ M_{r2}&=\frac{2}{3}\cdot\frac{N_r(y=H_{C1})}{H_{C1}}\cdot\frac{\delta_4}{\cos\alpha}\cdot H_{C2}^3\cdot\left(1-3\frac{\delta_2}{H_{C2}}+3\frac{\delta_2^2}{H_{C2}^2}-\frac{\delta_2^3}{H_{C2}^3}\right)\end{aligned} \tag{12}$$

where:

δ_4 - web thickness "4"

Resulting moment of web (M_r) is:

$$M_r = M_{r1} + M_{r2} \tag{13}$$

$$M_r = \frac{2}{3}\cdot\frac{N_r(y=H_{C1})}{H_{C1}}\cdot\left[\frac{\delta_3}{\cos\alpha}H_{C1}^3\left(1-3\frac{\delta_1}{H_{C1}}+3\frac{\delta_1^2}{H_{C1}^2}-\frac{\delta_1^3}{H_{C1}^3}\right)+\frac{\delta_4}{\cos\alpha}H_{C2}^3\left(1-3\frac{\delta_2}{H_{C2}}+3\frac{\delta_2^2}{H_{C2}^2}-\frac{\delta_2^3}{H_{C2}^3}\right)\right] \quad (14)$$

The resulting moment (M_{uk}), which carries the observed carrier under the influence of the external moment (M) is:

$$M_{uk} = M_p + M_r \quad (15)$$

Resulting internal load corresponds to the external load, so:

$$M_{uk} = M \quad (16)$$

From the previous expression the force per unit area is defined (N_r) for $y = H_{C1}$.

$$N_r(y=H_{C1}) = \frac{H_{C1}}{B_1H_{C1}^2\delta_1\cdot\left(1-\frac{\delta_1}{H_{C1}}+\frac{\delta_1^2}{3H_{C1}^2}\right)+B_2H_{C2}^2\delta_2\cdot\left(1-\frac{\delta_2}{H_{C2}}+\frac{\delta_2^2}{3H_{C2}^2}\right)+\frac{2}{3}\cdot\left[\frac{\delta_3}{\cos\alpha}H_{C1}^3\left(1-3\frac{\delta_1}{H_{C1}}+3\frac{\delta_1^2}{H_{C1}^2}-\frac{\delta_1^3}{H_{C1}^3}\right)+\frac{\delta_4}{\cos\alpha}H_{C2}^3\left(1-3\frac{\delta_2}{H_{C2}}+3\frac{\delta_2^2}{H_{C2}^2}-\frac{\delta_2^3}{H_{C2}^3}\right)\right]}\cdot M \quad (17)$$

The moment of upper flange (M_{p1}) is defined according to the equation:

$$M_{p1} = \frac{B_1H_{C1}^2\delta_1\cdot\left(1-\frac{\delta_1}{H_{C1}}+\frac{\delta_1^2}{3H_{C1}^2}\right)}{B_1H_{C1}^2\delta_1\cdot\left(1-\frac{\delta_1}{H_{C1}}+\frac{\delta_1^2}{3H_{C1}^2}\right)+B_2H_{C2}^2\delta_2\cdot\left(1-\frac{\delta_2}{H_{C2}}+\frac{\delta_2^2}{3H_{C2}^2}\right)+\frac{2}{3}\cdot\left[\frac{\delta_3}{\cos\alpha}H_{C1}^3\left(1-3\frac{\delta_1}{H_{C1}}+3\frac{\delta_1^2}{H_{C1}^2}-\frac{\delta_1^3}{H_{C1}^3}\right)+\frac{\delta_4}{\cos\alpha}H_{C2}^3\left(1-3\frac{\delta_2}{H_{C2}}+3\frac{\delta_2^2}{H_{C2}^2}-\frac{\delta_2^3}{H_{C2}^3}\right)\right]}\cdot M \quad (18)$$

The moment of upper flange (M_{p2}) is defined according to the equation:

$$M_{p2} = \frac{B_2H_{C2}^2\delta_2\cdot\left(1-\frac{\delta_2}{H_{C2}}+\frac{\delta_2^2}{3H_{C2}^2}\right)}{B_1H_{C1}^2\delta_1\cdot\left(1-\frac{\delta_1}{H_{C1}}+\frac{\delta_1^2}{3H_{C1}^2}\right)+B_2H_{C2}^2\delta_2\cdot\left(1-\frac{\delta_2}{H_{C2}}+\frac{\delta_2^2}{3H_{C2}^2}\right)+\frac{2}{3}\cdot\left[\frac{\delta_3}{\cos\alpha}H_{C1}^3\left(1-3\frac{\delta_1}{H_{C1}}+3\frac{\delta_1^2}{H_{C1}^2}-\frac{\delta_1^3}{H_{C1}^3}\right)+\frac{\delta_4}{\cos\alpha}H_{C2}^3\left(1-3\frac{\delta_2}{H_{C2}}+3\frac{\delta_2^2}{H_{C2}^2}-\frac{\delta_2^3}{H_{C2}^3}\right)\right]}\cdot M \quad (19)$$

Web moment (M_r) is defined according to the equation:

$$M_r = \frac{\frac{2}{3}\cdot\left[\frac{\delta_3}{\cos\alpha}H_{C1}^3\left(1-3\frac{\delta_1}{H_{C1}}+3\frac{\delta_1^2}{H_{C1}^2}-\frac{\delta_1^3}{H_{C1}^3}\right)+\frac{\delta_4}{\cos\alpha}H_{C2}^3\left(1-3\frac{\delta_2}{H_{C2}}+3\frac{\delta_2^2}{H_{C2}^2}-\frac{\delta_2^3}{H_{C2}^3}\right)\right]}{B_1H_{C1}^2\delta_1\cdot\left(1-\frac{\delta_1}{H_{C1}}+\frac{\delta_1^2}{3H_{C1}^2}\right)+B_2H_{C2}^2\delta_2\cdot\left(1-\frac{\delta_2}{H_{C2}}+\frac{\delta_2^2}{3H_{C2}^2}\right)+\frac{2}{3}\cdot\left[\frac{\delta_3}{\cos\alpha}H_{C1}^3\left(1-3\frac{\delta_1}{H_{C1}}+3\frac{\delta_1^2}{H_{C1}^2}-\frac{\delta_1^3}{H_{C1}^3}\right)+\frac{\delta_4}{\cos\alpha}H_{C2}^3\left(1-3\frac{\delta_2}{H_{C2}}+3\frac{\delta_2^2}{H_{C2}^2}-\frac{\delta_2^3}{H_{C2}^3}\right)\right]}\cdot M \quad (20)$$

Distributions of moments of attack for monotonous load (M=1) are given on the following diagrams ($x=\delta_1/H_{C1}$; $y=\delta_2/H_{C2}$):

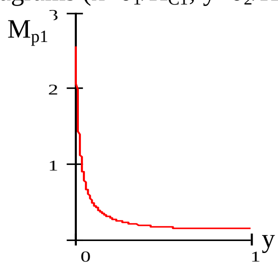


Fig.4. Diagram of flange moment distribution *(M_{p1})*

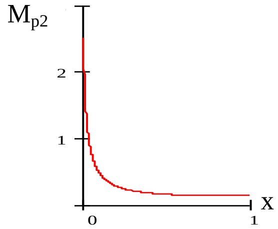


Fig.5. Diagram of flange moment distribution *(M_{p2})*

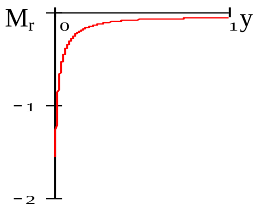


Fig.6. Diagram of web moment distribution *(M_r)*

Diagrams on the fig.2, fig.3 and fig. 4 show that the increase of relation δ_1/H_{C1} and δ_2/H_{C2} results in decrease of moment that is transferred by the flanges, whereas the difference is taken over by the webs of the carrier.

Expressions (17-20) can be simplified if the values $x=(\delta_1/H_{C1})$ and $y=(\delta_2/H_{C2})$ are small sizes in relation to other members in the abovementioned equations. Then these expressions get more simplified form that is suitable for practical application, so we have:

Force per unit areas (N_r) for $y = H_{C1}$.

$$N_r(y=H_{C1}) \approx \frac{H_{C1}}{B_1H_{C1}^2\delta_1+B_2H_{C2}^2\delta_2+\frac{2}{3}\cdot\left[\frac{\delta_3}{\cos\alpha}H_{C1}^3+\frac{\delta_4}{\cos\alpha}H_{C2}^3\right]}\cdot M \quad (21)$$

Moment of upper flange (M_{p1}) is determined according to the equation:

$$M_{p1} \approx \frac{B_1 H_{C1}^2 \delta_1}{B_1 H_{C1}^2 \delta_1 + B_2 H_{C2}^2 \delta_2 + \frac{2}{3} \cdot \left[\frac{\delta_3}{\cos\alpha} H_{C1}^3 + \frac{\delta_4}{\cos\alpha} H_{C2}^3 \right]} \cdot M \qquad (22)$$

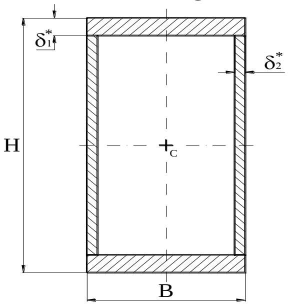
Moment of the lover flange (M_{p2}) is determined according to the equation:

$$M_{p2} \approx \frac{B_2 H_{C2}^2 \delta_2}{B_1 H_{C1}^2 \delta_1 + B_2 H_{C2}^2 \delta_2 + \frac{2}{3} \cdot \left[\frac{\delta_3}{\cos\alpha} H_{C1}^3 + \frac{\delta_4}{\cos\alpha} H_{C2}^3 \right]} \cdot M \qquad (23)$$

Web moment (M_r) is determined according to the equation:

$$M_r \approx \frac{\frac{2}{3} \cdot \left[\frac{\delta_3}{\cos\alpha} H_{C1}^3 + \frac{\delta_4}{\cos\alpha} H_{C2}^3 \right]}{B_1 H_{C1}^2 \delta_1 + B_2 H_{C2}^2 \delta_2 + \frac{2}{3} \cdot \left[\frac{\delta_3}{\cos\alpha} H_{C1}^3 + \frac{\delta_4}{\cos\alpha} H_{C2}^3 \right]} \cdot M \qquad (24)$$

In some special cases, when we have the box carries with rectangular cross section, we get:



$H_{C1} = H_{C2} = H/2$
$B_1 = B_2 = B$
$\delta_1 = \delta_2 = \delta_1^*$
$\delta_3 = \delta_4 = \delta_2^*$
$\alpha_3 = \alpha_4 = 0$

Fig. 7. Rectangular cross section of carrier

Force per unit area N_r for y=H/2 is:

$$N_r(y = \frac{H}{2}) = \frac{1}{B_1 H \delta_1^* + \frac{1}{3} H^2 \delta_2^*} \cdot M \qquad (25)$$

Web moment (M_r) is:

$$M_r = \frac{2 \cdot \frac{\delta_2^* H^3}{12}}{\frac{BH^2 \delta_1^*}{2} + 2 \cdot \frac{\delta_2^* H^3}{12}} \cdot M = \frac{I_r}{I_p + I_r} \cdot M \qquad (26)$$

where:

I_r - axial moment of inertia of the web
I_p - axial moment of flange
I - axial moment of inertia of the whole carrier

$I = I_r + I_p$ (27)

Moment of carrying flanges:

$M_p = M - M_r$ (28)

3 CONCLUDING REMARKS

The criterion for the application of equations (17-20) depends on the required accuracy of calculation and it is in the function of the cross sectional geometry, i.e. Of the sizes δ_1/H_{C1} or δ_2/H_{C2}. For the classical calculation, tolerated error in the moment of attack must be less than 10%, which corresponds to the size δ_1/H_{C1} (δ_2/H_{C2}) < 0,05. Analysis of the distribution of bending moments based on the linear distribution can be used for materials with approximately linear characteristic in the domain of elastic behaviour. Conducted on the basis of identification, we are able to carry deformational stress calculations, the global and local stability of the carrier [4], complex structures, using the attack load (moment) of the exact values that correspond to the real ones. Analysis of an exact determination of moments of attack on the carrying elements is particularly important in structures subjected to high loads (bridges, truck crane, etc.) with a very strong effect of buckling plate. A further aspect of the application of this analysis is reflected in the rationalization of weight of carrying elements of the transport equipment, applied especially to serve in distribution supply chains. Given the fact that in warehouse centers a working process is characterized with a large number of cycles, reduction of mass of the transport equipment without reduction of carrying capacity, significantly effect on the reduction of energy costs, and therefore on the total costs of the distribution process.

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