Overview of Wheel-rail Rolling Contact Theories

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In the contact area between wheel and rail there are intensive action and reaction forces that have a key influence on the dynamic behaviour of railway vehicles and track. These forces play a key role in supporting, guiding, traction and braking of railway vehicles. Bearing in mind that the wheel-rail contact problem is in fact the problem of rolling of two nonlinear profiles which occurs under the influence of many parameters, its solution and exact determination of wheel-rail contact forces belongs to the most complex tasks in railway engineering. Basically, solving the wheel-rail contact problem is reduced on solution of two types of contact problems - normal and tangential. These mutually coupled problems are solved from many researchers and today there are several different theories. Simpler theories are based on the assumptions under which the normal and tangential contact problem are solved completely separately. This results in a significantly simpler and faster calculation, but with greater error. On the other hand, more complex theories are based on coupled or iterative solving of given problems. As a result, they are characterized by greater accuracy, but also with more complicated calculation that requires significantly more computing time. In that sense, the aim of this paper is to analyse the wheel-rail rolling contact theories for solving the normal and tangential contact problems, with especial emphasize on determination of wheel-rail contact forces. Established conclusions are of particular importance for research of dynamic behaviour of railway vehicles.

Keywords: Wheel-rail contact, Rolling contact theories, Wheel-rail contact forces.

1. INTRODUCTION

Accurate determination of the wheel-rail contact forces is one of the most important tasks in the analysis of the dynamic behavior of railway vehicles. These forces are directly connected to the key functions of realization of railway traffic such as supporting, guiding, traction and braking of railway vehicles. Accordingly, the wheel-rail contact forces are the key influential parameters and indicators of the quality of the dynamic behavior of railway vehicles. Their values are the basis for evaluation of safety against the derailment which is obligatory in the phase of certification of all types of newly designed or modified railway vehicles. The international standards prescribe exclusively experimental tests as the way for determination of the wheel-rail contact forces in the certification process [1, 2]. Given in mind that these experimental tests are very expensive, certain simulations of dynamic behavior of railway vehicles must be performed, especially in the phase of development of railway vehicles. In such simulations, the wheel-rail contact forces and other parameters of dynamic behavior of railway vehicles must be determined on analytical or numerical way. Considering the complexity and stochasticity of geometry of the wheel-rail contact [3], analytical determination of forces in that contact is very complex task which is solved from many researchers over the past years. The task is related to the solving the problem of rolling of one nonlinear profile (wheel profile) via another nonlinear profile (rail profile). At the same time, the intensity of pressure between these profiles is changeable (under the influence of wear), the shapes of these profiles are changeable and mutual position between these profiles is changeable. During such dynamic process, in the contact surface between mentioned profiles of wheel and rail there are normal and tangential contact forces that depend on large number of influential parameters and that have a key influence on the dynamic behavior of railway vehicles. In contemporary analysis of dynamic behavior of railway vehicles, the analytical determination of these forces is based on solving of two types of mutually coupled problems – normal and tangential contact problem [4–6]. The different ways of solution of these problems are basis for numerical determination of wheel-rail contact forces and other parameters of dynamic behavior of railway vehicles. They are incorporated in all software tools for simulation of dynamic behavior of railway vehicles. Some of the most useful and most powerful software in this area of are Adams, Simpack, Vampire, Gensys, Nucars, etc. Thus, the aim of this paper is to give the main postulates and to analyze the wheel-rail rolling contact theories, with especial accent on their reliability in simulations of dynamic behavior of railway vehicles.

2. NORMAL CONTACT PROBLEM

Normal contact problem is related to determination of shape and size of contact surface, normal stress or contact pressure and its distribution in the contact surface which is caused by the action of normal force. The solution of normal contact problem between wheel and rail is usually based on application of Hertz static theory of contact of elastic bodies [7, 8]. This theory can be applied to the wheel-rail contact under the following assumptions: displacements and strains are small; contact surface is small in comparison to the dimensions of wheel and rail or rolling radius of the wheel (semi-space assumption); area in vicinity of contact surface is described with constant curve; surfaces of contact of wheel and rail are smooth (roughness is neglected); there are only elastic deformations; materials of wheel and rail are homogenous and isotropic [4]. Therefore, a key assumption implies that the bodies in contact (wheel and rail) are geometrically and elastically the same or quasi-identical. This assumption enables that normal and tangential contact problem can be solved separately.

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According to the Hertz theory the contact surface between wheel and rail compressed with normal force N has an elliptical shape with semi-axis a_e in direction of movement and semi-axis b_e in lateral direction (Fig. 1).

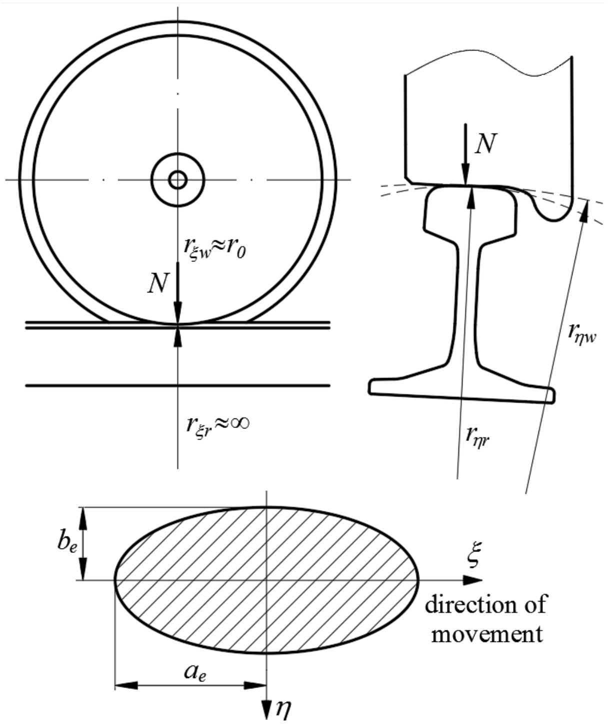

Figure 1: The contact surface between wheel and rail according to the Hertz theory

Semi-axes of elliptical contact surface can be calculated from the following expressions:

$$a_e = m \cdot \sqrt[3]{\frac{3}{2} \cdot \frac{N\left(1-\nu_P^2\right)}{E\left(A+B\right)}}$$
$$b_e = n \cdot \sqrt[3]{\frac{3}{2} \cdot \frac{N\left(1-\nu_P^2\right)}{E\left(A+B\right)}} \quad (1)$$

where:
E – modulus of elasticity,
ν_P – Poisson's ratio,
N – normal force between wheel and rail,
m, n – constants,
A, B – functions that are dependent on the radii of wheel and rail curvatures.

Functions A and B are defined with the following expressions:

$$A = \frac{1}{2}\left(\frac{1}{r_{\eta r}} - \frac{1}{r_{\eta w}}\right)$$
$$B = \frac{1}{2}\left(\frac{1}{r_{\xi r}} + \frac{1}{r_{\xi w}}\right) \approx \frac{1}{2r_{\xi w}} \approx \frac{1}{2r_0} \quad (2)$$

where:
$r_{\xi w}$ – radius of the wheel curvature in longitudinal direction (direction of movement),
r_0 – rolling radius of the wheel,
$r_{\eta w}$ – radius of the wheel curvature in lateral direction,
$r_{\xi r}$ – radius of the rail curvature in longitudinal direction,
$r_{\eta r}$ – radius of the rail curvature in lateral direction.

The constants m and n are determined from the appropriate tables, on the basis of value of the following function:

$$\theta = \arccos\left(\frac{A-B}{A+B}\right) \quad (3)$$

The surface of the ellipse or the contact area between wheel and rail is:

$$A_e = \pi \cdot a_e \cdot b_e \quad (4)$$

The distribution of contact pressure or normal stress in contact surface has the shape of semi-ellipsoid (Fig. 2), and it is defined with the following expression:

$$\sigma_\zeta\left(\xi,\eta\right) = \frac{3}{2} \cdot \frac{N}{A_e} \cdot \sqrt{1-\left(\frac{\xi}{a_e}\right)^2 - \left(\frac{\eta}{b_e}\right)^2} \quad (5)$$

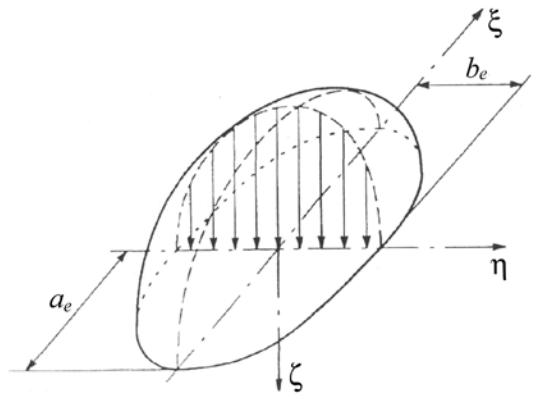

Figure 2: The distribution of contact pressure in wheel-rail contact surface according to the Hertz theory [4]

The numerical example of application of previously exposed Hertz theory for solution of normal wheel-rail contact problem in case of 4-axled freight waggon is given in the Table 1.

Table 1: The solution of normal contact problem between wheel profile UIC-ERRI S1002 and rail profile 60E1

Modulus of elasticity	E=21000 kN/cm^2	
Poisson's ratio	ν_P=0.3	
Radius of the wheel curvature in longitudinal direction	$r_{\xi w}$=r_0=46 cm	
Radius of the wheel curvature in lateral direction	$r_{\eta w}$=32 cm	
Radius of the rail curvature in longitudinal direction	$r_{\xi r}$=∞	
Radius of the rail curvature in lateral direction	$r_{\eta r}$=30 cm	
Function	A=0.104	
Function	B=1.087	
Function	θ=145°	
Constant	m=0.53	
Constant	n=2.4	
Normal force	N=40 kN	N=110 kN
Semi-axis	a_e=0.32 cm	a_e=0.45 cm
Semi-axis	b_e=1.45 cm	b_e=2.02 cm
Surface	A_e=1.46 cm^2	A_e=2.85 cm^2
Normal stress	$\sigma_{\zeta max}$=41.1 kN/cm^2	$\sigma_{\zeta max}$=57.9 kN/cm^2

It is important to emphasize that Hertz theory can be applied only on the cases of non-conformal contact between wheel and rail.

3. CREEP AND PARAMETERES IN WHEEL-RAIL CONTACT SURFACE

Due to the elasticity of the material, during the rolling along the rail, there is very small difference between the tangential velocity and the transnational velocity or velocity of the wheel progression (Fig. 3). This phenomenon is called creep and it has very significant influence on the dynamic behavior of railway vehicles which was first discovered from Carter [9, 10]. The proper analysis of creep is crucial for solution of tangential contact problem between wheel and rail.

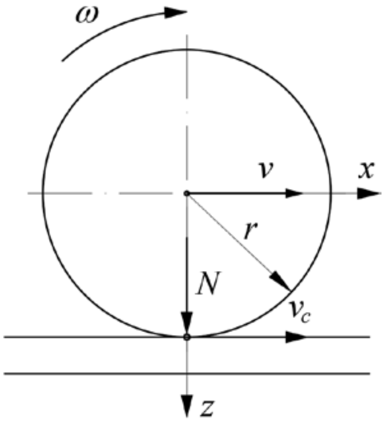

Figure 3: *The rolling of the wheel along the rail and creep velocity*

The creep velocity v_c represents a difference between tangential velocity of wheel with radius r which is revolving with angular velocity ω, and transnational velocity of wheel v:

$$v_c = r \cdot \omega - v \tag{6}$$

Due to a large number of influential parameters, ideal rolling of wheelset is very rare in practice, especially at running through the curves. Therefore, ideal radial steering of wheelset is extremely rare. During the running in the curves, wheelset is usually positioned in some under-radial position when guiding wheel under certain angle ψ is attacking outer rail (Fig. 4).

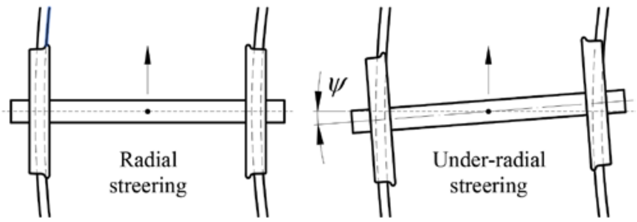

Figure 4: *The radial and under-radial steering of wheelset in the curves*

As a consequence of the previous considerations it can be observed that there is not only creep in running direction as shown in Fig. 3, but there is some resulting creep that can have an arbitrary direction and which, in addition to transnational, contains certain rotational component. These two components cause the occurrence of tangential force and moment in the contact surface between the wheel and rail. Therefore, parameters in wheel-rail contact surface are shown in Fig. 5.

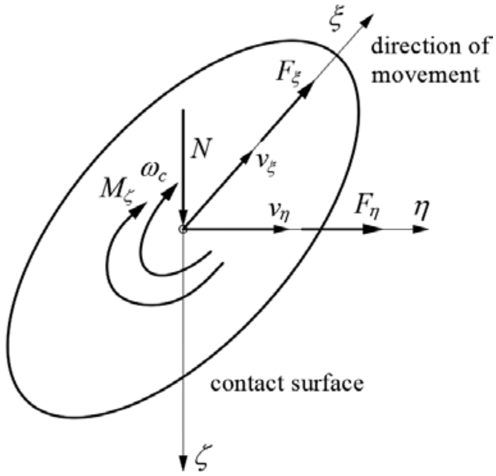

Figure 5: *The parameters in wheel-rail contact surface*

In Fig. 5 are:

N – normal force,

v_ξ – velocity of longitudinal creep,

v_η – velocity of lateral creep,

ω_c – angular velocity of rotational creep,

F_ξ – tangential force of longitudinal creep,

F_η – tangential force of lateral creep,

M_ζ – moment of rotational creep.

The determination of given tangential forces and moment is based on reduced creepages which are obtained when the velocities of the longitudinal and lateral creep and angular velocity of the rotational creep are reduced to velocity of wheel progression v:

$$\upsilon_\xi = \frac{v_\xi}{v} \qquad \upsilon_\eta = \frac{v_\eta}{v} \qquad \phi = \frac{\omega_c}{v} \tag{7}$$

Thus, total translational reduced creep is:

$$\upsilon = \sqrt{\upsilon_\xi^2 + \upsilon_\eta^2} \tag{8}$$

The components of reduced creepages play a key role in all methodologies that dealing with solving of the tangential contact problem.

4. TANGENTIAL CONTACT PROBLEM

The tangential contact problem is related to the determination of tangential forces and stresses which are generated in wheel-rail contact surface due to the presence of friction and creepage. For solving the tangential contact problem, there are various methodologies such as: simplified theory, exact theory, dynamical theory, quasi-static theory, three-dimensional theory, two-dimensional theory, etc. One of the most important reviews of these theories is made from Kalker in his research [11, 12].

Key problems in solving of the tangential contact problem are the existence of creep between wheel and rail, as well as the nonlinear character of the change of tangential force which is shown in Fig. 6. Consequently, the exact determination of tangential forces in wheel-rail contact must be based on the nonlinear theory. However, at smaller values of creep the change of tangential force has almost linear character and under such assumption its calculation can be performed with application of linear theory. This fact had a great impact on the development of the theories for solving the tangential contact problem. The most important theories are given below.

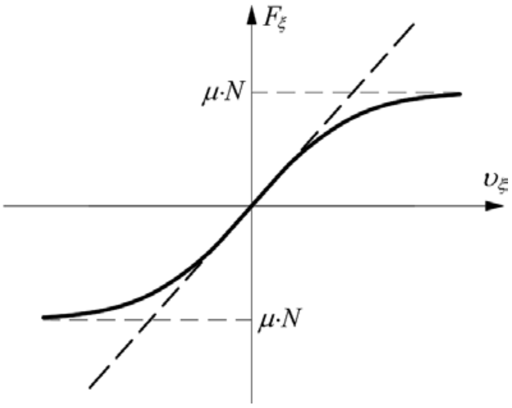

Figure 6: *The change of tangential force in function of creep*

4.1. Carter's theory

In 1926 Carter is developed a theory for solving the problems of rolling with friction which is applied in solution of the tangential contact problem [9, 10]. This theory is especially important because establishes analytical relation between longitudinal creep and tangential force. The wheel is considered as a cylinder and rail as thin plate, under assumption that radius of the wheel is much higher in comparison with the dimensions of the contact surface. In this way, the problem is solved under the assumption of half-space, wherein in the contact surface, in addition to normal stress, there is tangential stress in the longitudinal direction. The distribution of tangential stress and change of tangential force in contact surface according to Carter's theory are shown in Fig. 7.

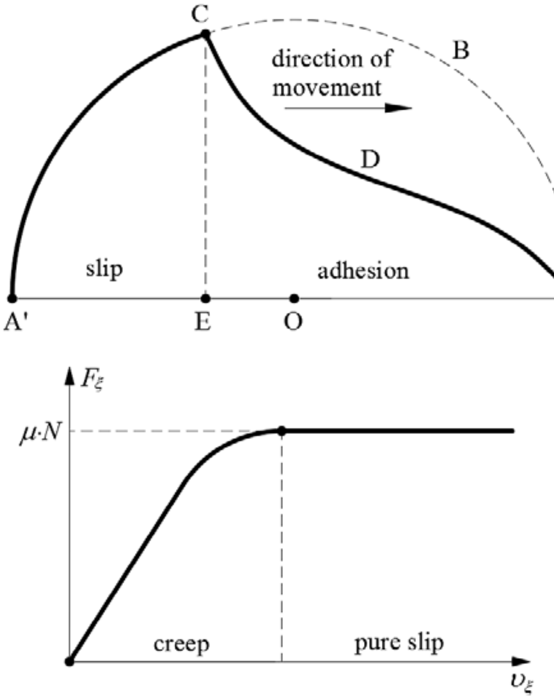

Figure 7: *The distribution of tangential stress and change of tangential force in contact surface according to Carter's theory [13]*

In wheel-rail contact surface according to Carter's theory there are two areas – adhesion area and slip area. A'EOA line in Fig. 7 represents the contact surface during the rolling of wheel over the rail in the longitudinal direction. In the point A observed piece of material comes in contact with the rail, while its contact with the rail ceases in the point A'. Curve ABCA' defines the distribution of limit value of tangential stress in the cross section of the contact surface. Curve ADCA' defines the real distribution of tangential stress in the cross section of the contact surface or in adhesion area and slip area. In the adhesion area (area below curve ADC) there are is absolutely adhesion and no mutual displacement of material particles of the wheel and rail. Thereby, the value of the tangential force is less than the limit value which is determined with the force of Coulomb friction in which there is completely sliding of the wheel along the rail ($F_\xi < \mu N$). Analogously, the tangential stress is less than the limit stress which is determined with the curve ABCA'. In slip area (area below the curve A'C) there is mutual displacement of pieces of material of the wheel and rail. At the same time, the value of the tangential force reaches limit value where is $F_\xi = \mu N$, and tangential stresses are equal to the limit stresses. In the case of the complete sliding of the wheel along the rail, the tangential force in the whole contact surface reaches a limit value at which the tangential stresses are equal to the limit tangential stresses (line ABCA'). This situation usually occurs in cases of traction or braking of railway vehicles or in cases of blocking of wheels for other reasons.

The main disadvantage of Carter's theory is related to the neglecting of lateral and rotational creep. That is why application of this theory in analysis of dynamic behavior of railway vehicles is very limited. Nevertheless, this theory is of a great importance in the study of phenomena related to the wheel-rail contact and forms the basis for the development of advanced theories for solving the tangential contact problem.

4.2. Theory of Johnson and Vermeulen

In 1985 Johnson is expanded Carter's theory on the case of contact of two spheres that roll on one another, wherein in addition to the longitudinal, lateral creep is taken into account [14]. After that he in 1964, together with Vermeulen, defined the theory which establishes analytical relation between creep in longitudinal and lateral direction and appropriate tangential forces [15]. The theory is based on the assumption that between wheel and rail there is only translational creep, while rotational creep is neglected. In contact surface there are two areas – slip area and adhesion area. The adhesion area has an elliptical form which touches the ellipse of contact surface in its headmost point (Fig. 8).

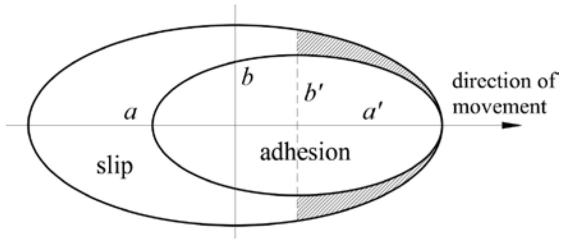

Figure 8: *The areas in contact surface according to the theory of Johnson and Vermeulen [11]*

The experimental tests have shown that error in calculation of the resulting tangential force according to the theory of Johnson and Vermeulen is less than 25%. This is certainly consequence of assumption that adhesion area has an elliptical shape. This theory is limited to the cases where there are only longitudinal and lateral creep, while rotational creep does not exist. This is the main disadvantage because this theory has limited application in analysis of dynamic behavior of railway vehicles.

4.3. Theory of strips of Halling, Haines and Ollerton

In 1963 Halling, Haines and Ollerton are developed an approximate theory for solving the tangential contact problem in case of the elliptical contact surface and pure longitudinal creep [16, 17]. In accordance with this theory, area of contact is divided on a certain number of strips which are parallel to the direction of movement (Fig. 9). Every strip is analyzed by using Carter's theory while mutual interaction between strips is neglected.

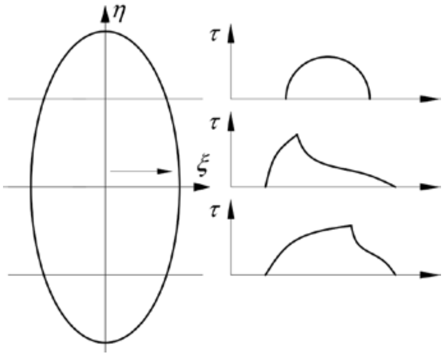

Figure 9: *The contact surface according to the theory of strips of Halling, Haines and Ollerton [11]*

The correctness of the theory of Halling, Haines and Ollerton has been confirmed by experimental tests using the method of photo-elasticity. The results of tests have shown that shape of the adhesion area is very similar to the shape which is assumed with the theory. The main drawback of this theory is limitation on the case of pure longitudinal creep. That is why its using in rail vehicles dynamics is very rare.

4.4. Kalker's extended theory of strips

In 1967 Kalker is expanded the theory of strips of Halling, Haines and Ollerton, whereby, in addition to the longitudinal, lateral and small rotational creep are taken into account [18]. The areas of adhesion and slip according to this theory are shown in Fig. 10.

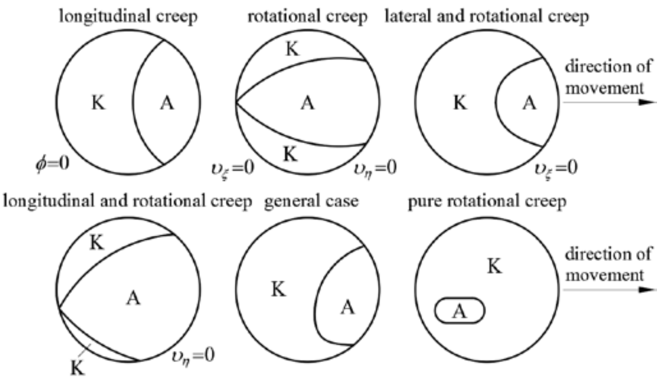

Figure 10: *The areas in contact surface according to the extended theory of strips [11]*

The main drawback of extended theory of strips is limitation on cases with small values of rotational creep and cases in which the longer axis of contact surface is directed in lateral direction regard to the direction of movement. These are the reasons why this theory, despite a relatively accurate determination of the shape and size of the areas of adhesion and slip, are rarely used in analyzing the dynamic behavior of railway vehicles.

4.5. Kalker's linear theory

In 1967 Kalker is developed linear theory of rolling of two elastic bodies in conditions of dry friction [19]. This theory is based on assumption of DePater that for small values of creepages slip area in the contact surface is so small that its influence can be completely neglected [20]. The consequence of this assumption is that in the whole contact surface there is the state of adhesion. The research has shown that Kalker's linear theory gives satisfactory results if the following condition is fulfilled [4]:

$$\left|\upsilon\right| + \left|\phi / 1000\right| \le 0{,}002 \tag{9}$$

In accordance with the Kalker's linear theory, longitudinal and lateral tangential forces and moment of creep are determined from the following expressions:

$$\begin{aligned} F_\xi &= -\kappa_{33} \cdot \upsilon_\xi \\ F_\eta &= -\kappa_{11} \cdot \upsilon_\eta - \kappa_{12} \cdot \phi \\ M_\zeta &= \kappa_{12} \cdot \upsilon_\eta - \kappa_{22} \cdot \phi \end{aligned} \tag{10}$$

The previous expression show that, according to Kalker, longitudinal tangential force does not depend on the lateral and rotational creep, while the lateral tangential force and moment do not depend on the longitudinal creep. The coefficients of creep κ_{11}, κ_{12}, κ_{22}, and κ_{33} are determined on the basis of values of semi-axis of ellipse of contact surface a and b, determined on the basis of Hertz's contact theory and Kalker's coefficients C_{11}, C_{22}, C_{23} and C_{33}:

$$\begin{aligned} \kappa_{11} &= \left(a \cdot b\right) \cdot G \cdot C_{22} \\ \kappa_{12} &= \left(a \cdot b\right)^{3/2} \cdot G \cdot C_{23} \\ \kappa_{22} &= \left(a / b\right)^{2} \cdot G \cdot C_{33} \\ \kappa_{33} &= \left(a \cdot b\right) \cdot G \cdot C_{11} \end{aligned} \tag{11}$$

The coefficients C_{11}, C_{22}, C_{23} and C_{33} are determined from Kalker's table on the basis of Poison's ratio ν_P and ratio of semi-axis of ellipse of contact surface a and b [13]. It is very important to emphasize that Kalker's coefficients mean the existence of dry friction between wheel and rail whereby the coefficient of friction is $\mu \approx 0{,}6$ [4]. However, the practical experiences have shown that there are not always the conditions of dry friction between wheel and rail, or the conditions under which Kalker's linear theory is formulated are not always fulfilled. On a certain sections of the track, due to presence of various impurities such as oil, dirt, dust, moisture, ice, snow, etc., there is reduction of coefficient of friction or reduction of value of limit tangential force at which pure slipping of the wheel along the rail occurs. Taking into account this stochastic change of friction coefficient, this influence is very complex for analytical describing. It is usually taken into account through corrections of creep coefficient in Kalker's linear theory with certain correction factors. Despite of significant approximations under which is defined, Kalker's linear theory has very large application in rail vehicles dynamics, especially in analysis of lateral dynamic stability of railway vehicles at their running on the tangent track.

4.6. Kalker's nonlinear exact numerical theory

All the previously discussed theories have certain limitations and do not provide a complete solution of the tangential contact problem. From this reason Kalker is developed a nonlinear exact numerical theory which is based on numerical solution of the problem by using computers [11, 12]. This theory is based on the condition that real total tangential stress in the contact surface must meet the Coulomb's inequality $\tau \leq \mu \cdot \sigma$. Based on the Kalker's nonlinear theory, in 1967 has been developed a computer program CONTACT which allows universal numerical solution of all types of half-space contact problems. After that, in 1978 has been developed a computer program DUVOROL which is based on the assumption of equal constants of elasticity of bodies in contact [21]. The results of experimental tests conducted on British railways have confirmed that, in conditions without presence of impurities on contact surfaces of wheels and rails, the program DUVOROL gives very reliable results in solution of the tangential contact problem. The main disadvantage of computer programs CONTACT and DUVOROL is very large computing time. That is why these programs are not suitable for usage in analysis of dynamic behavior of railway vehicles.

4.7. Kalker's empirical theory

Kalker is developed empirical theory which establishes relationship between longitudinal and lateral creep and total tangential force in contact surface between wheel and rail [22]. Somewhat simpler empirical expression with reduced accuracy in relation to the Kalker's are defined by Johnson and Vermeulen [15]. In empirical theory, at solving the tangential contact problem, the total normalized creep υ_n is used. It is determined on the basis of components of creepages υ_ξ and υ_η, semi-axis of ellipse of contact surface a and b, shear modulus G, coefficient of friction between wheel and rail μ, normal force N and coefficients of normalization c_1 and c_2. Coefficients of normalization are read from tables on the basis of ratio of semi-axes of ellipse of contact surface a and b, combined Poison's ratio for materials of wheel and rail ν, and elliptical integrals defined in theory of Johnson and Vermeulen [13]. Kalker's empirical theory gives very good results for all eccentricities of ellipse of contact surface and can be used in solving the contact problems in cases when materials of wheel and rail have different constants of elasticity. However, due to the neglecting of rotational creep this theory has very limited application in analysis of dynamic behavior of railway vehicles.

4.8. Approximate nonlinear theory (heuristic nonlinear model)

Researches in area of theory of wheel-rail contact have shown that is necessary to form considerably complex models in order to obtain more accurate results [23]. In this sense, Shen, Hedrick and Elkins are formed an approximate nonlinear theory which is based on heuristic nonlinear model [24]. The theory is based on calculation of tangential forces by using Kalker's linear theory and determination of their limit values by using the theory of Johnson and Vermeulen, wherein the rotational creep is neglected. Input data in the model are ratio of semi-axis of ellipse of contact surface a and b, and normalized creepages $\upsilon_{\xi n}$, $\upsilon_{\eta n}$ and ϕ_n. The ratio of semi-axis of ellipse of contact surface is function of radii of curves of elastic bodies which are in contact which is calculated with Hertz's theory, while normalized creepages are functions of normal force in contact surface between wheel and rail. The researches have shown that heuristic nonlinear model enables accurate description of nonlinear relations in wheel-rail contact in cases when two from three creepages are equal to zero [13]. Despite the fact that neglects the influence of rotational creep to the tangential forces in wheel-rail contact, approximate nonlinear theory is often used in analysis of dynamic behavior of railway vehicles.

4.9. Kalker's simplified theory and computer programs

The main motivation for development of Kalker's approximated theory was very long computer time for numerical solution of the tangential contact problem by using the exact numerical theory. Consequently, Kalker is defined simplified theory on the basis of which a few computer programs for determination of tangential forces and moment in wheel-rail contact are developed. The most famous are SIMROL, ROLCON and FASTSIM [25, 26]. The main version of program SIMROL has been written from the Kalker in ALGOL-60 programming language. Later, it is translated on the programming language FORTRAN-IV by Goree [27]. The program ROLCON has been written in 1978 by the Knothe and colleagues in programing language FORTRAN-IV [28]. Its speed of calculation is five times larger in relation to the program SIMROL. The program FASTSIM also has been written from the Kalker in 1980. It is very simple and its speed of calculation is 25 times larger in regard to the program SIMROL. The main advantage of Kalker's simplified theory is in fact that it enables satisfactory accurate description and solution of the tangential contact problem, while the time of computer's calculations is significantly shorter in regard to the exact numerical theory. Therefore, this theory has by far the greatest application in modeling the wheel-rail contact and analysis of the dynamic behavior of railway vehicles.

5. CONCLUSION

Condition in the contact surface or wheel-rail contact forces during the running of railway vehicles along the track are affected by large number of influential parameters. In addition to the geometries of profiles of the contact surfaces, the significant influence has the friction coefficient between wheel and rail or behavior of adhesion and slip. The important influence have the characteristics of material of the wheel and rail, roughness of contact surfaces, wear, as well as presence of water, oil, dirt, dust, mud, snow, ice and other environmental factors. All these influential parameters during the running along the track have stochastic character which is not possible to describe by analytical procedures in a reliable way. Consequently, the theory of wheel-rail contact is based on large number of approximations which are necessary for analytical formulation and numerical solution of the problem by using some of the previously analyzed methodology. The solving the contact problem and determination of the wheel-rail contact forces by using all mentioned methodology is based on the previous knowledge of creepages in the contact surface. Due to the nonlinearity of the wheel-rail contact geometry and stochasticity of change of parameters which determining the track geometry, accurate determination of creepages in contacts of wheels and rails and its change during the running along the track is very hard and complex. In the analytical models wheelset is usually considered as rigid body with six degrees of freedom. Forming of analytical expressions for determination of longitudinal, lateral and rotational creep in such models is based on a large number of assumptions and approximations without which it is not possible to provide mathematical description and solution of the problem. The most of analysis of dynamic behavior of railway vehicles is based on numerical calculation of creepages on basis of formed analytical models and recorded data about track geometry, after which the theory of contact is used for solution of normal and tangential contact problem and calculation of wheel-rail contact forces. This approach is applied in all modern software packages for simulation of dynamic behavior of railway vehicles. These software packages enable analyzing of dynamic behavior for certain operating conditions still under development phase of new or modification of existing railway vehicles. In this way very expensive experimental tests can be avoided. The simulations are very useful in analysis of certain phenomena, especially in quasi-static conditions of moving. However, due to the large number of simplifications and approximations their results are not enough reliable for derivation of final conclusion about quality of dynamic behavior of railway vehicles. These facts are very important and must be taken into account in any analysis of dynamic behavior or railway vehicles. It can be concluded that the best and the most reliable way for determination of parameters of dynamic behavior of railway vehicles, especially those most important such as wheel-rail contact forces, is experimental testing in exploitation conditions [29]. This fact is confirmed from relevant standards UIC 518 and EN 14363 which prescribe experimental tests in the certification of railway vehicles, while simulations are allowed only in certain cases when validity of developed models is experimentally confirmed.

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