Oscillation of Reservoir of Tank-wagon in Dynamic Longitudinal Load

Dragan Petrović*, Milan Bižić
Faculty of Mechanical and Civil Engineering in Kraljevo, University of Kragujevac, Kraljevo (Serbia)

Tank-wagons are typically designed to satisfy the static forces as well as their enlargement which is caused by the dynamic loads. However, sudden longitudinal impacts may cause a very rapid deformations of structural elements which can lead to the very complex physical phenomena. Apart from the oscillations of structural elements, changes of rheological properties of material, temperature changes, chemical changes, etc., may arise. During these occurrences, the behaviour of the structure can be completely different from the behaviour at the static load. Construction fails to obtain displacements that correspond to rapid changes in load, which can lead to sudden deformation of the structure. The sudden action of longitudinal load causes radial oscillations of reservoir. Thereby, if the load p_x is less than a certain value $p_{x,kr}$, these oscillations will have no increase in the amplitude of deflection around the equilibrium position. On the contrary, if the load p_x is higher than the value $p_{x,kr}$, amplitude of deflection increases with time and the reservoir loses stability. Besides, these oscillations may adversely affect the quality of running and safety of the train. Accordingly, this paper shows an example of tank-wagon and provides a method of determining the critical load from which an unlimited increase in the amplitude of deflection of the tank arises.

Keywords: Oscillation, Reservoir, Tank-wagon, Dynamic longitudinal load

1. INTRODUCTION

The study of behavior of elastic bodies at impact loads is very interesting both from the theoretical and practical aspects. In that case, the most expressed dynamic process of deformation is appeared, which may be characterized by a minimal number of variable parameters. Knowledge of the impact and values of dynamic parameters provides better design of the structure. In addition, the study of the behavior of structure at the dynamic load is necessary for the proper planning, execution and analysis of experimental research.

The reservoir of the wagon is made in the form of a circular cylinder of radius R, of constant thickness h, and length l (Fig. 1). The reservoir is over the edges supported on the underframe, so the supports on one of the ends usually have freedom of movement in the direction of the longitudinal axis. In this way, it is partially protected from the effects of horizontal axial forces that comes from the buffers at the wagons impact.

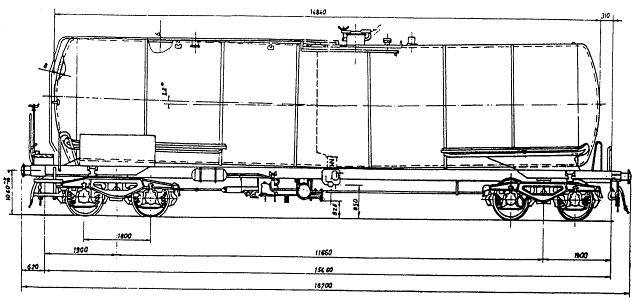


Figure 1: The tank-wagon

During maneuvering and changing of the running regime (acceleration, braking), or impacts caused by the track geometry, significant loads may arise at impact of fluid on the bottom of the reservoir. These loads can cause stresses and strains which can be a threat for the structure of the tank-wagon (Fig. 2).

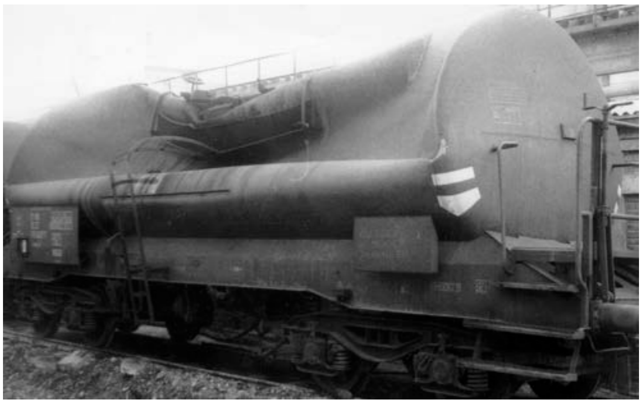


Figure 2: The deformed tank-wagon

2. BEHAVIOR OF ELASTIC BODIES AT IMPACT

The behavior of an elastic body loaded by the forces that do not change over time belongs to the field of statics. Besides, this includes quasi-static cases where the load changes over time is slow. If the load changes over time are fast, as in the case of impact loads, then it belongs to the field of dynamics. In this case, equations of static equilibrium of elastic body should be replaced with the equations of motion. In this case, the effect of dynamic (impulse) load are not transferred immediately in all points of the body. In loaded area there are waves of stresses and strains that have a finite speed of propagation. As in the famous case of propagation of sound in the air, a certain point will be excited only when the wave reaches it. In elastic body there is not one but several types of waves and they have different speeds of propagation. Rapid changes in strains and stresses caused by the impact cannot be precisely determined without considering the wave processes. Therefore, where possible, in the theoretical study of behavior of the elastic body in impact the wave character of propagation of deformation is analyzed.

However, for railway vehicles, where the complex geometry of the supporting structure is present and speeds

*Corresponding author: Dositejeva 19, 36000 Kraljevo, Serbia, petrovic.d@mfkv.kg.ac.rs

of impact are not so large, the model of elastic body which is insensitive to the speed of deformation can be formed. In this way, the local effects related to the triaxial stress state can be avoided. Thereby, the impact is defined with a certain speed of one of the cross-sections of the rod or shell and a mass ratio of the observed element and the load.

Consideration of the impact phenomena in this manner is different from the case where the change of several physical factors is present and wherein the material structure changes are dominant. So, most of the structures subjected to the effects of impact can be treated in this way.

3. DIFFERENTIAL EQUATIONS OF MOTION OF RESERVOIR

Today's constructions of railway vehicles have a supporting structure that is part of the polygonal shell which is combined with elements of beams and plate. Design of these structures requires a detailed study of their behavior. In that sense, the considerations of dynamic problems of railway vehicles can be found in the works of Timoshenko [1, 2].

The reservoir of tank-wagon is loaded as shown in Fig. 3.

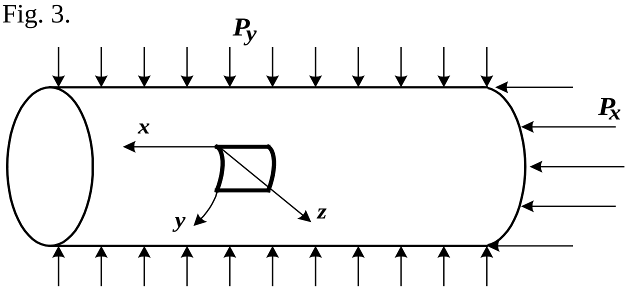


Figure 3: The loads of reservoir of the tank-wagon

The general equations of motion of the cylindrical shell given by the following expressions are used for analysis [3]:

$$\frac{D}{h}\nabla^4(w-w_0)=L(w,\Phi)+\nabla_k^2\Phi+\frac{q}{h}-\frac{\gamma}{g}\frac{\partial^2 w}{\partial t^2} \tag{1}$$

$$\frac{1}{E}\nabla^4\Phi=-\frac{1}{2}\left[L(w,w)-L(w_0,w_0)\right]-\frac{1}{R}\frac{\partial^2}{\partial x^2}(w-w_0) \tag{2}$$

where:
h – thickness of reservoir,
D – cylindrical rigidity of shell,
∇^4 – double Laplace operator,
w – deflection of reservoir,
w_o – initial deflection of reservoir,
Φ – function of stress,
γ – specific gravity of material,
g – gravity acceleration,
t – time,
q – cross load of shell,
E – modulus of elasticity,
R – radius of cylinder.

By its structure the previously given system of equations describing the propagation of waves of tension-compression and bending-slipping. Given that the exact method of integrating of given equations have not yet been developed, their approximate solutions are found in the form of order. So, the solution of defelction of reservoir can be asumed as the folowing order:

$$w=f(t)\cdot\left(sin\frac{m\pi x}{l}\cdot sin\frac{ny}{R}\right) \tag{3}$$

where:
$f(t)$ – amplitude of deflection,
l – lenght of cylinder,
R – radius of cylinder,
m – number of half-waves per length of cylinder,
n – number of waves per radius.

Experimental tests have shown that deflections of reservoir towards the center of curvature and from the center of curvature are not the same. Deflections directed towards the center of curvature are greater than the deflections directed from the center of curvature. Therefore, for assumed displacements w, equation (3) is amended with member which takes into account the asymmetry of amplitude of deflection:

$$w=f(t)\cdot\left(sin\frac{m\pi x}{l}\cdot sin\frac{ny}{R}+\psi\cdot sin^2\frac{m\pi x}{l}\right) \tag{4}$$

where:
ψ – time function of correction of amplitude.

Additionally it is assumed that the reservoir has an initial deflections w_0, amplitudes of deflection $f_0(t)$, or the irregularities of the same character as well as the total deflection w:

$$w_0=f_0(t)\cdot\left(sin\frac{m\pi x}{l}\cdot sin\frac{ny}{R}+\psi\cdot sin^2\frac{m\pi x}{l}\right) \tag{5}$$

Assumption that $\alpha=m\pi/l$ and $\beta=n/R$ leads to the following assumed total and initial deflections:

$$\begin{aligned} w &= f(t)\cdot(sin\,\alpha x\cdot sin\,\beta y+\psi\cdot sin^2\alpha x)\\ w_0 &= f_0(t)\cdot(sin\,\alpha x\cdot sin\,\beta y+\psi\cdot sin^2\alpha x)\end{aligned} \tag{6}$$

The operators $L(w,\Phi)$, $L(w,w)$, $L(w_0,w_0)$. ∇^4 i ∇_k^2 have the following shape:

$$\begin{aligned} L(w,\Phi) &= \frac{\partial^2 w}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2\Phi}{\partial x^2}-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y}\\ L(w,w) &= 2\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\\ L(w_0,w_0) &= 2\left[\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}-\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2\right]\\ \nabla^4 &= \nabla^2\nabla^2=\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^4}\\ \nabla_k^2 &= k_y\frac{\partial^2}{\partial x^2}+k_x\frac{\partial^2}{\partial y^2}\end{aligned} \tag{7}$$

Depending on the oscillation form of the reservoir, the parameters m and n are changing (Fig. 4). These two parameters can be influenced by structural modifications or installation of rings on the reservoir (Fig. 5).

From the previous considerations, it can be concluded that in the case of plates and shells there is mixed stress state or there are stresses from bending and forces (Figs. 6 and 7). Thereby, two extreme cases can be distinguished:

- no-moment stress state, when the bending stresses are negligible in comparison to the stresses induced by the forces (Fig. 6),
- pure moment condition, when the stresses induced by the forces are negligible in comparison to the bending stresses (Fig. 7).

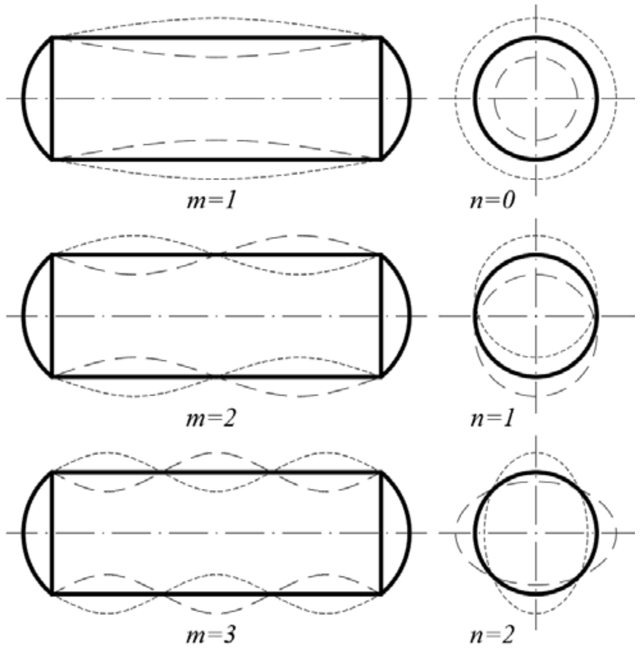


Figure 4: The different forms of oscillations of reservoir

Figure 5: The rings on reservoir of tank-wagon

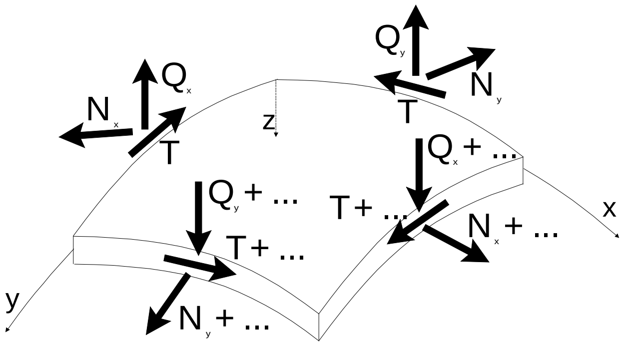


Figure 6: The normal forces N, lateral forces Q and tangential forces T on the element of shell

If the radii of curvature along the *x* and *y* directions are denoted with ρ_x and ρ_y, the curvature of the corresponding surfaces will be $k_x=1/\rho_x$ and $k_y=1/\rho_y$.

Given in mind that their thickness is smaller in comparison to the other two dimensions, plates and shells are sensitive to the bending where there are large stresses and deflections. Such condition is very dangerous in practice, so it is natural that it should be avoided. The no-moment condition is much more convenient, where plates and shells are uniformly loaded over the entire thickness while the external load is the most rationally transferred on the supports.

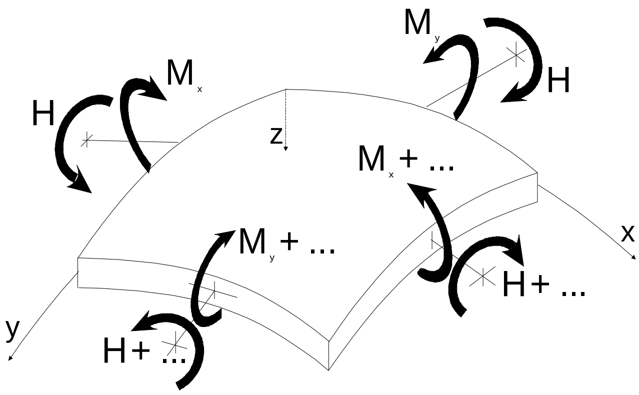


Figure 7: The bending moments M_x and M_y, and torque moment H on the element of shell

In cases where no-moment condition cannot be realized it is possible to find alternative solutions. The stress state of a mixed type is localized at the ends of plates and shells (eg. on the bottom of the reservoir), while elsewhere there is no-moment stress state which significantly simplifies the mathematical operations.

This is in connection with the term "edge effect" in which at moving away from the edge of shell or plate the stress state of mixed type significantly decreases. It is important to note that the source of "edge effect" can be not only the edge of plate and shell but also any line of middle surface with radical change of curve or thickness of plate and shell.

4. STRESS FUNCTION

The shell of constant thickness *h* with a coordinate system in the central plane of the shell is considered in this chapter (Figs. 6, 7 and 8). In order to transformation of three-dimensional into two-dimensional problem, the hypothesis of undeformed normals (Kirchhoff–Love hypothesis) is applied [3].

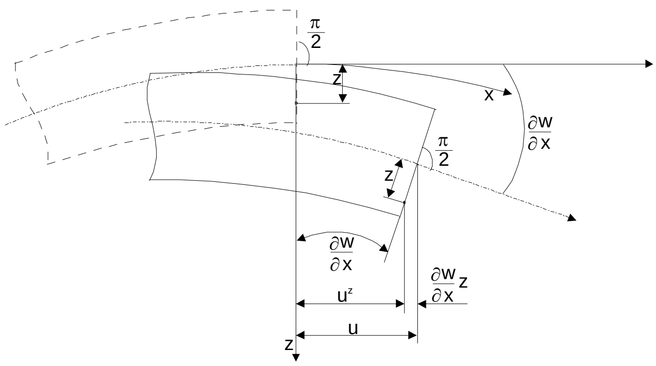


Figure 8: The deformation of shell element according to the Kirchhoff–Love hypothesis

The basis of this hypothesis is assumption that each fiber which is perpendicular to the middle plane after deforming remains right and normal to the median plane, which causing that the length of the fiber along the length of the shell remains unchanged. Therefore, the normal stresses in direction of normal on the median plane can be neglected in comparison with the basic stresses. In the theory of the shell, the basic stresses are related to the normal and tangential stresses in the central plane and in layers which are parallel to the central plane. This hypothesis is considered as the model of the first approximation. It is very suitable for solving of many static and dynamic problems while the obtained results are suitable for the practical use.

In order to simplification of writing and solving the equations of motion of shell (1) and (2), the stress function Φ is introduced in the central plane of the shell. It is assumed that the stresses acting on the unit length of the shell. The relations of normal stresses (σ_x, σ_y), tangential stresses (τ), normal forces (N_x, N_y), tangential forces (T) and thickness of the shell h, with stress function Φ are given with the following expresions [3]:

$$\sigma_x = \frac{N_x}{h} = \frac{\partial^2 \Phi}{\partial y^2} \tag{8}$$

$$\sigma_y = \frac{N_y}{h} = \frac{\partial^2 \Phi}{\partial x^2} \tag{9}$$

$$\tau = \frac{T}{h} = -\frac{\partial^2 \Phi}{\partial x \partial y} \tag{10}$$

5. STABILITY OF RESERVOIR OF TANK-WAGON IN DYNAMIC LONGITUDINAL LOAD

The theoretical considerations from the previous chapters are applied on concrete example of the tank-wagon (Fig. 1) with the following characteristics:

E=21·10^{10} N/m^2 – elastic modulus of material of reservoir,
ν =0,3 – Poisson's ratio,
h= 6 mm – thickness of reservoir,
R=1450 mm – radius of cylinder,
l=14200 mm – length of reservoir.

For solution of equations (1) and (2), the method of Bubnov–Galerkin is applied [4]. In general case, the loads of the reservoir along the directions x, y and z are as follows:

q – lateral load which is normal to the medium surface,
p_x – pressure or tension load which is acting along the x direction,
p_y – pressure or tension load which is acting along the y direction.

External loads for the reservoir of tank-wagon along the directions x and y are as follows:

$$\overline{p}_x = p_x \cdot R / 2h \qquad \overline{p}_y = p_y \cdot R / h \tag{11}$$

Solving the equations (1) and (2) is performed by applying the Bubnov–Galerkin method [4], wherein X is the following function:

$$X = \frac{D}{h}\nabla^4(w-w_o) - L(w,\Phi) - \frac{1}{R}\frac{\partial^2\Phi}{\partial x^2} - \frac{q}{h} + \frac{\gamma}{g}\frac{\partial^2 w}{\partial t^2} \tag{12}$$

Function X must be equal to zero for all points of the reservoir and must satisfy the following equations:

$$\int_0^{\ell}\int_0^{2\pi R} X \sin\alpha x \cdot \sin\beta y dxdy = 0 \tag{13}$$

$$\int_0^{\ell}\int_0^{2\pi R} X \sin^2 \alpha x dxdy = 0 \tag{14}$$

By using the boundary and initial conditions, the equation (12) is solved, wherein is obtained the following square of frequency of natural oscillations of the reservoir:

$$\omega_0^2 = c^2\left[\frac{\alpha^4}{R^2\left(\alpha^2+\beta^2\right)^2} + \frac{h^2\left(\alpha^2+\beta^2\right)^2}{12\left(1-\nu^2\right)}\right] \tag{15}$$

The speed of propagation of elastic waves is:

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{Eg}{\gamma}} \tag{16}$$

As is known, the reservoir has an infinite number of degrees of freedom. This means that the number of natural frequencies is infinitely large, while each frequency has its own form of oscillation.

Solution of the equation (15) will be periodic if ω_o^2 <0, or periodic if ω_o^2 >0, wherein the amplitudes of the deflection will increasing with time, which leads to the losing of stability of the reservoir.

The diagrams of changes of the natural oscillations ω_o [Hz] of the reservoir of tank-wagon in dependence of change of number of waves per radius n and in dependence of number of half-waves per length of cylinder m, as well as the diagram of change of ω_o in function of change of parameters m and n are shown in Fig. 9.

Considering that $\overline{p}_y$ =0, from the condition $\omega_o^2 - \frac{\alpha^2}{\rho}\overline{p}_{kr}$ =0, the critical load is determined as:

$$\overline{p}_{kr} = E\left[\frac{\alpha^2}{R^2\left(\alpha^2+\beta^2\right)^2} + \frac{h^2\left(\alpha^2+\beta^2\right)^2}{12\left(1-\nu^2\right)\alpha^2}\right] \tag{17}$$

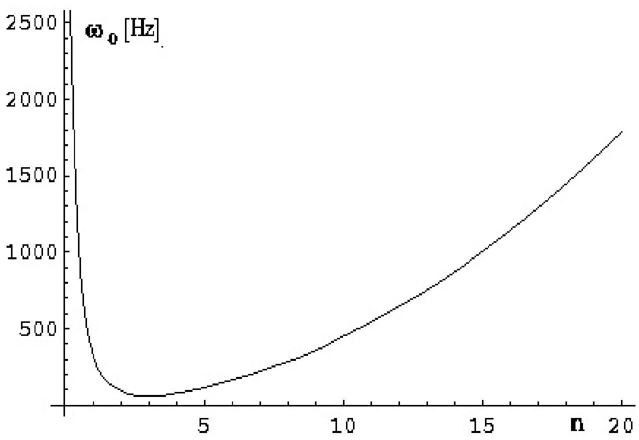
In order of analytical determining of the minimum of function $\overline{p}_{kr}$, the first derivative of the function should be equated with zero:

$$\overline{p}_{kr}{}^{min} = E\frac{h}{R}\frac{1}{\sqrt{3\left(1-\nu^2\right)}} \tag{18}$$

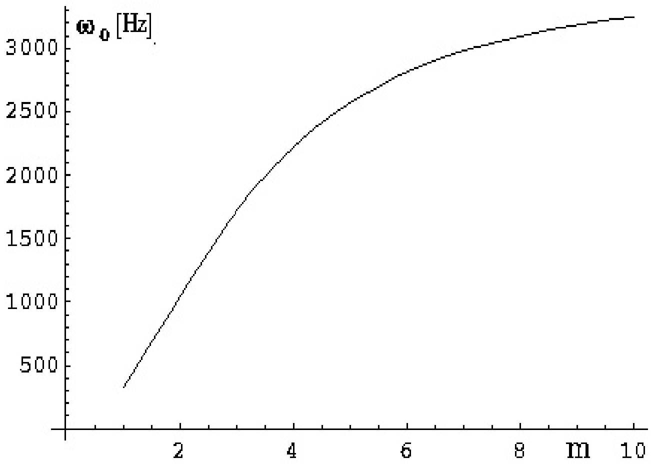
For the observed reservoir of the tank-wagon is $\overline{p}_{kr}{}^{min}$=5,259 10^8 N/m^2, wherein minimal value of stress in the direction x caused by the external load is determined from the expression (11), as follows:

$$p_{x,kr}{}^{min} = \overline{p}_{kr}^{min} \cdot \frac{2h}{R} = 43{,}52 \cdot 10^5 \quad \frac{N}{m^2} \tag{19}$$

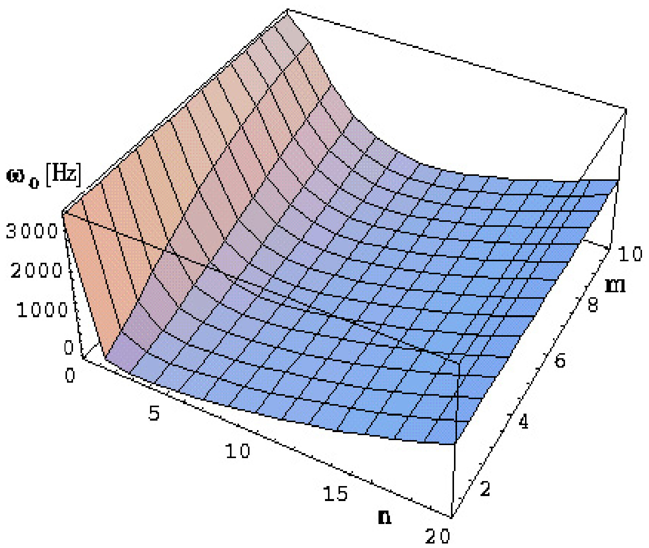
The diagram of change of the critical load in function of change of parameters m and n is shown in Fig. 10, where the minimum of function $\overline{p}_{kr}$ is marked with dart.



a)



b)



c)

Figure 9: The change of frequency of natural oscillations of reservoir of tank-wagon in function of change of parameter n (a), parameter m (b) and parameters m and n (c)

In the case of a longitudinal impact load $p_x(t)$ reservoir will lose stability when active load reaches a value $p_{x,kr}=p_{x,kr}^{min}$. Thereby, from a set of possible types of losses of stability, reservoir will be distorted with forming of m and n half-waves, which are given in the above diagram. If any external cause excites the distortion in some another form, the critical value of load $p_{x,kr}$ will be greater from the minimal critical load $p_{x,kr}^{min}$ along the direction *x*.

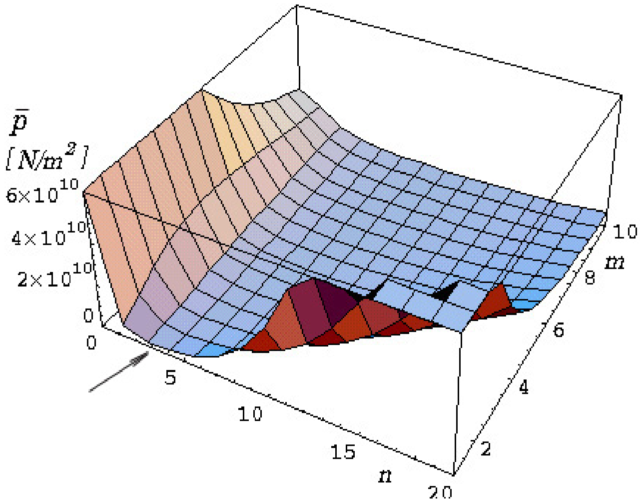


Figure 10: The change of critical load $\bar{p}_{kr}$ in function of parameters m and n

At impact dynamic loading, the analysis of influences of shapes of oscillation on the occurrence of lost of stability has also been made, and the results are shown in Fig. 11. From the diagram it can be concluded that at dynamic action of load that arises at the impact of wagons at the speed of 19.5 m/s, critical load obtains at the number of semi-waves *m*=1 and *n*=4. In addition to that, the value of minimum critical load which the tank can stand is for higher at dynamic load in relation to static load.

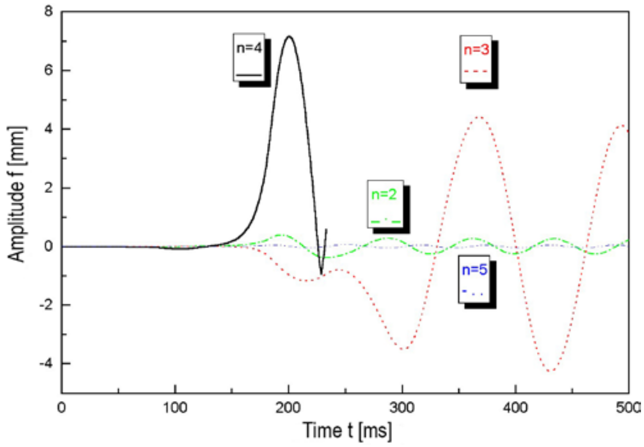


Figure 11: Influence of shapes of oscillation on amplitude of deflection of the tank for m=1

6. CONCLUSION

The aim of the research presented in this paper is related to the stability of the reservoir of tank-wagon at action of impulse loads. In forming the mathematical model, the theoretical assumptions of the nonlinear dynamics of plates and shells are used. Established mathematical model is described via the coupled partial differential equations which are solved by using the Runge-Kutta method of fourth order. The convergence of the obtained solutions is checked and realistic picture of the physical phenomena is obtained. The minimal critical load at which amplitude of deflection suddenly increase or when the reservoir losing stability at the dynamic longitudinal load, is defined.

In forming the mathematical model, the influence of initial deflection and oscillation forms on the stability of the reservoir is taken into account. If there is a possibility of adjusting the number of half-waves *m* and *n*, then it is possible to enlarge the dynamic load that can withstand a reservoir, without loss of the stability.

The developed model of oscillation of the reservoir of tank-wagon with appropriate initial and boundary conditions can be applied to all types of reservoirs exposed to the dynamic loads.

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