Proceedings

The 7th International Congress of Serbian Society of Mechanics

Sremski Karlovci, June 24-26, 2019

Edited by:

Mihailo Lazarević
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Andjelka Hedrih
Bojan Jeremić

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Foreward

The present volume contains plenary lectures, abstracts and papers of young authors competing for the „*Rastko Stojanović''* award at the 7th International Congress of Serbian Society of Mechanics. The objectives of this Congress, to be held at in Sremski Karlovci during the period 24th -26th June 2019, are to review and discuss some of the latest trends in various fields of theoretical and applied mechanics as well as it aims to bring together the scientific communities of theoretical and applied mechanics in an effort to facilitate the exchange of ideas on topics of mutual interests, and to serve as a platform for establishing links between research groups with complementary activities.

We are happy to report that the number of accepted papers to be presented at the 7th Congress is 119. In addition, among them, 8 invited plenary lectures were presented by the authors from Italy,China, Greece, Croatia,Hungary and Serbia. Also, we have 4 invited speakers for Mini-Simposia. Accepted papers were grouped in the following sections General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies, Control and Robotics, and Interdisciplinary and Multidisciplinary Areas. Also, the three Minisymposia were organized with following topics: Nonlinear Dynamics, Bioengineering,Turbulence, Waves and diffusion in complex media and Biomechanics and Mathematical Biology.

The Editors would like to express their thanks to all participants of the 7th Congress of Mechanics.First, to the authors of the papers whose quality work is the essence of this event. Next, to of the papers whosethe distinguished invited lecturers who kindly accepted the invitation to come to Congress and helped make it success. We owe great thanks to the reviewers of the papers, to the members of the Scientific and Organizing Committee. Also, special thanks to the organizers of the Mini-symposia on Nonlinear Dynamics, Bioengineering,Turbulence, Waves and diffusion in complex media and Biomechanics and Mathematical Biology. The support of the members of Steering Committee of Serbian Society of Mechanics in organizing this event is also appreciated. Finally, special thanks are also due to those organizations which supported financially this Congress: Serbian Society of Mechanics, Ministry of Education, Science and Technological Development of the Republic of Serbia, Faculty of Mechanical Engineering, University of Belgrade, Belgrade and Serbian Academy of Sciences and Arts- Branch in Novi Sad, Provincial Secretariat for Higher Education and Scientific Research.

It is our great pleasure to welcome you with us at the 7th Congress International Congress of Serbian Society of Mechanics. We would like to wish all participants of this Congress a warm welcome to our country, our Serbian Society of Mechanics and Venue Congress place at *the Karlovci Gymnasium*, Sremski Karlovci, Serbia.

Sremski Karlovci, June, 2019

The Editors
Mihailo Lazarević, Srboljub Simić
Damir Madjarević,Ivana Atanasovska
Anđelka Hedrih,Bojan Jeremić

Technical program

SUNDAY, June 23, 2019

19:30 *Welcoming Coctail* (Hotel Prezident, Main Hall)

MONDAY, June 24, 2019

8:00 – 8:45 *Registration of participants* (*Main Hall, The Karlovci Gymnasium)*

Chairs: Srboljub Simić, Mihailo Lazarević

8:45 – 9:20 (*Congress hall, The Karlovci Gymnasium)*

- Radovan Kovačević, *Director of the Karlovci Gymnasium, Welcome address*
- Academician Teodor Atanacković*, Novi Sad branch of SASA, Welcome address*
- Aleksandar Stojkecić, *Historical notes - Sremski Karlovci*
- Prof.Mihailo P. Lazarević, *the President of Serbian Society of Mechanics ,Welcome address*

Plenary Lectures (*Congress Hall*)
Chairman: Katica *R.* (Stevanović) Hedrih

9:20 - 10:05 P-1 Walter Lacarbonara
ASYMPTOTIC RESPONSE OF SYSTEMS AND MATERIALS WITH HYSTERESIS

10:05 - 10:50 P-2 Zdravko Terze, et al.
LIE GROUP DYNAMICS OF MULTIBODY SYSTEM IN VORTICAL FLUID FLOW

10:50 - 11:15 *Coffee Break* (*Main Hall*)

11:15 - 13:00 Parallel Sessions

Session	G1	S1	M4	C1
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*
11:15	G1a	S1a	M4a	C1a
11:35	G1b	S1b	M4b	C1b
11:55	G1c	S1c	M4c	C1c
12:15	G1d	S1d	M4d	C1d
12:35	G1e	S1e	M4e	C1e
12:55		S1f	M4f	

13:10 - 14:40 *Lunch* *(Restaurant Bermet*)

Plenary Lecture (*Congress hall*)

Chairman: Srboljub Simić

14:40 - 15:25 P-3 HongGuang Sun, Yong Zhang
ANOMALOUS DIFFUSION: MODELING AND APPLICATION

15:30 -17:30 *Social program* (*excursion to Monasteries at Fruska Gora)*

17:30 - 19:10 Parallel Sessions

Session	G2	S2	M4	M1	M1
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*	
17:30	G2a	S2a	M4g	M1p*	17:30
17:50	G2b	S2b	M4h	M1a	18:00
18:10	G2c	S2c	M4i	M1b	18:20
18:30	G2d	S2d	M4j	M1c	18:40
18:50	G2e	S2e	M4k	M1d	19:00
19:10		S2f	M4l	M1e	19:20

TUESDAY, June 25, 2019

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Chairman: Zdravko Terze

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CONTINUUM MECHANICS AND NONEQUILIBRIUM THERMODYNAMICS

9:45 - 10:30 P-6 Dušan. Zorica
HEREDITARINESS AND NON-LOCALITY IN WAVE PROPAGATION MODELLING

10:30 - 10:50 *Coffee Break* (*Main Hall*)

10:50 - 13:00 Parallel Sessions

Session	G3	M1	M2	M3	M3
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*	
10:50	G3a	M1f	M2a	M3a	10:50
11:10	G3b	M1g	M2b	M3b	11:20
11:30	G3c	M1h	M2c	M3c	11:40
11:50		M1i	M2d	M3d	12:00
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12:30		M1k	M2f		

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Plenary Lecture (*Congress hall*)
Chairman: Dušan Zorica

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15:20 - 17:20 Parallel Sessions

Session	S3	M1	M2	M3
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*
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15:40	S3b	M1m	M2h	M3g
16:00	S3c	M1n	M2i	M3h
16:20	S3d	M1o	M2j	M3i
16:40	S3e	M1r	M2k	M3j
17:00	S3f			

17:00 - 18:00 Round table: HARMONIZATION AND MODERNIZATION OF THE CURRICULUM IN ENGINEERING MECHANICS

17:00-17:15 Katica R. (Stevanović) *Hedrih, Academisian LJUBOMIR KLERIĆ (June 29, 1844- January 21, 1910); Dedicated to Jubilee 175 years from birthday*

18:00 - 19:00 General Assembly Meeting of Serbian Society of Mechanics (*Congress Hall*)

19:00-19:30 W*ine tasting (* winery "Bajilo")

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Session	I1	S4	M2	M5	M5
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*	
10:30	I1a	S4a	M2l	M5a	10:30
10:50	I1b	S4b	M2m	M5b	11:00
11:10	I1c	S4c	M2n	M5c	11:20
11:30	I1d	S4d	M2o		
11:50	I1e	S4e	M2p		

12:10 - 12:35 *Coffee Break* (*Main Hall*)

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Session	F1	S5	M2	M5
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*
13:00	F1a	S5a	M2r	M5d
13:20	F1b	S5b	M2s	M5e
13:40	F1c	S5c	M2t	M5f
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G2c: Sreten Mastilović
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G2d**:** Sreten Mastilović
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G3b: Emina Dzindo, Simon A. Sedmak, Milan Travica
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Organizers: *Katica R. (Stevanović) Hedrih, Ivana Atanasovska*
Mathematical Institute SASA, Belgrade

M1_1 *Chairs: Katica R. (Stevanović) Hedrih, Ivana Atanasovska*

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M1_2 *Chairs: Alexander Prokopenya, Mirjana Filipović*

M1f: Stevan R. Maćešić, Željko D. Čupić, Milorad M. Anđelković, Ana D. Stanojević, Vladimir M. Marković, Ljiljana Z. Kolar-Anić
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CORRECTION OF THE STRAIN FIELD OF LINEAR TETRAHEDRAL FINITE ELEMENT USING STRAIN SMOOTHING METHOD

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Abstract

The purpose of this paper was analyzing the performance of the linear tetrahedral finite element. As 4-node tetrahedrons are low-order elements that have a very stiff behavior with a constant strain field in bending problem analysis, this paper presents the Strain Smoothed Element Method [1], which gives considerably better performances of the linear elements comparing to standard formulation, without introducing significant changes in FEM. Taking constant strains of the adjacent elements into consideration, using this method the linear strain field is created in the observed element, which achieves greater precision and hence lowering the discontinuity of the stress on the borders between elements.

In the second part of the paper, numerical examples that encompass two plane stress problems were used to examine the performances of the linear tetrahedral element, which has already been implemented within the program package PAK, in comparison with the performances of the element with a corrected strain field, shown in [1]. As mentioned problems that we encounter in modeling with linear elements are overcome in practice with higher order approximation, performance of tetrahedral element with parabolic interpolation of the displacement field is also analyzed with numerical examples. The comparatively presented results obtained from the program packages PAK and Nastran display great agreement of these two software packages and significantly bigger precision of elements with a corrected strain field compared to the constant strain element. Upon comparing the parabolic and corrected elements, a better behavior was observed in the parabolic element. However, as this is an element that has greater number of degrees of freedom, the obtained results are fully justified.

Key words: low-order elements, tetrahedral elements, stiffness behavior, strain smoothed element method, finite element method

1. Introduction

Due to the elementary formulation which refers to linear interpolation of the displacement field, linear tetrahedral elements belong to the category of low-order finite elements, so they represent introductory elements of the FEM concept. Also, a crucial advantage of these elements, in regard to other finite elements, is the possibility of automatic mesh generation in most software packages that implement the finite element method, even for very complex geometries. Because of all of the above, these elements are very convenient for software implementation and structure analysis.

On the other hand, the approximation of the displacement field with linear functions, which define constant strain and stress field within each element is the main disadvantage of using linear tetrahedral finite element. One example of the constraints in using these elements are bending problems in which modeling with low order elements leads to a discretization model which is stiffer than the real problem, so the resulting deformations are smaller than expected. This is partly caused by the constant stress field of these elements, which is linear in bending problems, so with a small number of low-order elements it isn't possible to accurately approximate linear varying stress field of bending problem. Except overly stiff behavior, volumetric locking is also a known problem and it happens for materials whose Poisson's ratio values are close to one half. The material then exhibits incompressible behavior, i.e. it doesn't change its volume with the changing of stress. In the discretized model this is manifested as mesh locking – the inability of the mesh to be deformed.

Listed limitations indicate that a large number of these elements is necessary for adequate approximation of nonlinear physical quantity field. Thus, a question regarding big strain and stress discontinuity on the borders between elements arises, which the displacement approach in FEM includes for a coarse mesh.

In order to overcome these disadvantages, numerous numerical methods for increasing the accuracy of low-order elements' coarse mesh have been proposed in literature for decades back [2]. Method that has been proven to be very efficient in FEM is the strain smoothing method [3,4], which without introducing crucial changes to the FEM concept leads to much better convergence behavior and accuracy of linear elements solutions. As early as 2000s a whole family of smoothing methods have been developed, which have significantly expanded the application domain of low order elements [3-8]. Method for smoothing strain field [1], which has proven to be very efficient in solving overly stiff behavior problem of linear tetrahedral elements, is presented in this paper.

In the sections that follow, first of all the isoparametric formulation of linear tetrahedral elements is briefly presented. In section 3 the strain smoothing element method is described. At the end of the paper, numerical examples which include two plane stress problems showed abovementioned element disadvantages, that are comparatively presented in regard to the performance of the element with the corrected strain field.

2. Linear tetrahedral finite element

In this chapter, elementary equations of simplest 3D finite element, 4-node tetrahedron, will be presented. In literature it is often called the constant strain element, because the displacement field is interpolated with a linear polynomial function. Depending on the coordinate system, multiple formulation approaches have been developed [9,10], so the most efficient, isoparametric, formulation will be briefly presented.

2.1 Isoparametric formulation of linear tetrahedral finite element

Linear tetrahedral element is shown at Figure 1a in the Cartesian coordinate system. Geometry interpolation for 4-node tetrahedral element is in the form of:

$$x_i = \sum_{k=1}^{4} h_k X_i^k \qquad i = 1,2,3 \tag{1}$$

in which $x_i(r,s,t)$ are the coordinates of tetrahedral element's material points, X_i^k are coordinates of nodal point k in global Cartesian coordinate system and $h_k(r,s,t)$ are interpolation functions of nodal point k, where $i=1$ corresponds to axis x, $i=2$ to axis y and $i=3$ to axis z.

Similar to the geometry interpolation, displacement of the linear tetrahedral element's material points $u_i(r,s,t)$ is calculated as follow:

$$u_i = \sum_{k=1}^{4} h_k U_i^k \qquad i=1,2,3 \tag{2}$$

in which U_i^k is the displacement of node k in direction of axis i in global Cartesian coordinate system x,y,z.

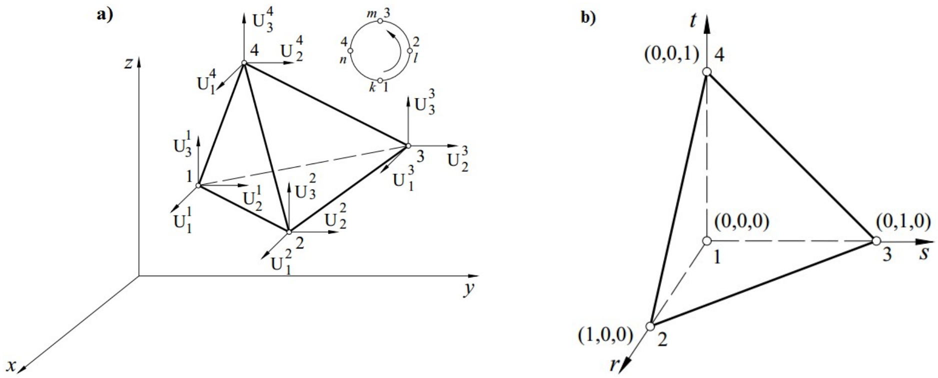

Figure 1. a) Linear tetrahedral element in global Cartesian coordinate system b) Natural coordinate system of 4-node tetrahedral element

Interpolation functions $h_k(r,s,t)$ are defined in the natural coordinate system of the element, Figure 1b, whereby in case of the linear tetrahedral element they have the following form [9]:

$$h_1 = 1-r-s-t,\;\; h_2 = r,\;\; h_3 = s \;\text{ и }\; h_4 = t \tag{3}$$

In each material point of the tetrahedral element we may define all of six strain components, whereby the strain tensor can be presented in the form of:

$$\mathbf{e} = \mathbf{BU} = \left[\mathbf{B}^1\; \mathbf{B}^2\; \mathbf{B}^3\; \mathbf{B}^4\right]\left[\mathbf{U}^1\; \mathbf{U}^2\; \mathbf{U}^3\; \mathbf{U}^4\right]^T \tag{4}$$

in which $\mathbf{U}$ represents total displacement vector of element's nodes, $\mathbf{U}^k$ is displacement vector of node k, and $\mathbf{B}$ is matrix of derivatives of interpolation functions.

Submatrices $\mathbf{B}^k$ of node k of the linear tetrahedral element are equal to:

$$\mathbf{B}^k = \frac{1}{6V}\begin{bmatrix} b_k & 0 & 0 \\ 0 & c_k & 0 \\ 0 & 0 & d_k \\ c_k & b_k & 0 \\ 0 & d_k & c_k \\ d_k & 0 & b_k \end{bmatrix} \qquad k,l,m,n = 1,2,3,4$$

$$b_k = (-1)^k \begin{vmatrix} 1 & X_2^l & X_3^l \\ 1 & X_2^m & X_3^m \\ 1 & X_2^n & X_3^n \end{vmatrix},\; c_k = (-1)^k \begin{vmatrix} X_1^l & 1 & X_3^l \\ X_1^m & 1 & X_3^m \\ X_1^n & 1 & X_3^n \end{vmatrix},\; d_k = (-1)^k \begin{vmatrix} X_1^l & X_2^l & 1 \\ X_1^m & X_2^m & 1 \\ X_1^n & X_2^n & 1 \end{vmatrix} \tag{5}$$

Coefficients b_k, c_k and d_k are obtained by cycling permutation of index k,l,m,n, i.e. - $klmn \rightarrow lmnk \rightarrow mnkl \rightarrow nklm$.

3. Strain smoothing element method

Smoothed finite element method [3,4] represents an approach of smoothing strain field which is applied in standard finite element analysis primarily in order to improve performance of low-order elements. Combining the technique of strain smoothing, which was originally applied for stabilization of nodal integration in the Galerkin's mesh free method [11], and the finite element concept, a method which significantly improves characteristics and convergence behavior of low-order elements was created. Domain discretization, i.e. mesh of elements, is created in a same manner as in the standard FEM, while the primary idea of this method is based in spatial averaging of the standard strain gradient field, or so-called smoothing functions. Thus, primary difference in comparison to the finite element method is the way of obtaining the stiffness matrix, which is not calculated by integration over the element, yet over the smoothing domain. This way, depending on the way of forming integration domains i.e. smoothing domains, several types of these methods have been formulated, i.e.: cell-based smoothed finite element method – CS-FEM [3,4], node-based smoothed finite element method – NS-FEM [5], edge-based smoothed finite element method – ES-FEM [6,7], face-based smoothed finite element method – FS-FEM [8] and others.

The first applications of strain smoothing in FEM occurred in 2006, when Liu et al. [3] recommended smoothing of the strain field of the 4-node quadrilateral element. He named the created method cell/element-based smoothed finite element method (SFEM or CS-FEM). Cell-wise strain smoothing technique was applied in the standard formulation of these elements with the purpose of overcoming the limit related to the usage of highly distorted elements and the coordinate mapping from a Cartesian into a natural coordinate system. As presented in [3], in comparison to FEM, this method achieves much greater solution accuracy and, as there isn't an element shape constraint, the method is much simpler and more robust then FEM. The only limitation of the method is material incompressibility, i.e. volumetric locking. After quadrilateral elements, this method was applied to general n - sided polygonal elements [12], with the purpose of reducing the sensitivity to volumetric locking. In 2012 Nguyen-Xuan et al. expanded this method to 8-noded hexahedral elements [13].

NS-FEM method, for which the stiffness matrix is calculated based on smoothing domains created using elements' nodes, primarily was used on the mesh of polygonal elements [14]. Other than enabling the usage of n-sided polygonal elements, this method lowers the sensitivity of elements to volumetric locking. The method was later extended to linear tetrahedral elements [5], which achieved higher accuracy and convergence of the stress solution. The main disadvantages of this method compared to FEM are greater computational expenses, due to greater bandwidth of stiffness matrix. FS-FEM method has shown to be very effective in decreasing the sensitivity of 3D elements to volumetric locking [8]. This method is a natural extension of the ES-FEM method for 2D problems. By applying this method to tetrahedral elements, much better performances are achieved compared to standard FEM, even in the domain of nonlinear analysis. Due to its simplicity, this method was applied even with visco-elastoplastic analysis of 3D problems [15]. But, as is the case with NS-FEM, the computational expenses here are also greater than with standard FEM. ES-FEM method is known in literature as the most effective method of strain smoothing. This method calculates the stiffness matrix along smoothing domains created based on element edges, and its primarily used for strain field smoothing in triangular elements [16]. Due to its simple formulation, this method was extended to quadrilateral, polygonal and tetrahedral finite elements [7]. Application of this method lowers the stiffness of the model created by using tetrahedral elements, i.e. as shown in [7] the method possesses a close-to-exact stiffness of the continuous system.

3.1 New strain smoothing method for the 4-node tetrahedral element

In contrast to previously mentioned methods that construct a constant strain field in the created smoothing domains, in this chapter a tetrahedral element strain smoothing method [1], which forms a linear strain field inside the element by taking into account the constant strains of surrounding elements, will be presented. The other thing that makes this method very efficient is the fact that the smoothing domains are not created and that the linearity is achieved without introducing additional parameters and degrees of freedom. So, the domain discretization, interpolation function and degrees of freedom remain unchanged in comparison to the standard formulation of these elements. Based on the numerical examples analyzed in paper [1], it was shown that this method achieves better performances, even when compared to the edge based strain smoothing method, which is in literature known as the most efficient method.

A linear tetrahedral element is observed, k^{th} edge and the adjacent elements that share this edge with the observed element, Figure 2. Let us note that the configuration of the elements along the common edge of the tetrahedral element is not unambiguously defined and is closely related to the generated mesh of finite elements.

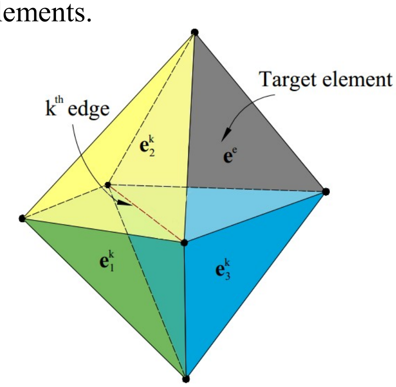

Figure 2. Position of the target element and adjacent elements that also contain the k^{th} edge

Along each edge of the tetrahedron the strains are smoothed, and for the observed elements a strain vector is defined [1]:

$$\hat{\mathbf{e}}^k = \frac{1}{V^e + \sum_{i=1}^{n_k} V_i^k}\left(V^e \mathbf{e}^e + \sum_{i=1}^{n_k} V_i^k \mathbf{e}_i^k\right) \qquad k = 1,2,3,4,5,6 \tag{6}$$

where n_k is the number of elements which share the common edge k with the observed element (for the observed configuration $n_k = 3$), $\mathbf{e}^e$ and $\mathbf{e}_i^k$ represent strain vectors of the observed element and the i^{th} element that contains the k^{th} edge, and V^e and V_i^k represent their volumes, respectively.

The main difference between smoothing method and standard finite element method is shown in the way of determining the elements' stiffness matrix, which in target elements is calculated using the smoothing strain (6). For generating the elements' stiffness matrix four integration points are considered, and their position in the natural coordinate system r,s,t is shown in Figure 3. Smoothed strains are directly assigned to the Gauss integration points, so the following equations are true [1]:

$$\begin{aligned} \mathbf{e}^a &= \frac{1}{5}\left(\hat{\mathbf{e}}^1 + \hat{\mathbf{e}}^2 + \hat{\mathbf{e}}^3 + \hat{\mathbf{e}} + \mathbf{e}^e\right) & \mathbf{e}^d &= \frac{1}{5}\left(\hat{\mathbf{e}}^3 + \hat{\mathbf{e}}^5 + \hat{\mathbf{e}}^6 + \hat{\mathbf{e}} + \mathbf{e}^e\right) \\ \mathbf{e}^b &= \frac{1}{5}\left(\hat{\mathbf{e}}^1 + \hat{\mathbf{e}}^4 + \hat{\mathbf{e}}^6 + \hat{\mathbf{e}} + \mathbf{e}^e\right) & \hat{\mathbf{e}} &= \frac{1}{6}\sum_{k=1}^{6}\hat{\mathbf{e}}^k \\ \mathbf{e}^c &= \frac{1}{5}\left(\hat{\mathbf{e}}^2 + \hat{\mathbf{e}}^4 + \hat{\mathbf{e}}^5 + \hat{\mathbf{e}} + \mathbf{e}^e\right) \end{aligned} \tag{7}$$

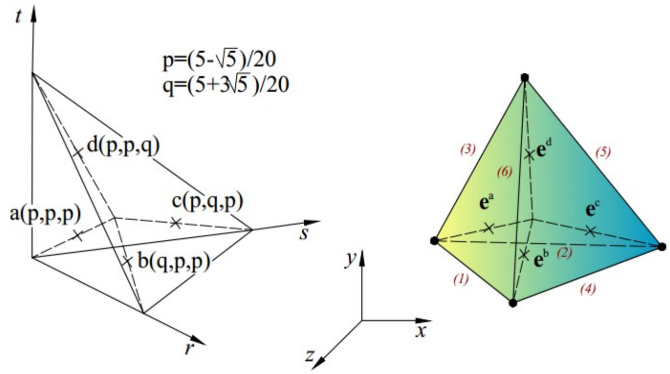

Figure 3. Position of Gauss integration points of a tetrahedral element with a smoothed strain field

We can notice that for the edge located on the boundary of the discretized domain, which belongs to only one element, strain field retains the standard form, i.e. $\hat{\mathbf{e}}^k = \mathbf{e}^e$. Also, it is important to note that for determining the stiffness matrix of the element with a smoothed strain field only the element's volume and corresponding matrices of derivatives interpolation functions, defined by equations (5), are necessary.

4. Numerical examples

This chapter presents the numerical analysis performed in order to check linear and tetrahedral elements with midside nodes, that are implemented in the software package PAK [17], and to compare their accuracy, i.e. convergence of solutions with those of the element with corrected strain field given in [1]. Stress-strain calculation was also performed using the software package Nastran, so the performance comparison of the elements implemented into these two software packages will be presented. Pre- and post-processing is done using the Femap environment [18].

4.1 Cook's skew beam problem

Cook's skew beam problem is given in the literature as an example of bending the variable cross-section tapered beam clamped on the left edge, while the right edge is subjected to the distributed shearing force of total value $\mathrm{P} = 1\,\mathrm{N}$. Dimensions and material properties of the skew beam corresponding to the plane stress conditions are shown in Figure 4a.

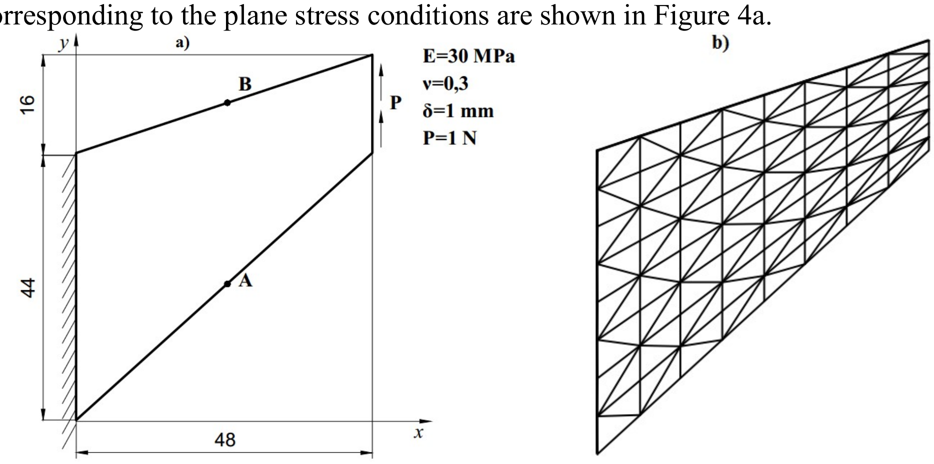

Figure 4. a) Bending of the variable cross-section skew beam b) Cook's skew beam problem finite element mesh for the case of $N = 8$ elements along edges

In order to adequately present the plane stress state in xy plane, half of the skew beam thickness is modeled with boundary condition $u_z = 0$ for the nodes of the beam's central plane. Displacement along the x-axis is prevented, i.e. $u_x = 0$ for the nodes in zy plane, as well as vertical displacement $u_y = 0$ for the central node of the FE model's central plane, i.e. node located in the central line, in order to accurately model the strain of the clamped cross-section.

This example is used for analysis of the convergence of the von Mises stress numerical solution at point A, for different mesh densities $N \times N\left(N = 2, 4, 8\right)$. Along the skew beam thickness two rows of elements were used. Figure 4b shows the Cook's skew beam problem finite element mesh.

Tables 1 and 2 provide an overview of the numerical results obtained by software packages PAK and Nastran compared to the reference solution given in [1]. The error of obtained results is calculated in percentages using the relation $\left|\sigma_{ref} - \sigma_f\right| / \sigma_{ref} \times 100$, where the indices ref and f refer to the reference and obtained numerical solution, respectively. Since there is continuity of displacements in finite element method implemented in PAK, and not of their derivatives, the stresses in the node A of the beam are calculated as the arithmetic mean of the stress value of all the elements that contain the observed node.

Stress [MPa]	PAK					
	Linear element		Parabolic element		Corrected element	
N	[MPa]	[%]	[MPa]	[%]	[MPa]	[%]
2	0.0727	69.34	0.1853	21.85	0.1209	48.99
4	0.1772	25.26	0.2297	3.12	0.2105	11.19
8	0.2112	10.92	0.2278	3.92	0.2290	3.40
Reference solution: 0.2371						

Table 1. Von Mises stress at point A obtained by software package PAK

Stress [MPa]	Nastran					
	Linear element		Parabolic element		Corrected element	
N	[MPa]	[%]	[MPa]	[%]	[MPa]	[%]
2	0.0727	69.34	0.2388	0.72	0.1209	48.99
4	0.1772	25.26	0.2352	0.8	0.2105	11.19
8	0.2112	10.92	0.2356	0.63	0.2290	3.40
Reference solution: 0.2371						

Table 2. Von Mises stress at point A obtained by software package Nastran

From the presented results, a significantly slower convergence of linear elements compared to parabolic elements can be noticed. Of course, by increasing the mesh density, linear elements converge to a reference solution. On the other hand, parabolic elements due to the linear strain and stress fields that are fully described by the bending problem, even in a case of very coarse mesh, have a significantly higher accuracy compared to linear elements.

By comparing the linear element implemented in software package PAK and the corrected element presented in the paper [1], a significantly faster convergence of the corrected element can be noticed.

For the same mesh density, the parabolic element achieves faster convergence compared to the corrected element. As the parabolic element uses greater number of degrees of freedom and higher-order interpolation functions, the results are completely justified.

Observing the results obtained by using PAK and Nastran for the case of linear elements, these two software packages completely agree. Deviation of the numerical results obtained in these two software packages occurs in the case of parabolic elements. Namely, the software package Nastran calculates nodal stress values by extrapolation of values from integration points, and therefore the extrapolated nodal values are higher. On the other hand, values obtained in the software package PAK represent the exact values obtained at the integration points.

4.2 Block subjected to distributed force

In this example, the block subjected to a distributed compression force that acts along the block's right half of the upper edge of the total value $\mathrm{P}=1\,\mathrm{N}$ is analyzed. Dimensions and material properties of the block are shown in Figure 5a.

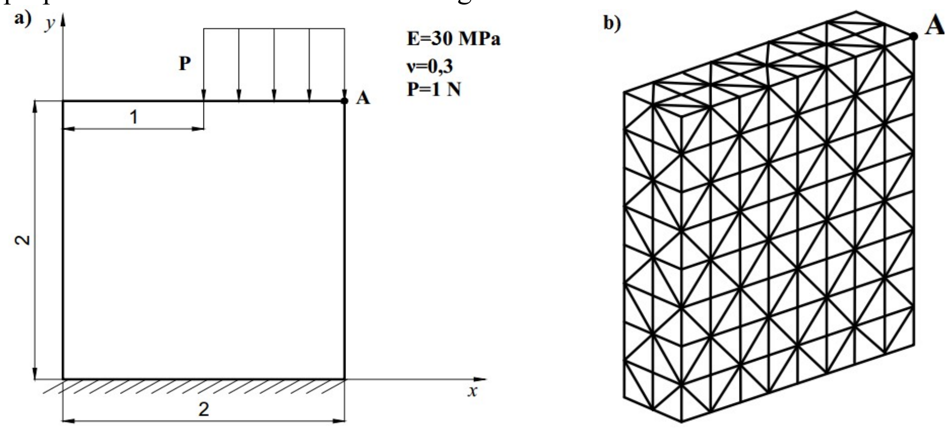

Figure 5. a) Bending of the block subjected to distributed force b) Finite element mesh of the block for the case of $N=8$ elements along edges

The bottom edge of the block is clamped, wherein for modeling of the plane stress state in xy plane, half of the block thickness is modeled with boundary condition $u_z=0$ for the nodes at the central plane of the block. In order to accurately model the strain of the clamped cross-section, displacement outside the zx plane is prevented, i.e. $u_y=0$ for the nodes of this cross section, as well as horizontal displacement $u_x=0$ for the central node located in the FE model's central line.

This example is used for analysis of the convergence of the numerical solution of vertical displacement at point A, for different mesh densities $N\times N\left(N=2, 4, 8\right)$, whereby the mesh of discretized model of the block is shown in Figure 5b. Along the block thickness two rows of elements were used.

Table 3 shows an overview of the numerical results of vertical displacement at point A obtained in PAK and Nastran using linear and tetrahedral elements with midside nodes, compared to the reference solution of the corrected element taken from [1]. The error of obtained results is calculated in percentages using the relation $\left|u_{ref}-u_f\right|/u_{ref}\times 100$, where the indices ref and f refer to the reference and obtained numerical solution, respectively.

Displacement [$\times 10^{-8}$ m]	PAK-Nastran					
	Linear element		Parabolic element		Corrected element	
N	[m]	[%]	[m]	[%]	[m]	[%]
2	-6.4882	17.21	-7.993	1.98	-8.1969	4.59
4	-7.4795	4.56	-7.9107	0.93	-7.8770	0.51
8	-7.7654	0.91	-7.8949	0.73	-7.8431	0.07
Reference solution: -7.8372						

Table 3. Vertical displacement at point A obtained by software packages PAK and Nastran

The error of linear element displacement in case of a very coarse mesh, $N = 2$, is as high as 17,21%, while the corrected element displacement has the error of only 4,59%. Therefore, by using the element with corrected strain field, a very good agreement of the obtained results with the reference ones is noticed with a relatively coarse mesh, which shows the convenience of using this element in cases of displacement field nonlinearity. Comparing the parabolic and corrected elements for the same mesh density, the element with midside nodes has a slightly better convergence in case of very coarse mesh. Results obtained by the both software packages completely agree.

5. Conclusion

In order to overcome the existing problems in the standard formulation of linear tetrahedral elements, primarily the very stiff behavior and the discontinuity of stress at the element boundaries, the strain smoothing method, which has considerably better elements performance for the same mesh density without introducing additional parameters and degrees of freedom, is presented in the paper. Using the constant strain of the surrounding elements, the strain field is smoothed along the common edge, and a linear corrected field is formed in elements. In this way, by retaining the same interpolation functions, we obtain the less stiff element and therefore achieve significantly faster convergence.

The abovementioned problems are shown by numerical analysis on concrete examples. Two plane stress problems were considered with the main goal to analyze the linear tetrahedral element performance already implemented in the software package PAK, and then to compare its convergence rate with the corrected element presented in [1]. As the smoothing method relates primarily to linear tetrahedral elements, the main goal was to compare performances of linear and corrected element. Since the problems arising in the linear elements' application are solved by the higher order approximation, i.e., by using tetrahedral elements with midside nodes, the performance of these elements was also analyzed.

Very stiff behavior of elements with a constant strain field is confirmed by bending problem analysis. By smoothing the strain field using the method presented in this paper, improvement of the characteristics of the 4-node tetrahedral element in bending was noticed. Also, given the good agreement of the obtained results with the reference solution in case of a very coarse mesh of parabolic tetrahedral elements, the fact of the suitability of applying these elements in nonlinear physical quantity field problems is confirmed.

As the goal in the finite element method is to formulate an element with good performances even in case of a very coarse mesh, it can be concluded that this problem is partially solved by the smoothing method presented in this paper. The obtained element is less stiff in bending problems and for the same mesh density achieves less stress discontinuity on the borders between elements due to the linear strain field. Considering the observed benefits of using the elements with corrected strain field in bending problems, a further research will primarily focus on implementation of the corrected element in the software package PAK and testing its performance on the concrete examples.

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