Proceedings

The 7th International Congress of Serbian Society of Mechanics

Sremski Karlovci, June 24-26, 2019

Edited by:

Mihailo Lazarević
Srboljub Simić
Damir Madjarević
Ivana Atanasovska
Andjelka Hedrih
Bojan Jeremić

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Ivana Atanasovska
Anđelka Hedrih
Bojan Jeremić

Foreward

The present volume contains plenary lectures, abstracts and papers of young authors competing for the „*Rastko Stojanović*'' award at the 7th International Congress of Serbian Society of Mechanics. The objectives of this Congress, to be held at in Sremski Karlovci during the period 24th -26th June 2019, are to review and discuss some of the latest trends in various fields of theoretical and applied mechanics as well as it aims to bring together the scientific communities of theoretical and applied mechanics in an effort to facilitate the exchange of ideas on topics of mutual interests, and to serve as a platform for establishing links between research groups with complementary activities.

We are happy to report that the number of accepted papers to be presented at the 7th Congress is 119. In addition, among them, 8 invited plenary lectures were presented by the authors from Italy,China, Greece, Croatia,Hungary and Serbia. Also, we have 4 invited speakers for Mini-Simposia. Accepted papers were grouped in the following sections General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies, Control and Robotics, and Interdisciplinary and Multidisciplinary Areas. Also, the three Minisymposia were organized with following topics: Nonlinear Dynamics, Bioengineering,Turbulence, Waves and diffusion in complex media and Biomechanics and Mathematical Biology.

The Editors would like to express their thanks to all participants of the 7th Congress of Mechanics.First, to the authors of the papers whose quality work is the essence of this event. Next, to of the papers whosethe distinguished invited lecturers who kindly accepted the invitation to come to Congress and helped make it success. We owe great thanks to the reviewers of the papers, to the members of the Scientific and Organizing Committee. Also, special thanks to the organizers of the Mini-symposia on Nonlinear Dynamics, Bioengineering,Turbulence, Waves and diffusion in complex media and Biomechanics and Mathematical Biology. The support of the members of Steering Committee of Serbian Society of Mechanics in organizing this event is also appreciated. Finally, special thanks are also due to those organizations which supported financially this Congress: Serbian Society of Mechanics, Ministry of Education, Science and Technological Development of the Republic of Serbia, Faculty of Mechanical Engineering, University of Belgrade, Belgrade and Serbian Academy of Sciences and Arts- Branch in Novi Sad, Provincial Secretariat for Higher Education and Scientific Research.

It is our great pleasure to welcome you with us at the 7th Congress International Congress of Serbian Society of Mechanics. We would like to wish all participants of this Congress a warm welcome to our country, our Serbian Society of Mechanics and Venue Congress place at *the Karlovci Gymnasium*, Sremski Karlovci, Serbia.

Sremski Karlovci, June, 2019

The Editors

Mihailo Lazarević, Srboljub Simić
Damir Madjarević,Ivana Atanasovska
Anđelka Hedrih,Bojan Jeremić

Technical program

SUNDAY, June 23, 2019

19:30 *Welcoming Coctail* (Hotel Prezident, Main Hall)

MONDAY, June 24, 2019

8:00 – 8:45 *Registration of participants* (*Main Hall, The Karlovci Gymnasium)*

Chairs: Srboljub Simić, Mihailo Lazarević

8:45 – 9:20 (*Congress hall, The Karlovci Gymnasium)*

- Radovan Kovačević, *Director of the Karlovci Gymnasium, Welcome address*
- Academician Teodor Atanacković*, Novi Sad branch of SASA, Welcome address*
- Aleksandar Stojkecić, *Historical notes - Sremski Karlovci*
- Prof.Mihailo P. Lazarević, *the President of Serbian Society of Mechanics ,Welcome address*

Plenary Lectures (*Congress Hall*)
Chairman: Katica *R.* (Stevanović) Hedrih

9:20 - 10:05 P-1 Walter Lacarbonara
ASYMPTOTIC RESPONSE OF SYSTEMS AND MATERIALS WITH HYSTERESIS

10:05 - 10:50 P-2 Zdravko Terze, et al.
LIE GROUP DYNAMICS OF MULTIBODY SYSTEM IN VORTICAL FLUID FLOW

10:50 - 11:15 *Coffee Break* (*Main Hall*)

11:15 - 13:00 Parallel Sessions

Session	G1	S1	M4	C1
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*
11:15	G1a	S1a	M4a	C1a
11:35	G1b	S1b	M4b	C1b
11:55	G1c	S1c	M4c	C1c
12:15	G1d	S1d	M4d	C1d
12:35	G1e	S1e	M4e	C1e
12:55		S1f	M4f	

13:10 - 14:40 *Lunch* *(Restaurant Bermet*)

Plenary Lecture (*Congress hall*)

Chairman: Srboljub Simić

14:40 - 15:25 P-3 HongGuang Sun, Yong Zhang
ANOMALOUS DIFFUSION: MODELING AND APPLICATION

15:30 -17:30 *Social program* (*excursion to Monasteries at Fruska Gora)*

17:30 - 19:10 Parallel Sessions

Session	G2	S2	M4	M1	M1
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*	
17:30	G2a	S2a	M4g	M1p*	17:30
17:50	G2b	S2b	M4h	M1a	18:00
18:10	G2c	S2c	M4i	M1b	18:20
18:30	G2d	S2d	M4j	M1c	18:40
18:50	G2e	S2e	M4k	M1d	19:00
19:10		S2f	M4l	M1e	19:20

TUESDAY, June 25, 2019

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Chairman: Zdravko Terze

9:00 - 9:45 P-4 Peter Van
CONTINUUM MECHANICS AND NONEQUILIBRIUM THERMODYNAMICS

9:45 - 10:30 P-6 Dušan. Zorica
HEREDITARINESS AND NON-LOCALITY IN WAVE PROPAGATION MODELLING

10:30 - 10:50 *Coffee Break* (*Main Hall*)

10:50 - 13:00 Parallel Sessions

Session	G3	M1	M2	M3	M3
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*	
10:50	G3a	M1f	M2a	M3a	10:50
11:10	G3b	M1g	M2b	M3b	11:20
11:30	G3c	M1h	M2c	M3c	11:40
11:50		M1i	M2d	M3d	12:00
12:10		M1j	M2e	M3e	12:20
12:30		M1k	M2f		

12:50 - 14:15 *Lunch* (*Restaurant Bermet*)

Plenary Lecture (*Congress hall*)
Chairman: Dušan Zorica

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15:20 - 17:20 Parallel Sessions

Session	S3	M1	M2	M3
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*
15:20	S3a	M1l	M2g	M3f
15:40	S3b	M1m	M2h	M3g
16:00	S3c	M1n	M2i	M3h
16:20	S3d	M1o	M2j	M3i
16:40	S3e	M1r	M2k	M3j
17:00	S3f			

17:00 - 18:00 Round table: HARMONIZATION AND MODERNIZATION OF THE CURRICULUM IN ENGINEERING MECHANICS
17:00-17:15 Katica R. (Stevanović) *Hedrih, Academisian LJUBOMIR KLERIĆ (June 29, 1844- January 21, 1910); Dedicated to Jubilee 175 years from birthday*

18:00 - 19:00 General Assembly Meeting of Serbian Society of Mechanics (*Congress Hall*)

19:00-19:30 W*ine tasting (* winery "Bajilo")

20:00 - 22:30 *Gala Dinner* (*Restaurant Pasent*)

WEDNESDAY, June 26, 2019

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Chairman: HongGuang Sun

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10:30 - 12:10 Parallel Sessions

Session	I1	S4	M2	M5	M5
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*	
10:30	I1a	S4a	M2l	M5a	10:30
10:50	I1b	S4b	M2m	M5b	11:00
11:10	I1c	S4c	M2n	M5c	11:20
11:30	I1d	S4d	M2o		
11:50	I1e	S4e	M2p		

12:10 - 12:35 *Coffee Break* (*Main Hall*)

12:35 - 12:55 B. Popkonstatinović,N.Mladenović, M.Stojićević, *Faculty of Mech. Eng., Belgrade , Presentation book* ESCAPEMENT DYNAMICS AND HOROLOGICAL ERRORS, *(Congress Hall*)

13:00 – 15:00 Parallel Sessions

Session	F1	S5	M2	M5
Hall	*Classroom 1*	*Classroom 2*	*Classroom 24*	*Classroom 25*
13:00	F1a	S5a	M2r	M5d
13:20	F1b	S5b	M2s	M5e
13:40	F1c	S5c	M2t	M5f
14:00			M2u	
14:20			M2v	
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G2c: Sreten Mastilović
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G2d: Sreten Mastilović
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I1e: Nemanja D. Zorić, Radoslav D. Radulović, Vladimir M. Jazarević
DEVELOPMENT OF SMALL ELECTRIC FIXED-WING vtol uav

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Organizers: *Katica R. (Stevanović) Hedrih, Ivana Atanasovska*
Mathematical Institute SASA, Belgrade

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PENALTY METHOD APPLIED TO STRUCTURAL STRENGTH ASSESSMENT OF THE AXIAL BALL JOINT

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Abstract:

In this paper, the application of the penalty method to solve 3D contact problem is presented. Standard procedures are used for the detection of contact and the application of displacement constraints. The friction forces are based on Coulomb's law. The implemented model includes the linearization of virtual work which enables high robustness of finite element techniques. The presented method is used for the procedure of the structural strength and fatigue assessment of the spring seat in the axial ball joint. The Finite Element Method software is used to analyze the stress state under the prescribed extreme loading conditions. The target number of loading cycles is set to 100.000, and the producer prescribes the amplitude of compressive and tension loading forces. The material parameters are adopted according to literature, and the fatigue resistance is compared to the computed stress values. It is concluded that the observed axial ball joint satisfies the targeted structural and fatigue strength.

Key words: penalty method, contact problems, finite element method, fatigue strength, ball joint

1. Introduction

Contact between the bodies often occurs in nature and engineering praxis: an interaction between soil and foundations in civil engineering, a general bearing problem, a bolt and a screw joint, a prosthetics in biomedical engineering, pneumatic tires with better handling characteristics in automotive engineering, but also the collision of cars, metal forming etc. [1].

The practical application of finite element (FE) contact solutions requires a high level of experience. This paper aims to provide a procedure based on the penalty method for structural strength assessment of specific contact problem with friction. Contact can occur between: a deformable body and a rigid obstacle, between two deformable bodies or as a self-contact. In this paper, contact between two deformable bodies is considered, and the numerical results are used for fatigue strength assessment [2].

2. Formulation of the multi-body frictional contact problem

Considering two bodies $B^{(1)}$ and $B^{(2)}$ in Figure 1, one can notice pairs of contact surfaces involved in the problem as slave $\Gamma_C^{(1)}$ and master $\Gamma_C^{(2)}$ surfaces. The condition is that any slave particle cannot penetrate the master surface [1].

The projection point of the current position of the slave node $\mathbf{x}^k$ onto the current position of the master surface $\Gamma_C^{(2)}$ is $\bar{\mathbf{x}}$, defined as [1]:

$$\frac{\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)}{\left\|\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)\right\|} \cdot \bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2) = 0\,, \tag{1}$$

where $\bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2)$ are the tangent covariant base vectors at the point $\bar{\mathbf{x}}$. The penetration g_N for slave node k is defined as the distance between current positions of this node to the master surface $\Gamma_C^{(2)}$ [1]:

$$g_N = (\mathbf{x}^k - \bar{\mathbf{x}}) \cdot \bar{\mathbf{n}}\,, \tag{2}$$

where $\bar{\mathbf{n}}$ is the normal to the master face $\Gamma_C^{(2)}$ at point $\bar{\mathbf{x}}$.

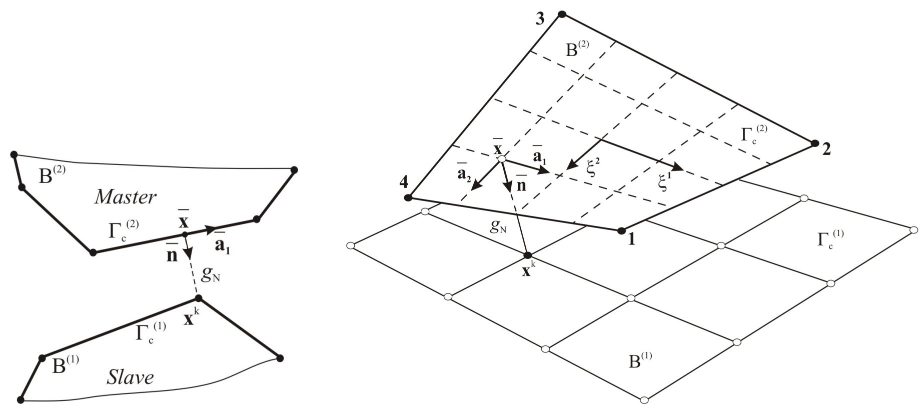

Fig. 1. The geometry of the 2D node-to-segment and the 3D node-to-surface contact element [1]

This gap gives the non-penetration conditions as follows [1]:

$$\begin{cases} g_N = 0 & \text{perfect contact} \\ g_N > 0 & \text{no contact}\,, \\ g_N < 0 & \text{penetration} \end{cases} \tag{3}$$

If the friction is modeled [3], a tangential relative displacement must be introduced. The sliding path of the node $\mathbf{x}^k$ over the contact surface $\Gamma_C^{(2)}$ is described by total tangential relative displacement as:

$$g_T = \int_{t_0}^{t} \|\dot{\mathbf{g}}_T\|\,dt = \int_{t_0}^{t} \left\|\dot{\bar{\xi}}^\alpha \bar{\mathbf{a}}_\alpha\right\| dt = \int_{t_0}^{t} \sqrt{\dot{\bar{\xi}}^\alpha \dot{\bar{\xi}}^\beta a_{\alpha\beta}}\,dt\,, \tag{4}$$

in the time interval from t_0 to t. The time derivatives of the parameter $\bar{\xi}^\alpha$ in eq. (4) can be computed from the eq. (1), which gives the relative tangential velocity at the contact point as [1]:

$$\dot{\mathbf{g}}_T = \dot{\bar{\xi}}^\alpha \bar{\mathbf{a}}_\alpha = \dot{g}_{T\alpha} \bar{\mathbf{a}}^\alpha\,, \tag{5}$$

A contact stress vector $\bar{\mathbf{t}}$ of the current contact interface $\Gamma_C^{(2)}$ can be split into a normal and tangential part:

$$\bar{\mathbf{t}} = \bar{\mathbf{t}}_N + \bar{\mathbf{t}}_T = \bar{t}_N \bar{\mathbf{n}} + \bar{t}_{T\alpha} \bar{\mathbf{a}}^\alpha\,, \tag{6}$$

where $\bar{\mathbf{a}}^\alpha$ is the contravariant base vector. The stress acts on both surfaces, so according to the action-reaction principle: $\bar{\mathbf{t}}(\bar{\xi}^1,\bar{\xi}^2) = -\mathbf{t}$ in the contact point $\bar{\mathbf{x}}$.

In the tangential direction, there is a difference between the stick and slip. As long as there is no sliding between two bodies, the tangential relative velocity and displacement are equal to zero. This state is called the stick case [1]:

$$\dot{\mathbf{g}}_T = \mathbf{0} \iff \mathbf{g}_T = \mathbf{0} . \tag{7}$$

Relative displacement between two bodies occurs if the loading is large enough to overcome the static friction resistance. Therefore, the relative sliding velocity and the sliding displacement are in the opposite direction concerning the friction force. The tangential stress vector is [1]:

$$t_{T\alpha}^{sl} = -\mu \left|\mathbf{t}_N\right| \frac{\dot{g}_{T\alpha}^{sl}}{\left\|\dot{\mathbf{g}}_T^{sl}\right\|} , \tag{8}$$

where μ is the friction coefficient.

An indicator function which determines whether stick or slip takes place is given concerning the Coulomb's model for frictional interface law [1-3]:

$$f = \begin{cases} \left\|\mathbf{t}_T\right\| \text{-} \mu \left|t_N\right| \le 0 & \rightarrow \text{Stick} \\ \left\|\mathbf{t}_T\right\| \text{-} \mu \left|t_N\right| > 0 & \rightarrow \text{Slip} \end{cases} . \tag{9}$$

Use of the penalty method for normal stress, a constitutive equation can be formulated as:

$$t_N = \varepsilon_N g_N , \tag{10}$$

where ε_N is the normal penalty parameter.

For the stick case, a simple linear constitutive model can be used to describe the tangential stress:

$$t_{T\alpha}^{stick} = \varepsilon_T g_{T\alpha} , \tag{11}$$

where ε_T is the tangential penalty parameter. For the slip case, the tangential stress is given by the constitutive law for frictional sliding in eq. (8). A backward Euler integration scheme and return mapping strategy are employed to integrate the friction eq. (9). If the stick is assumed, the trial values of the tangential contact pressure vector $t_{T\alpha}$, and the indicator function f at load step $n+1$ can be expressed in terms of their values at load step n as follows [1]:

$$t_{T\alpha\ n+1}^{trial} = t_{T\alpha\ n} + \varepsilon_T \Delta g_{T\alpha\ n+1} = t_{T\alpha\ n} + \varepsilon_T \bar{a}_{\alpha\beta} \Delta \xi_{n+1}^{\beta} , \tag{12}$$

$$f_{Tn+1}^{trial} = \left\|\mathbf{t}_{Tn+1}^{trial}\right\| - \mu \left|t_{Nn+1}\right| . \tag{13}$$

The return mapping is completed by:

$$t_{T\alpha\ n+1} = \begin{cases} t_{T\alpha\ n+1}^{trial} & \text{if } f \le 0 \\ \mu \left|t_{Nn+1}\right| n_{T\alpha\ n+1}^{trial} & \text{if } f > 0 \end{cases} , \tag{14}$$

with

$$n_{T\alpha\ n+1}^{trial} = \frac{t_{T\alpha\ n+1}^{trial}}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|} . \tag{15}$$

3. Finite element formulation

The virtual work is formulated for one slave node k as [4,5]:

$$\delta A_c = \mathbf{F}_N \delta \mathbf{g}_N + \mathbf{F}_T \delta \mathbf{g}_T = \mathbf{t}_N A_k \delta \mathbf{g}_N + \mathbf{t}_T A_k \delta \mathbf{g}_T = \mathbf{t}_N A_k \delta \mathbf{g}_N + t_{T\alpha} A_k \delta \bar{\xi}^{\alpha} , \tag{16}$$

where $F_N = t_N A_k$ is the normal force, $F_{T\alpha} = t_{T\alpha} A_k$ is the tangential force, A_k is the surface area of the contact element, δg_N is the virtual normal displacement (gap), and $\delta g_{T\alpha}$ is the virtual tangential displacements. The matrix formulation of the variations of the gap and the tangential displacements is given as [4,5]:

$$\delta g_N = \delta \mathbf{u}_c^T \cdot \mathbf{N} , \qquad \delta \bar{\xi}^{\alpha} = \delta \mathbf{u}_c^T \cdot \mathbf{D}^{\alpha} , \tag{17}$$

where:

$$\delta\mathbf{u}_c^T = \left\{\delta\mathbf{u}^k \quad \delta\mathbf{u}_1 \quad \delta\mathbf{u}_2 \quad \delta\mathbf{u}_3 \quad \delta\mathbf{u}_4\right\}, \tag{18}$$

and

$$\mathbf{N} = \begin{Bmatrix} \bar{\mathbf{n}} \\ -H_1\bar{\mathbf{n}} \\ -H_2\bar{\mathbf{n}} \\ -H_3\bar{\mathbf{n}} \\ -H_4\bar{\mathbf{n}} \end{Bmatrix}, \qquad \mathbf{T}_\beta = \begin{Bmatrix} \bar{\mathbf{a}}_\beta \\ -H_1\bar{\mathbf{a}}_\beta \\ -H_2\bar{\mathbf{a}}_\beta \\ -H_3\bar{\mathbf{a}}_\beta \\ -H_4\bar{\mathbf{a}}_\beta \end{Bmatrix}, \qquad \mathbf{D}^\alpha = \bar{a}^{\alpha\beta}\mathbf{T}_\beta . \tag{19}$$

```
LOOP over all contact segments or surfaces k
 (check for contact) IF g_N ≤ 0 THEN
      (the first iteration)      IF i=1 THEN
                                   set all active nodes to stick state,
                                   t_{Tn+1}, compute matrix K_T^{stick}
                                 ELSE
                                   Compute trial state: t^{trial}_{Tα n+1} and f^{trial}_{Tn+1}
                                   IF  f^{trial}_{Tn+1} ≤ 0 THEN
                                     t_{Tα n+1} = t^{trial}_{Tα n+1} , compute matrix K_T^{stick}
                                     GO TO (a)
                                  ELSE
                                     t_{Tα n+1} = μ|t_{Nn+1}| n^{trial}_{Tα n+1} , compute matrix K_T^{slip}
                                  ENDIF
                                 ENDIF
                         ENDIF
(a)   END LOOP
```

Table 1. Frictional contact algorithm using penalty method [7]

The contact forces F_N and $F_{T\alpha}$ in $\mathbf{F}_c = \left[F_N\mathbf{N} + F_{T\alpha}\mathbf{D}^\alpha\right]$ can be obtained by multiplying the constitutive laws (10), (11), and (14) by the surface area of the contact element A_k. Finally, we obtain the global nonlinear FE equation extended by contact forces [6]:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}(t) - \mathbf{F}_c , \tag{20}$$

where the mass and the stiffness matrix are:

$$\mathbf{M} = \int_V \rho\mathbf{H}^T\mathbf{H}dV , \qquad \mathbf{K} = \int_V \mathbf{B}^T\mathbf{C}\mathbf{B}dV , \tag{21}$$

the vector $\mathbf{F}(t)$ corresponds to the external forces, $\mathbf{C}$ is the constitutive matrix, $\mathbf{H}$ contains the shape functions, and $\mathbf{B}$ is the strain-displacement matrix.

The nonlinear equilibrium equation (20) needs to be solved with inequality constraints (3) as a result of contact. In order to apply Newton's method to the system of equilibrium equation (20), a linearization of the contact contributions is necessary [6].

4. Structural Strength Assessment of the Axial Ball Joint

The strength and fatigue assessment of the ball joint spring seat includes an application of the penalty method theory. For this purpose, a 3D FE model was created according to manufacturer's documentation. The ball joint consists of a ball stud and a ball, a spring seat and a ball seat, Figure 2.

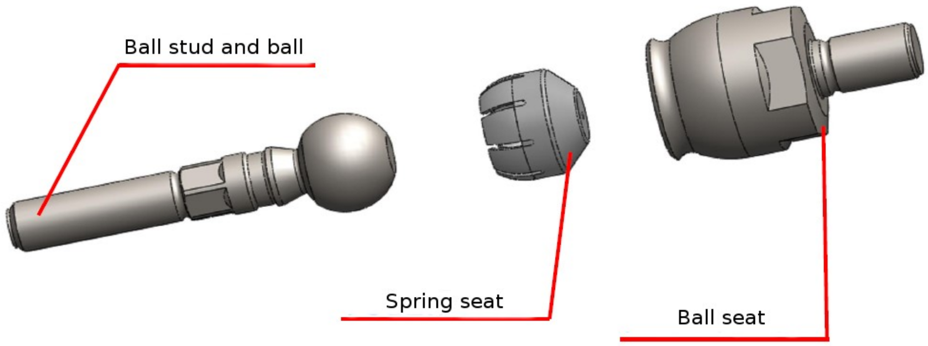

Fig. 2. The 3D FE geometry of the axial ball joint

The required target fatigue of spring seat is set to 100.000 cycles under the maximal loading force of $F^{t}_{MAX} = 4557 daN$ in tension and $F^{c}_{MAX} = -2278 daN$ in compression.

Due to axial symmetry of the model and loads, a cross-section is modeled. The FE mesh is shown in Figure 3. The model is generated using 8-node 3D solid elements with 5190 nodes. The number of elements per property is given in Table 2.

Part	Material	Number of elements
Ball Stud and Ball	Steel	1570
Spring seat	Delrin 100	200
Ball seat	Steel	648

Table 2 Number of elements per property

Three properties and the corresponding three materials are used. The material used for the ball stud and the ball is C.4181 (41CrS4), as well as for the ball seat. The yield stress of the used steel is 560MPa.

The spring seat is made of the plastic material Delrin 100. The adopted material parameters are taken from the literature [8]. The materials properties are given in the Table 3.

Part	Material type	ID	Young modulus, E [N/mm^2]	Poisson coefficient, ν	Yield stress, σ_e[MPa]	Ref. Temp., T[ºC]	Expansion coefficient, α[1/ºC]
Ball stud and ball	steel	1	2.1x10^5	0.3	560	0	-
Elastic seat	plastic	2	2.8x10^3	0.35	69	8	12.2 10^{-5}
Ball seat	steel	3	2.1x10^5	0.3	560	0	-

Table 3 The material properties of the joint ball parts

The boundary conditions include axial symmetry of the model. The nodes on the Z-axis are constrained in the X direction, and the nodes on the X-axis are constrained in the Z direction. Also, all nodes are permanently constrained in the normal Y direction to the cross-section plane.

During analysis, the contact type constraints are used between the surfaces of different parts of the ball joint as shown in Figure 4. The friction coefficient used between surfaces is set to $\mu = 0.4$.

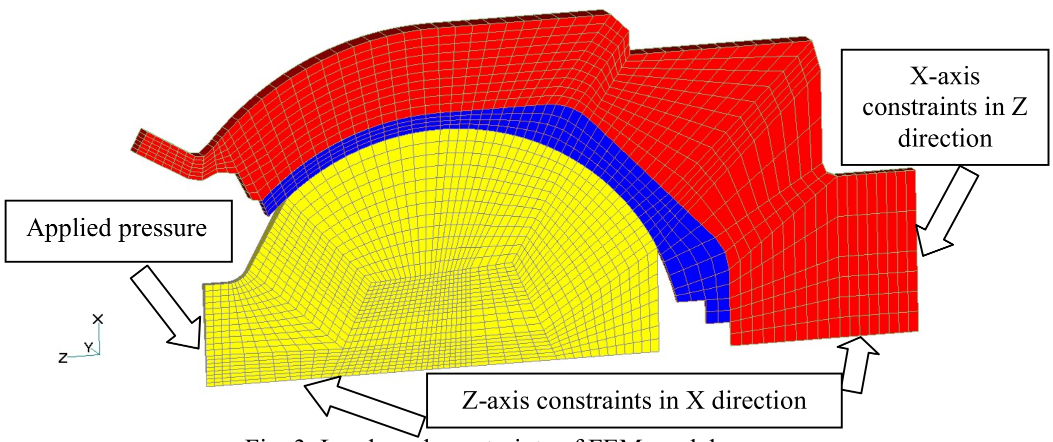

Fig. 3. Loads and constraints of FEM model

The pressure was applied at face normal to the cross section of the ball stud. The pressure value is $200.9 MPa$ in the tension case, and $-100.45 MPa$ in the compression case.

The manufacturer of the ball joint defined the two loading cases. The torsion balance moment, between the spring seat and the ball, is $40 Nm$, and the calculated initial stress field in the spring seat is at least $1.7 MPa$. The approximated surface area of contact between elastic seat and ball is calculated as $A = 3889.3 mm^2$. To satisfy the torsion moment, the necessary tangential force is $2666 N$, so the radial force between the ball and the elastic seat is calculated as $F_R = 6665 N$. Dividing by the contact surface area, the initial stress field in the spring seat approximately is $\sigma = 1.7 MPa$. In this case, the thermal strains are used to simulate the effect of residual stress after the mounting process.

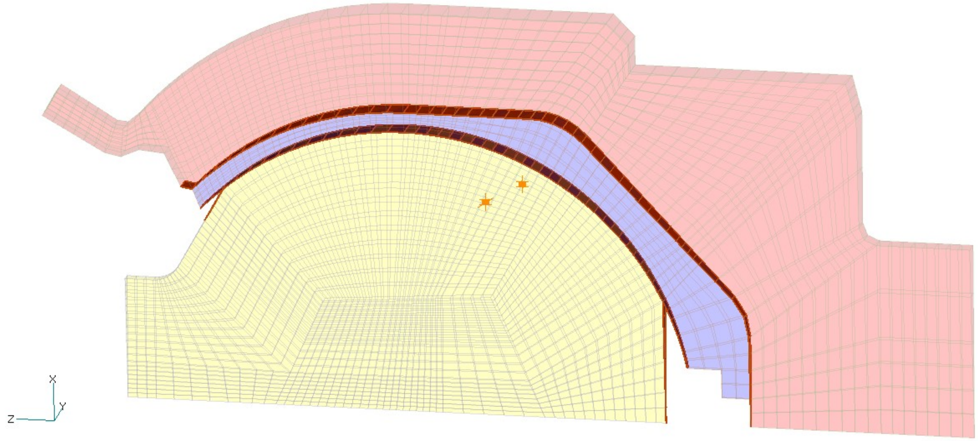

Fig. 4. Contact surfaces between the parts of the ball joint

In the first loading case, the force is defined as $F^c_{MAX} = -2278\ kN$. The force $F^c_{MAX} = -2278\ kN$ is applied as pressure $P^c_{MAX} = -100.45 MPa$ to the face shown in Figure 3 in the Z direction. For the second load case, the force is defined as $F^t_{MAX} = 4557\ kN$. The force

$F^t_{MAX} = 4557\ kN$ is applied as pressure $P^t_{MAX} = 200.9 MPa$ to the face shown in Figure 3 in the Z direction.

4.2 Proof of fatigue structural strength

The fatigue strength assessment was performed by the norm of amplitudes of stress components (the difference between stress tensors) from the two extreme load cases, in a fixed Cartesian coordinate system. The difference between stress tensor due to tension and stress tensor due to compression is defined as:

$$\Delta\boldsymbol{\sigma} = \boldsymbol{\sigma}^{ten} - \boldsymbol{\sigma}^{comp}, \qquad (22)$$

where $\boldsymbol{\sigma}^{ten}$ is stress tensor due to tension and $\boldsymbol{\sigma}^{comp}$ is stress tensor due to compression.

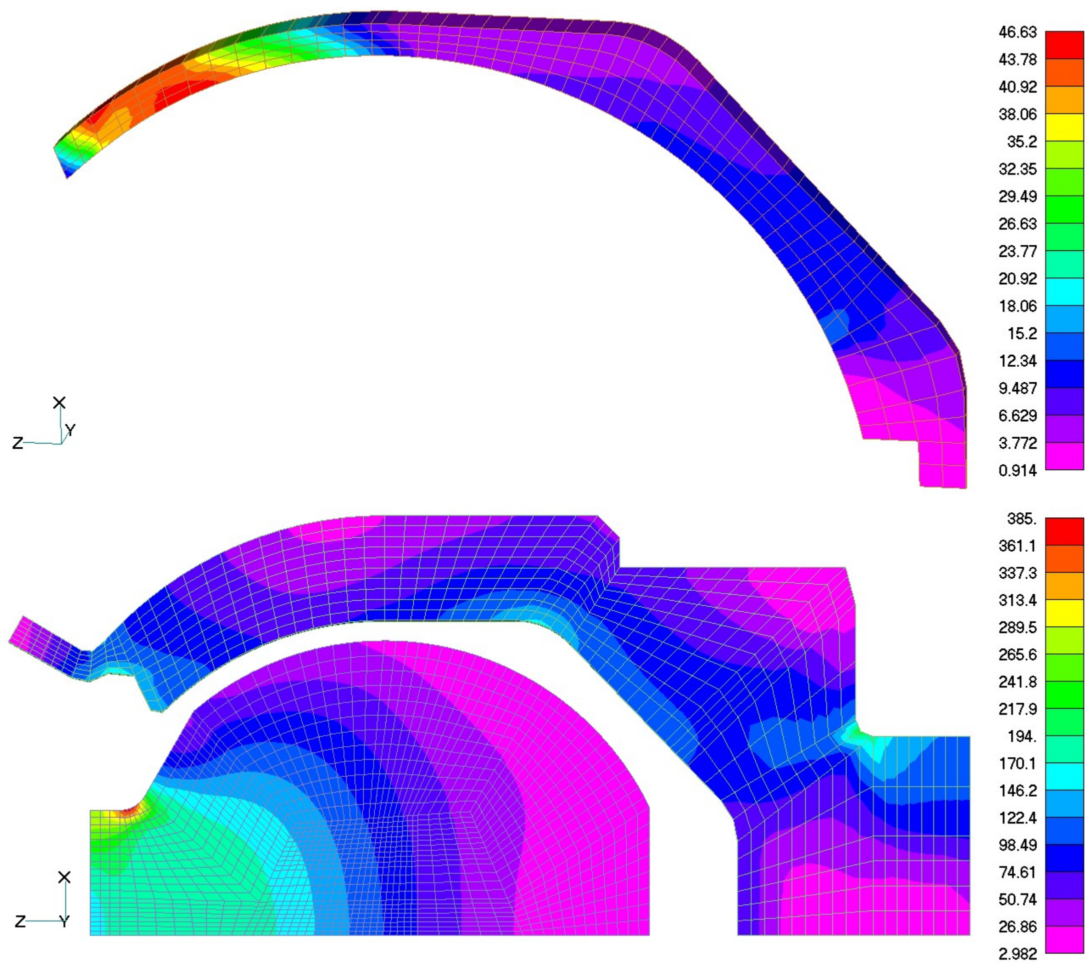

Fig. 5. Difference of effective stress fields $\Delta\sigma_{eqv}$ for fatigue strength assessment

The difference of effective stress fields is used for defining fatigue strength of the ball joint is:

$$\Delta\sigma_{eqv} = \sqrt{\frac{1}{2}\left[(\Delta\sigma_{xx}-\Delta\sigma_{yy})^2+(\Delta\sigma_{yy}-\Delta\sigma_{zz})^2+(\Delta\sigma_{zz}-\Delta\sigma_{xx})^2\right]+3(\Delta\sigma_{xy}^2+\Delta\sigma_{yz}^2+\Delta\sigma_{zx}^2)}\ . \qquad (23)$$

In Figure 5, the difference of effective stress fields is shown, and it will be used for fatigue strength assessment. According to the literature [8], for 100.000 cycles and uniaxial loading at room temperature, the difference of effective stress for Delrin 500 has fatigue resistance of $51.5MPa$. The maximum calculated fatigue strength is $46.63MPa$ so it can be concluded that the fatigue strength is achieved.

5. Conclusion

Many engineering problems which need optimization or strength analysis include contact between the bodies. The penalty method is one of the robust solutions for the analysis of contact problems. The implementation in FEM software gives the possibility to simulate the behavior of systems of rigid and deformable bodies. The theory and the algorithm for the application of such procedure on real problems are demonstrated for the case of an axial ball joint which consists of a ball with ball stud, ball seat, and the spring seat. The requirement was the assessment of fatigue strength for 100.000 cycles for spring seat made of plastic material Delrin 100. The loading cases are defined as extreme values of applied force for compressive and tensile stress state. The difference between the effective stress vectors is compared with the literature data. The conclusion is that the proposed structure satisfies the required loading conditions.

Acknowledgment

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